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Homework

1.

Table 1

$\bar{X}_i = \frac{621}{8} = 77.625$

$\bar{Y}_i = \frac{25.7}{8} = 3.2125$

Student	Y_i	X_i	Y_i^2	X_i^2	X_iY_i	x_i $(X_i-\bar{X})$	x_i^2 $(X_i-\bar{X})^2$
1	2.8	63	7.84	3,969	176.4	-14.625	213.8906
2	3.4	72	11.56	5,184	244.8	-5.625	31.6406
3	3.0	78	9	6,084	234	0.375	0.1406
4	3.5	81	12.25	6,561	283.5	3.375	11.3906
5	3.6	87	12.96	7,569	313.2	9.375	87.8906
6	3.0	75	9	5,625	225	-2.625	6.8906
7	2.7	75	7.29	5,625	202.5	-2.625	6.8906
8	3.7	90	13.69	8,100	333	12.375	153.1406
	25.7	621	83.59	48,717	2,012.4	0	511.8748
	ΣY_i	ΣX_i	ΣY_i^2	ΣX_i^2			

1.1.) Use OLS to find the estimator of β_1 and β_2.

First, let's find $\hat{\beta}_2$

$$\hat{\beta}_2 = \frac{n\Sigma X_iY_i - \Sigma X_i \Sigma Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2}$$

$$= \frac{8(2{,}012.4) - (621)(25.7)}{8(48{,}717) - (621)^2}$$

$$= \frac{16{,}099.2 - 15{,}959.7}{389{,}736 - 385{,}641}$$

$$\boxed{\hat{\beta}_2 = \frac{139.5}{4{,}095} = 0.0341} \quad \#$$

Then we calculate for $\hat{\beta}_1$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= 3.2125 - 2.6474$$

$$\boxed{\hat{\beta}_1 = 0.5655} \quad \#$$

Thus, $Y_i = 0.5655 + 0.0341 X_i + u_i$ ← SRF done.

$\hat{Y}_i = \cancel{0.5881} + 0.0341 X_i$

0.565

Interpretation

According to the regression model, we can interpret that if student increase his/her econometrics exam point, their GPA will increase for 0.0341. However, if they did 0 in the exam, their GPA will be 0.5655

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1.2) Find $\hat{Y}_i$ and $\hat{u}_i$

$\hat{Y}_1 = 0.5655 + 0.0341(63) = 2.7138$
$\hat{Y}_2 = 0.5655 + 0.0341(72) = 3.0207$
$\hat{Y}_3 = 0.5655 + 0.0341(78) = 3.2253$
$\hat{Y}_4 = 0.5655 + 0.0341(81) = 3.3276$
$\hat{Y}_5 = 0.5655 + 0.0341(87) = 3.5322$
$\hat{Y}_6 = 0.5655 + 0.0341(75) = 3.1230$
$\hat{Y}_7 = 0.5655 + 0.0341(75) = 3.1230$
$\hat{Y}_8 = 0.5655 + 0.0341(90) = 3.6345$

found $\hat{u}_i$: $Y_i = \hat{u}_i + \hat{Y}_i \rightarrow Y_i - \hat{Y}_i = \hat{u}_i$

$\hat{u}_1 : 2.8 - 2.7138 = 0.0862$
$\hat{u}_2 : 3.4 - 3.0207 = 0.3793$
$\hat{u}_3 : 3.0 - 3.2253 = -0.2253$
$\hat{u}_4 : 3.5 - 3.3276 = 0.1724$
$\hat{u}_5 : 3.6 - 3.5322 = 0.0678$
$\hat{u}_6 : 3.0 - 3.1230 = -0.1230$
$\hat{u}_7 : 2.7 - 3.1230 = -0.423$
$\hat{u}_8 : 3.7 - 3.6345 = 0.0655$

$\Sigma \hat{u}_i = -0.0001$

1.3) Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

$(\hat{u}_1)^2 = (0.0862)^2 = 0.0074$
$(\hat{u}_2)^2 = (0.3793)^2 = 0.1439$
$(\hat{u}_3)^2 = (-0.2253)^2 = 0.0508$
$(\hat{u}_4)^2 = (0.1724)^2 = 0.0297$
$(\hat{u}_5)^2 = (0.0678)^2 = 0.0046$
$(\hat{u}_6)^2 = (-0.1230)^2 = 0.0151$
$(\hat{u}_7)^2 = (-0.423)^2 = 0.1789$
$(\hat{u}_8)^2 = (0.0655)^2 = 0.0043$

