# DEMAND FOR RISKY ASSETS

- BASIC CONCEPT ABOUT A RANDOM VARIABLE.

A RANDOM VARIABLE (R.V.) $w$ TAKES VALUE

$w_1, w_2, w_3, \ldots, w_S$ W/ PROBABILITIES $\pi_1, \pi_2, \pi_3, \ldots, \pi_S$.

NOTE $\pi_1 + \pi_2 + \pi_3 + \ldots + \pi_S = 1$

- THE MEAN (EXPECTED VALUE) OF THE DISTRIBUTION IS AVERAGE VALUE OF THE R.V. :

$$E[w] = \mu_w = \sum_{s=1}^{S} \pi_s \cdot w_s$$

- THE DISTRIBUTION'S VARIANCE IS THE R.V.'S AVERAGE SQUARED DEVIATION FROM THE MEAN;

$$VAR[w] = \sigma_w^2 = \sum_{s=1}^{S} (w_s - \mu_w)^2 \cdot \pi_s$$

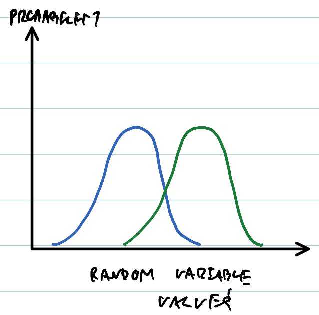
VARIANCE MEASURES THE R.V.'S VARIATION (SPREAD)

$$S.D. = \sigma_w = \sqrt{\sum_{s=1}^{S} (w_s - \mu_w)^2 \cdot \pi_s}$$

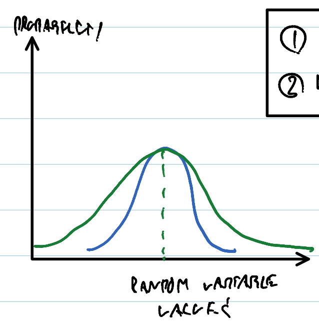
S.D CAN BE USED TO MEASURE THE R.V.'S VARIABILITY AS WELL.

NOTE: A DISTRIBUTION IS DESCRIBED BY

| |
|---|
| ① MEAN = CENTRAL TENDENCY |
| ② VARIANCE = SPREAD |




TWO DISTRIBUTIONS W/ THE SAME VARIANCE AND DIFFERENT MEANS




TWO DISTRIBUTION W/ THE SAME MEANS BUT DIFFERENT VARIANCES.

# PREFERENCE OVER RISKY ASSETS

**FACT #1** HIGHER "MEAN RETURN" IS PREFERRED
(OR EXPECTED RETURN)

SO, MEAN RETURN IS A GOOD : MORE IS BETTER,

**FACT #2** LESS VARIATION IN RETURN IS PREFERRED.
(LESS RISK)

SO, RISK IS A BAD : LESS IS PREFERRED TO MORE.

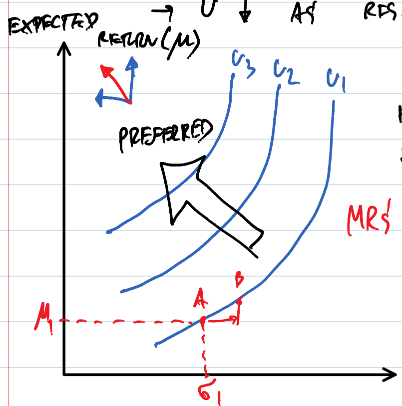
FACT#3 PREFERENCES ARE REPRESENTED BY A UTILITY FUNCTION:

$U(\mu, \sigma)$

$\mu$: EXPECTED RETURN, $\sigma$: RISK OF THE PORTFOLIO

→ $U \uparrow$ AS EXPECTED RETURN $\mu \uparrow$

→ $U \downarrow$ AS RISK $\sigma \uparrow$.




HIGHER $\mu$ IS A GOOD

HIGHER $\sigma$ IS A BAD,

$$MRS = \frac{d\mu}{d\sigma} = \frac{\partial U/\partial \sigma}{\partial U/\partial \mu} \quad \left[ = \frac{dU/d\sigma}{dU/d\mu} \right]$$

MU OF RISK

MU OF EXPECTED RETURN

PROOF HOW IS THE MRS COMPUTED?

$$U = f(\mu, \sigma)$$

$$dU = \frac{dU}{d\mu} d\mu + \frac{dU}{d\sigma} d\sigma = 0$$

$$\Rightarrow \frac{\partial U}{d\mu} d\mu = - \frac{dU}{d\sigma} d\sigma$$

$$\frac{d\mu}{d\sigma} = - \frac{\frac{dU}{d\sigma}}{\frac{dU}{d\mu}} \quad \begin{matrix} MU_\sigma \\ MU_\mu \end{matrix}$$

# BUDGET CONSTRAINTS FOR RISKY ASSETS

CONSIDER ① TWO ASSETS

- RISK-FREE ASSETS
- STOCK

② RISK-FREE ASSET'S RETURN = $r_f$.

