

1 market demand $P = 10 - Q^2$

$$Q^2 = 10 - P$$

$$Q = \pm\sqrt{10 - P}$$

$$Q = \sqrt{10 - P}, Q \geq 0$$

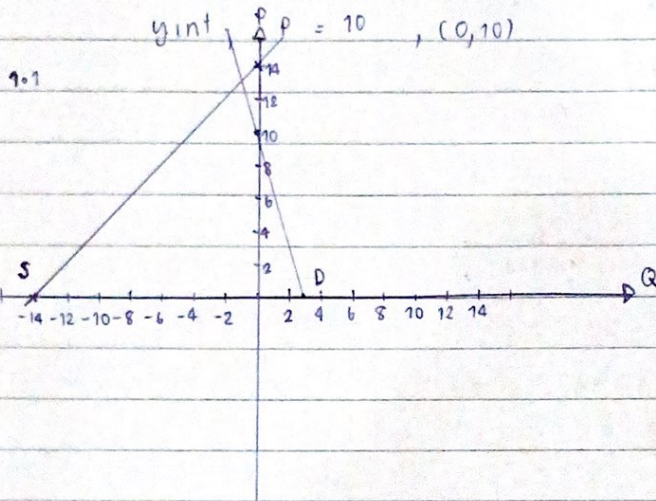
market supply $Q = -14 + P$

x-int, $Q = -14$ $(-14, 0)$

y-int, $P = 14$ $(0, 14)$

x-int, $Q = \sqrt{10}$, $(\sqrt{10}, 0)$

y-int, $P = 10$, $(0, 10)$



1.2 Solve P^* , Q^*

Demand $Q = \sqrt{10 - P}$

Supply $Q = -14 + P$

equilibrium when market Demand = market supply

$$\sqrt{10 - P} = -14 + P$$

$$(\sqrt{10 - P})^2 = (-14 + P)^2$$

$$10 - P = P^2 - 28P + 196$$

$$P^2 - 28P + 196 - 10 + P = 0$$

$$P^2 - 27P + 186 = 0$$

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$P = \frac{27 \pm \sqrt{(-27)^2 - 4(1)(186)}}{2}$$

P is undefined

Hence, no P^* and Q^*

1.3 if market supply changed from $Q_1 = -14 + P_1$ to $Q_2 = -12 + P_2$

The result of a increased is that Quantity will be increased and supply curve shifts right. This means quantity is increased while price is reduced.

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$$R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), \quad Q \geq 0$$

$$TR = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right)$$

$$\begin{aligned} \text{MR} = \frac{d(TR)}{dQ} &= \frac{2Q}{Q^2+1} + 3\left(\frac{1(Q+1) - Q(1)}{(Q+1)^2}\right) \\ \text{(Marginal Revenue)} &= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2} \end{aligned}$$

To find critical value

$$\frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2} = 0$$

$$2Q(Q^2+2Q+1) + 3Q^2+3 = 0$$

$$2Q^3+4Q^2+2Q+3Q^2+3 = 0$$

$$2Q^3+7Q^2+2Q+3 = 0$$

no critical value

$$\begin{aligned} \frac{d^2(TR)}{dQ^2} &= \frac{2(Q^2+1) - (2Q)(2Q)}{(Q^2+1)^2} + \frac{2(-2)}{(Q+1)^3} \\ &= \frac{2Q^2+2-4Q^2}{(Q^2+1)^2} - \frac{6}{(Q+1)^3} \end{aligned}$$

convex or increasing gradient when $\frac{d^2(TR)}{dQ^2} > 0$

$$\frac{-2Q^2+2}{(Q^2+1)^2} - \frac{6}{(Q+1)^3} > 0$$

$$\frac{-2Q^2+2}{(Q^2+1)^2} > \frac{6}{(Q+1)^3}, \quad Q \geq 0$$

concave or decreasing gradient when $\frac{d^2(TR)}{dQ^2} < 0$

$$\frac{-2Q^2+2}{(Q^2+1)^2} < \frac{6}{(Q+1)^3}, \quad Q \geq 0$$

3 $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$, solve profit maximizing output by
second derivative

$$\frac{d(\pi(Q))}{dQ} = -\frac{3}{3}Q^2 - 2Q + 8$$

$$0 = -Q^2 - 2Q + 8$$

$$Q^2 + 2Q - 8 = 0$$

$$(Q+4)(Q-2) = 0$$

$$Q = -4, \textcircled{2}$$

$$\frac{d^2(\pi(Q))}{dQ^2} = -2Q - 2$$

$$\text{Plug } Q = 2; -2(2) - 2 = -6 < 0$$

Hence profit maximize when quantity is 2 unit

$$\text{and } \max \pi(Q) = -\frac{1}{3}(2)^3 - 2^2 + 8(2) - 1$$

$$= \frac{25}{3}$$

$$4 \quad 4.1 \quad A+B = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$A+B$ does not exist because A and B have no equality of row \times column

$$4.2 \quad A \times B = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×2 2×3

$$= \begin{bmatrix} 8+36 & 16+45 & 24+54 \\ 10+44 & 20+55 & 30+66 \end{bmatrix}$$

$$= \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

$$4.3 \quad \det A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$$

$$= 88 - 90$$

$$= -2$$

4.4 $\det B$ does not exist because B is not square matrix

4.5 $\det C$

$$C = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{bmatrix}$$

$105 + 48 + 72 = 225$
 $45 + 84 + 96 = 225$

$$\det C = 225 - 225$$

$$= 0$$

5. $u(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$

$$\frac{\partial u}{\partial x} = a x^{a-1} y^b + \frac{1}{\frac{x}{x+y}} \left(\frac{1(x+y) - (1)(x)}{(x+y)^2} \right)$$

$$\frac{\partial u}{\partial x} = a x^{a-1} y^b + \left(\frac{x+y}{x} \right) \left(\frac{y}{(x+y)^2} \right)$$

$$\frac{\partial u}{\partial x} = a x^{a-1} y^b + \left(\frac{y}{x} \right) \left(\frac{1}{(x+y)} \right)$$

$$\frac{\partial u}{\partial y} = b x^a y^{b-1} + \frac{1}{\left(\frac{x}{x+y} \right)} \left(\frac{0 - (1)(x)}{(x+y)^2} \right)$$

$$= b x^a y^{b-1} + \left(\frac{x+y}{x} \right) \left(\frac{-x}{(x+y)^2} \right)$$

$$= b x^a y^{b-1} + \left(\frac{-1}{(x+y)} \right)$$