$\Sigma(\hat{u}_i^2) = 0.4347$

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Then we find $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}$

$$\boxed{= \frac{0.4347}{8-2} = \frac{0.4347}{6} = 0.0725 = \hat{\sigma}^2 = var(\hat{u}_i)}$$

Thus, we can able to find $Var(\hat{\beta}_1)$ and $Var(\hat{\beta}_2)$

$$\boxed{Var(\hat{\beta}_2) = \frac{\sigma_u^2}{\sum x_i^2} = \frac{0.0725}{\sum(X_i - \bar{X})^2} = \frac{0.0725}{511.8748} = 0.0001}$$

$$\boxed{Var(\hat{\beta}_1) = \frac{\sigma_u^2 \sum X_i^2}{n \sum x_i^2} = \frac{0.0725(48,717)}{8(511.8748)} = 0.8625}$$

2.

X_i	Y_i	X_i^2	Y_i^2	X_iY_i	x_i $(X_i-\bar{X})$	x_i^2 $(X_i-\bar{X})^2$	y_i $(Y_i-\bar{Y})$	y_i^2 $(Y_i-\bar{Y})^2$	x_iy_i
10	0	100	0	0	-10	100	-9.1	82.81	91
12	2	144	4	24	-8	64	-7.1	50.41	56.8
14	5	196	25	70	-6	36	-4.1	16.81	24.6
16	6	236	36	96	-4	16	-3.1	9.61	12.4
18	7	324	49	126	-2	4	-2.1	4.41	4.2
22	10	484	100	220	2	4	0.9	0.81	1.8
24	10	576	100	240	4	16	0.9	0.81	3.6
26	15	676	225	390	6	36	5.9	34.81	35.4
28	16	784	256	448	8	64	6.9	47.61	55.2
30	20	400	400	600	10	100	10.9	118.81	109

$\sum X_i = 200$	$\sum X_i^2 = 4,440$	$\sum x_i = 0$
$\bar{X} = \frac{200}{10} = 20$	$\sum Y_i^2 = 1,195$	$\sum x_i^2 = 440$
$\sum Y_i = 91$	$\sum x_i^2$	$\sum y_i = 0$
$\bar{Y} = 9.1$		$\sum y_i^2 = 366.9$
$\sum X_iY_i = 2,214$		$\sum x_iy_i = 394$

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2.1) From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$
Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

$$\hat{\beta}_2 = \frac{n\sum X_iY_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2}$$

$$= \frac{10(2{,}214) - (200)(91)}{10(4{,}440) - (200)^2}$$

$$= \frac{22{,}140 - 18{,}200}{44{,}400 - 40{,}000}$$

$$\boxed{\hat{\beta}_2 = \frac{3{,}940}{4{,}400} = 0.8955 \ \#}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 9.1 - 0.8955(20)$$

$$\boxed{\hat{\beta}_1 = 9.1 - 17.91 = -8.81 \ \#}$$

Regression Model
$\hat{Y}_i = -8.81 + 0.8955 X_i$

Interpretation
Based on the equation, if we increase X_i for 1 unit, $\hat{Y}_i$ will increase for 0.8955.
In case X_i is -,
$\hat{Y}_i$ will equal to -8.81 #

2.2) Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$\hat{Y}_1 = -8.81 + 0.8955(10) = 0.145$
$\hat{Y}_2 = -8.81 + 0.8955(12) = 1.936$
$\hat{Y}_3 = -8.81 + 0.8955(14) = 3.727$
$\hat{Y}_4 = -8.81 + 0.8955(16) = 5.518$
$\hat{Y}_5 = -8.81 + 0.8955(18) = 7.309$
$\hat{Y}_6 = -8.81 + 0.8955(22) = 10.891$
$\hat{Y}_7 = -8.81 + 0.8955(24) = 12.682$
$\hat{Y}_8 = -8.81 + 0.8955(26) = 14.473$
$\hat{Y}_9 = -8.81 + 0.8955(28) = 16.264$
$\hat{Y}_{10} = -8.81 + 0.8955(30) = 18.055$

$\sum \hat{Y}_i = 91$

(Y, X)

next, we find $\hat{u}_i$ on the next page.