RISKY STOCK'S RATE-OF-RETURN = $m_s$ IF STATE $s$ OCCURS, W/ PROB = $\pi_s$

SO, RISKY STOCK'S MEAN RATE-OF-RETURN IS

$$r_m = \sum_{s=1}^{S} m_s \pi_s .$$

A BUNDLE CONTAINING SOME OF THE RISKY STOCK AND SOME OF RISK-FREE ASSET IS "A PORTFOLIO"

SUPPOSE $x$ IS A FRACTION OF WEALTH USED TO BUY THE RISKY STOCK

$1-x$ IS A FRACTION OF WEALTH USED TO BUY NON-RISKY ASSET.

WITH A GIVEN $x$, THE AVERAGE RATE-OF-RETURN OF THE PORTFOLIO =

$$\boxed{r_x = x \cdot r_m + (1-x) r_f.} \quad \text{—— I}$$

IF $x = 0$, $r_x = r_f$ (WHEN PUTTING ALL MONEY INTO RISK-FREE ASSET)

IF $x = 1$, $r_x = r_m$ (WHEN PUTTING ALL MONEY INTO RISKY ASSET)

NOTICE THAT THE PORTFOLIO'S EXPECTED RATE-OF-RETURN RISES AS $x$ RISES. ( = MORE STOCK IN THE PORTFOLIO)

$$\sigma_x^2 = \sum_{s=1}^{S} \left( [x m_s + (1-x) r_f] - r_x \right)^2 \cdot \pi_s$$

$R_x = x r_m + (1-x) r_f$

$$\sigma_x^2 = \sum_{s=1}^{S} \left( x m_s + \cancel{(1-x) r_f} - x r_m - \cancel{(1-x) r_f} \right)^2 \cdot \pi_s$$

$$= \sum_{s=1}^{S} \left( x m_s - x r_m \right)^2 \cdot \pi_s$$

$$= x^2 \sum_{s=1}^{S} (m_s - r_m)^2 \pi_s$$

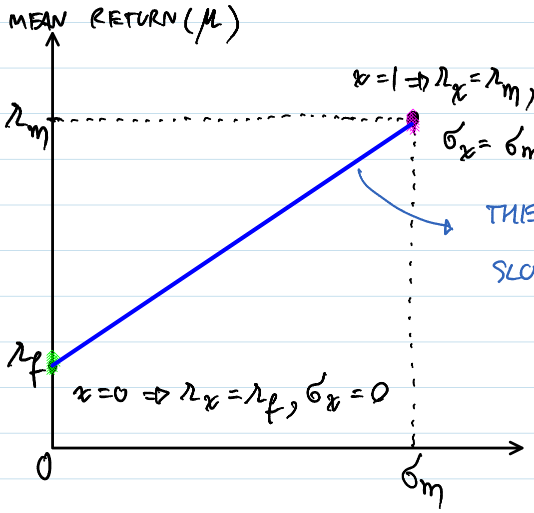
$$\sigma_x^2 = x^2 \cdot \sigma_m^2$$

| | FRACTION | $\mu = E(w)$ | RISK |
|---|---|---|---|
| RISKY | $x$ | $r_m$ | $\sigma_m$ |
| RISK-FREE | $1-x$ | $r_f$ | $\sigma_f = 0$ |

$$\boxed{S.D. = \sigma_x = x \cdot \sigma_m} \quad \text{—— II}$$

- IF $x = 0$, $\sigma_x = 0$ (THE PORTFOLIO HAS 0 RISK)
- IF $x = 1$, $\sigma_x = \sigma_m$ (THE RISKINESS OF THE PORTFOLIO IS DETERMINED BY RISKINESS (VARIABILITY) OF THE RATE-OF-RETURN ON STOCK.

NOTE THAT RISK RISES WITH $x$ ( = MORE STOCK IN THE PORTFOLIO)




$x=1 \Rightarrow r_x = r_m,\ \sigma_x = \sigma_m \qquad r_x = x \cdot r_m + (1-x)\cdot r_f$ — (I)

$\sigma_x = x \cdot \sigma_m$ — (II)

