B.E. International Program

Faculty of Economics, Thammasat University

TU152 Fundamental Mathematics

Quiz #1 (Semester 2/2012)

Name…………………………………………………………….Student No…………………………….…….

1. (10 marks) Prove that if $q \vee \sim(u \vee p)$, $\sim(q \wedge r)$, and $\sim u \rightarrow (u \vee r)$, then $u \leftrightarrow \sim r$.

Proof: First, we need to show $u \rightarrow \sim r$. Assume u and show $\sim r$.

1) u	Assumption
2) $u \rightarrow (u \vee p)$	Law of Addition
3) $u \vee p$	1, 2, Rule of Modus Ponens
4) $q \vee \sim(u \vee p)$	Assumption
5) q	3, 4, Disjunctive Syllogism, Law of Double Negation, Rule of Substitution
6) $\sim(q \wedge r)$	Assumption
7) $\sim q \vee \sim r$	6, De Morgan's Law, Rule of Substitution
8) $\sim r$	5, 7, Disjunctive Syllogism

Hence, $u \rightarrow \sim r$.

Next, we prove $\sim r \rightarrow u$ by assuming $\sim r$ and show u. In this case we will show by using the method of indirect proof, that is we assume u to be false and we try to show the contradiction.

Note: Since we assume u to be false then $\sim u$ is assumed to be true.

1) $\sim u$	Assumption
2) $\sim r$	Assumption
3) $\sim u \wedge \sim r$	1, 2, Definition of Conjunction
4) $\sim(u \vee r)$	3, De Morgan's Law, Rule of Substitution
5) $\sim u \rightarrow (u \vee r)$	Assumption
6) u	4, 5, Rule of Modus Tollens, Law of Double Negation, Rule of Substitution
7) $u \wedge \sim u$	1, 6, Definition of Conjunction

Since $u \wedge \sim u$ is a contradiction, hence $\sim u$ is false and that is u is true. Therefore, $\sim r \rightarrow u$.

$\therefore$ We have proved that $u \leftrightarrow \sim r$.

2. (10 marks) Show that for every natural number n, $n < 2^n$.

Proof: Let S_n be the statement "$n < 2^n$ where n is any natural number".

1) S_1 is "$1 < 2^1$" which is true.

2) Assume S_k is true and we need to show that S_{k+1} is true.

That is we assume

$k < 2^k$ for every natural number k.

We need to show that $(k+1) < 2^{(k+1)}$.

Since $k < 2^k$

then $2 \cdot k < 2 \cdot 2^k$

$k + k < 2^{k+1}$

$k + 1 \le k + k < 2^{k+1}$ (Because $1 \le k$)

That is, $k + 1 < 2^{k+1}$

Hence, S_{k+1} is true.

Therefore, by Mathematical Induction we conclude that for every natural number n, $n < 2^n$.

Laws and Rules for Proving:

Theorem: For all statements *p*, *q*, and *r*, the following statements are tautologies.

1. Law of Identity: $p \leftrightarrow p$

2. Law of Double Negation: $p \leftrightarrow \sim(\sim p)$

3. Law of Excluded Middle: $p \vee \sim p$

4. Law of Contradiction: $\sim(p \wedge \sim p)$

5. Idempotent Laws: $(p \wedge p) \leftrightarrow p$, $(p \vee p) \leftrightarrow p$

6. Law of Addition: $p \rightarrow (p \vee q)$

7. Law of Equivalence: $[p \leftrightarrow q] \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$

8. Law of Contraposition: $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

9. Law of (Hypothetical) Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r]$

10. Commutative Laws: $(p \wedge q) \leftrightarrow (q \wedge p)$

    $(p \vee q) \leftrightarrow (q \vee p)$

11. Associative Laws: $[p \wedge (q \wedge r)] \leftrightarrow [(p \wedge q) \wedge r] \leftrightarrow [p \wedge q \wedge r]$

    $[p \vee (q \vee r)] \leftrightarrow [(p \vee q) \vee r] \leftrightarrow [p \vee q \vee r]$

12. Distributive Laws: $[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

    $[p \vee (q \wedge r)] \leftrightarrow [(p \vee q) \wedge (p \vee r)]$

13. Absorption Laws: $[p \wedge (p \vee q)] \leftrightarrow p$

    $[p \vee (p \wedge q)] \leftrightarrow p$

14. De Morgan's Laws: $\sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q)$

    $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$

15. Law of Implication: $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

    $(p \vee q) \leftrightarrow (\sim p \rightarrow q)$

16. Law of Negation for Implication: $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

Rules for Proving:

1. Rule of Modus Ponens or Rule of Implication

   If $p \to q$ and p, then q

2. Rule of Substitution

   2.1 Substitute the statement that is tautology (or contradiction) with the other statement.

   2.2 Substitute the statement with its equivalent statement.

3. Definition of Conjunction

   3.1 If $p \wedge q$, then p and q

   3.2 If p and q, then $p \wedge q$

4. Rule of Modus Tollens

   If $p \to q$ and $\sim q$, then $\sim p$

5. Disjunctive Syllogism

   5.1 If $p \vee q$ and $\sim p$, then q

   5.2 If $p \vee q$ and $\sim q$, then p

6. Constructive Dilemma

   If $(p \to q) \wedge (r \to s)$ and $p \vee r$, then $q \vee s$