

1 Cost Minimization: Example

- Continue example above: $y = f(L, K) = AK^\alpha L^\beta$

- Cost minimization:

$$\begin{aligned} \min wL + rK \\ \text{s.t. } AK^\alpha L^\beta = y \end{aligned}$$

- What is the return to scale for this example?
- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$
- How much does input increase?
 - Decreasing returns to scale: for all \mathbf{z} and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

– Constant returns to scale: for all \mathbf{z} and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

– Increasing returns to scale: for all \mathbf{z} and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Returns to scale depend on $\alpha + \beta \lesseqgtr 1$: $f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha+\beta} AK^\alpha L^\beta = t^{\alpha+\beta} f(K, L)$

- Solutions:

- Optimal amount of labor:

$$L^*(r, w, y) = \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w\alpha}{r\beta}\right)^{-\frac{\alpha}{\alpha+\beta}}$$

- Optimal amount of capital:

$$\begin{aligned} K^*(r, w, y) &= \frac{w\alpha}{r\beta} \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w\alpha}{r\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w\alpha}{r\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{aligned}$$

- Check various comparative statics:

- $\partial L^* / \partial A < 0$ (technological progress and unemployment)
- $\partial L^* / \partial y > 0$ (more workers needed to produce more output)

– $\partial L^*/\partial w < 0$, $\partial L^*/\partial r > 0$ (substitute away from more expensive inputs)

- Parallel comparative statics for K^*

- Cost function

$$\begin{aligned}
 c(w, r, y) &= wL^*(r, w, y) + rK^*(r, w, y) = \\
 &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left[w \left(\frac{w\alpha}{r\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r \left(\frac{w\alpha}{r\beta}\right)^{\frac{\beta}{\alpha+\beta}} \right]
 \end{aligned}$$

- Define $B := w \left(\frac{w\alpha}{r\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r \left(\frac{w\alpha}{r\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

- Cost-minimizing output choice:

$$\max py - B \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}}$$

- First order condition:

$$p - \frac{1}{\alpha + \beta} \frac{B}{A} \left(\frac{y}{A} \right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0$$

- Second order condition:

$$-\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left(\frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

- When is the second order condition satisfied?

- Solution:

- $\alpha + \beta = 1$ (CRS):

- * S.o.c. equal to 0

- * Solution depends on p

- * For $p > \frac{B}{A}$, produce $y^* \rightarrow \infty$

- * For $p = \frac{B}{A}$, produce any $y^* \in [0, \infty)$

- * For $p < \frac{B}{A}$, produce $y^* = 0$

– $\alpha + \beta > 1$ (IRS):

* S.o.c. positive

* Solution of f.o.c. is a minimum!

* Solution is $y^* \rightarrow \infty$.

* Keep increasing production since higher production is associated with higher returns

– $\alpha + \beta < 1$ (DRS):

* s.o.c. negative. OK!

* Solution of f.o.c. is an interior optimum

* This is the only "well-behaved" case under perfect competition

* Here can define a supply function

2 Cost Curves

- Nicholson, Ch. 10, pp. 341-349; Ch. 11, pp. 380-383

- Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization

$$p = MC = \partial c(w, r, y) / \partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?

$$\pi = py - c(w, r, y) > 0 \text{ iff}$$

$$\pi/y = p - c(w, r, y)/y > 0 \text{ iff}$$

$$c(w, r, y)/y = AC < p$$

- **Supply function.** Portion of marginal cost MC above average costs.(price equals marginal cost)

- Assume only 1 input (expenditure minimization is trivial)

- **Case 1.** Production function. $y = L^\alpha$

- Cost function? (cost of input is w):

$$c(w, y) = wL^*(w, y) = wy^{1/\alpha}$$

- Marginal cost?

$$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha}$$

- Average cost $c(w, y) / y$?

$$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$

- **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?
- **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?
- **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?

2.1 Supply Function

- Supply function: $y^* = y^*(w, r, p)$
- What happens to y^* as p increases?
- Is the supply function upward sloping?

- Remember f.o.c:

$$p - c'_y(w, r, y) = 0$$

- Implicit function:

$$\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y}(w, r, y)} > 0$$

as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.