Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p^d = a - bQ^d$; $a \geq 0, \quad b \leq 0.$

Supply : $p^s = c + dQ^s$; $d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

tax on producer
L $p^s = p^d - t$

tax on consumer
L $p^d = p^s + t$

E. will not changing

$p^s + t = a - b \cdot Q^d$

$p^s = a - t - b \cdot Q^d$

$p^s = c + dQ^s$

$Q^d = Q^c$

$p^s = p^s$

$\hookrightarrow a + t - b \cdot Q = c + d \cdot Q$

$a - t - c = (b+d)Q$

$Q^* = \dfrac{a+t-c}{b+d}$

$= \dfrac{a-c}{b+d} - \dfrac{t}{b+d}$

Derive t

$t = 0 \rightarrow Q = \dfrac{a-c}{b+d}$

$t > 0 ;\ Q = \dfrac{a-c}{b+d} - \dfrac{t}{b+d}$

$\Delta Q = Q$ After tax $- Q$ before tax

$= \dfrac{-t}{b+d}$

So, $\dfrac{\partial Q}{\partial t}$; $\dfrac{\Delta Q}{\Delta t}$

$= \dfrac{-1}{b+d}$

When $Q \downarrow$

$t \uparrow$

- Derive the excess burden formula for buyers and sellers

$$p^s = c + dQ^s$$
$$= c + d\left(\frac{a-c}{a-d} - \frac{t}{b+d}\right)$$
$$= c + \frac{d(a-c)}{b+d} - \frac{dt}{b+d}$$
$$p^{s*}_{(t)} = \frac{cb+da}{b+d} - \frac{dt}{b+d}$$

$$p^d = p^s + t$$
$$p^d = \frac{cb+da}{b+d} - \frac{dt}{b+d} + t$$
$$p^{d*} = \frac{cb-da}{b+d} + \left|\frac{b}{b+d} \cdot t\right|$$

p^s, p^d depends on tax

$t=0 \rightarrow t>0 \rightarrow |\Delta p^s| = \frac{dt}{b+d}$

$$|\Delta p^d| = \frac{bt}{b+d}$$

$$\left|\frac{\Delta p^s}{\Delta t}\right| = \frac{d}{b+d} = \frac{\frac{\Delta P}{\Delta Q^s}}{\frac{\Delta P}{\Delta Q^d} + \frac{\Delta D}{\Delta Q^s}} = \frac{\frac{1}{\frac{\Delta Q^s}{\Delta P}} \cdot \frac{1}{\frac{p^*}{Q^*}}}{\frac{1}{\Delta Q^d}\frac{1}{\frac{p^*}{Q^*}} + \frac{1}{\frac{\Delta Q^s}{\Delta P}} \cdot \frac{1}{\frac{p^d}{Q^*}}}$$

$$= \frac{\frac{1}{\varepsilon^s}}{\frac{1}{\varepsilon^d} + \frac{1}{\varepsilon^s}} = \frac{\frac{1}{\varepsilon^s}}{\frac{\varepsilon^d + \varepsilon^d}{\varepsilon^d \cdot \varepsilon^s}} = \frac{\varepsilon^d}{\varepsilon^d + \varepsilon^s}$$

Consumer burden

$$\left|\frac{\Delta p^d}{\Delta t}\right| = \frac{b}{b+d}$$
$$= \frac{\varepsilon^s}{\varepsilon^s + \varepsilon^d}$$

$$\frac{\Delta \text{ consumer burden}}{\Delta \varepsilon^d} = -\frac{\varepsilon^s}{(\varepsilon^s + \varepsilon^d)^2}$$

$$\frac{\Delta \text{ firm burden}}{\Delta \varepsilon^s} = -\frac{\varepsilon^d}{(\varepsilon^s + \varepsilon^d)^2}$$

$\therefore \varepsilon^s \downarrow \rightarrow \uparrow$ firm budget

- Calculate the tax rate that maximizes the tax revenue of government.

$$T = t \cdot Q$$

$$T = t \cdot \left(\frac{a-c}{b+d} - \frac{t}{b+d} \right)$$

$$T(t) = \left(\frac{a-c}{b+d} \right) \cdot t - \frac{t^2}{b+d}$$

$$= \frac{a-c}{b+d} - \frac{2t}{b+d}$$

t^* maximize $T(t)$

$$t^* = \frac{-\left(\frac{a-c}{b+d} \right)}{2\left(\frac{-1}{b+d} \right)}$$

$$= \frac{a-c}{2}$$