THE BUDGET LINE

SLOPE? $\Rightarrow$ $r_x = x r_m + (1-x) r_f$

$= r_f + x(r_m - r_f)$

$r_x = r_f + \dfrac{\sigma_x}{\sigma_m}(r_m - r_f)$

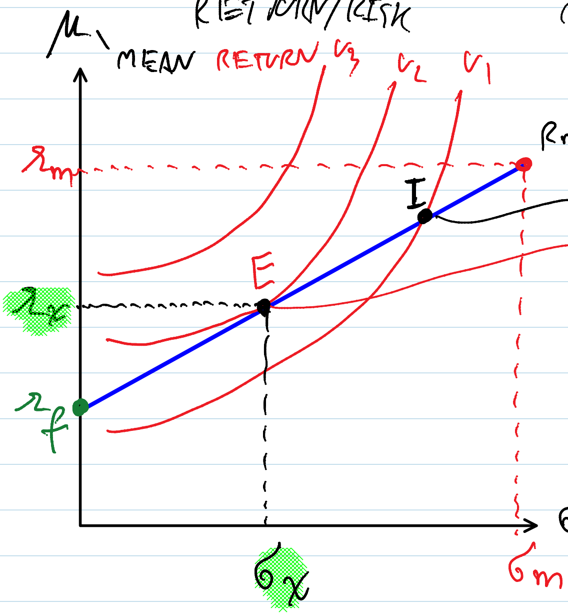
$$\boxed{\frac{d r_x}{d\sigma_x} = \frac{r_m - r_f}{\sigma_m}}$$

$\Rightarrow$ SLOPE OF THE BUDGET LINE

$\Rightarrow$ IT IS CALLED THE PRICE OF RISK RELATIVE TO MEAN RETURN.

NOTE $\dfrac{d r_x}{d\sigma_x} > 0$

Q? WHERE IS THE MOST PREFERRED RETURN/RISK COMBINATION?




$R_m = r_f + \dfrac{\sigma_x}{\sigma_m}(r_m - r_f)$

NOT AN OPTIMAL CHOICE (WHY?)

$$\boxed{MRS = \frac{r_m - r_f}{\sigma_m}}$$

"PRICE OF RISK"

$= \dfrac{\partial U / \partial \sigma \to MU_\sigma}{\partial U / \partial \mu \to MU_\mu}$

$\sigma$ (STANDARD DEVIATION OF THE RETURN)

DIY: EXPLAIN WHY CHOICE I IS NOT AN OPTIMAL CHOICE. (PROVIDE INTUITION)

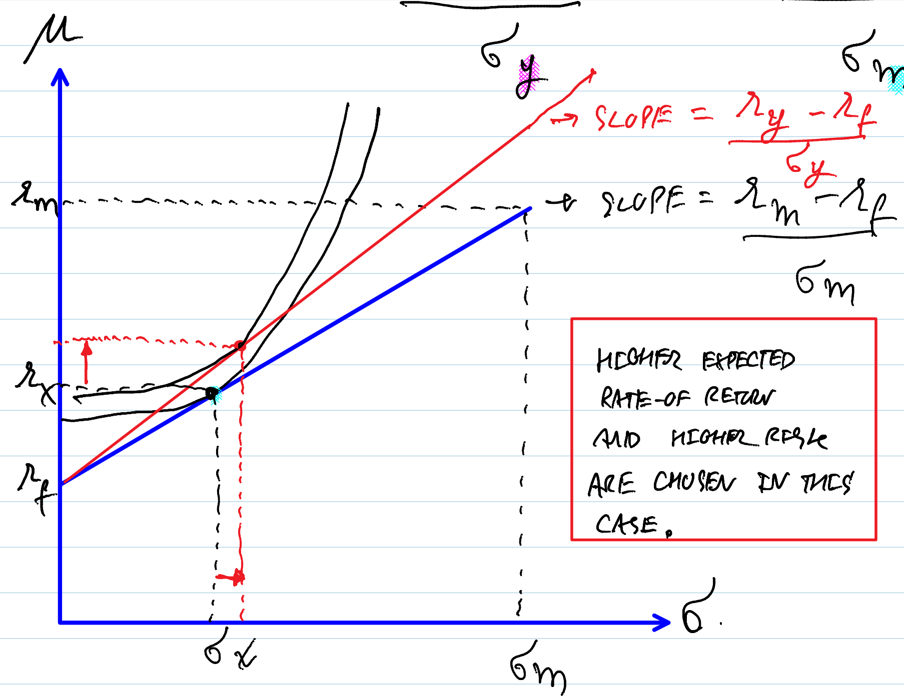
HINT: SLOPE OF IC > SLOPE OF BL AND EXPLAIN ????

# CHOOSING A PORTFOLIO

SUPPOSE A NEWLY RISKY ASSET APPEARS, W/ A MEAN RATE-OF-RETURN $r_y > r_m$ AND A STANDARD DEVIATION $\sigma_y > \sigma_m$.

WHICH ASSET IS PREFERRED?

SUPPOSE $\dfrac{r_y - r_f}{\sigma_y} > \dfrac{r_m - r_f}{\sigma_m}$.




HIGHER EXPECTED RATE-OF RETURN AND HIGHER RISK ARE CHOSEN IN THIS CASE.

EXERCISE

| | FRACTION | RETURN | RISK |
|---|---|---|---|
| RISKY ASSET | $x = 0.8$ | $r_m = 0.2$ | $\sigma_m = 3$ |
| RISK-FREE ASSET | $1 - x = 0.2$ | $r_f = 0.04$ | $\sigma_f = 0$ |

FIND $r_x$ AND $\sigma_x$.

TODAY'S MAKE UP 15:30 – 18:30: PRODUCTION THEORY