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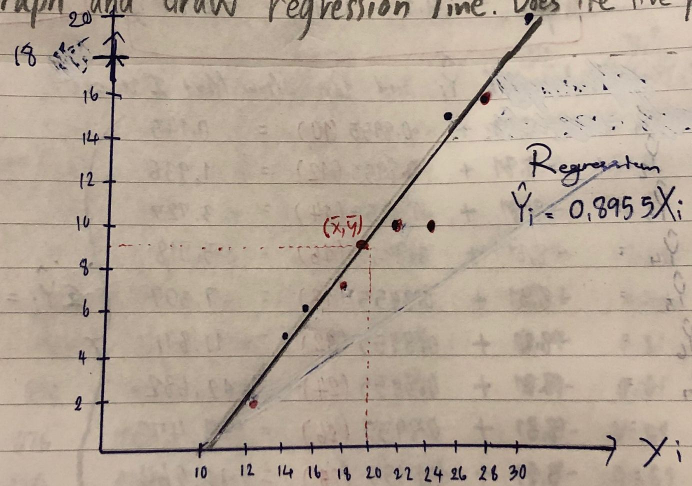
Let's find $\hat{u}_i$ from the equation: $Y_i = \hat{u}_i + \hat{Y}_i \longrightarrow Y_i - \hat{Y}_i = \hat{u}_i$

$$\left.\begin{array}{lrcrcr}
\hat{u}_1 : & 0 & - & 0.145 & = & -0.145 \\
\hat{u}_2 : & 2 & - & 1.936 & = & 0.064 \\
\hat{u}_3 : & 5 & - & 3.727 & = & 1.273 \\
\hat{u}_4 : & 6 & - & 5.518 & = & 0.482 \\
\hat{u}_5 : & 7 & - & 7.309 & = & -0.309 \\
\hat{u}_6 : & 10 & - & 10.891 & = & -0.891 \\
\hat{u}_7 : & 10 & - & 12.682 & = & -2.682 \\
\hat{u}_8 : & 15 & - & 14.473 & = & 0.527 \\
\hat{u}_9 : & 16 & - & 16.264 & = & -0.264 \\
\hat{u}_{10} : & 20 & - & 18.055 & = & 1.945
\end{array}\right\} \sum \hat{u}_i = 0$$

2.3.) Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



Yes, the line passes $(\bar{X}, \bar{Y})$

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2.4.) If $X_i = 18$, what is the predicted Y ? (Find $\hat{Y}$)

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$
$$\hat{Y} = -8.81 + (0.8955)(18)$$
$$\boxed{\hat{Y} = 7.309}$$

∴ If $X_i = 18$, predicted Y is 7.309

2.5) Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

First, we find $var(\hat{u}_i)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2}$$; but we don't have $\Sigma \hat{u}_i^2$, so we need to find it first.

Finding $\Sigma \hat{u}_i^2$

$(\hat{u}_1)^2 : (-0.145)^2 = 0.0210$
$(\hat{u}_2)^2 : (0.064)^2 = 0.0041$
$(\hat{u}_3)^2 : (1.273)^2 = 1.6205$
$(\hat{u}_4)^2 : (0.482)^2 = 0.2323$
$(\hat{u}_5)^2 : (-0.309)^2 = 0.0955$
$(\hat{u}_6)^2 : (-0.891)^2 = 0.7939$
$(\hat{u}_7)^2 : (-2.682)^2 = 7.1931$
$(\hat{u}_8)^2 : (0.527)^2 = 0.2777$
$(\hat{u}_9)^2 : (-0.264)^2 = 0.0697$
$(\hat{u}_{10})^2 : (1.945)^2 = 3.7830$

$\Sigma \hat{u}_i^2 = 14.0908$

After that, we input it in equation to calculate $var(\hat{u}_i)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{14.0908}{8} = 1.7614$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}_u^2}{\Sigma x_i} = \frac{\hat{\sigma}_u^2}{\Sigma (X_i - \bar{X})^2} = \frac{1.7614}{440} = 0.004$$

$$var(\hat{\beta}_1) = \frac{\sigma_u^2 \Sigma X_i^2}{n \Sigma x_i^2} = \frac{\sigma_u^2 \Sigma X_i^2}{n \Sigma (X_i - \bar{X})^2} = \frac{(1.7614)(4,440)}{10(440)} = 1.7774$$

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3.) Consider the below regression function:
consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i \sim NIID(0, \sigma^2)$
Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.

$$\hat{\beta}_1 = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$= \bar{Y} - \hat{\beta}_2 \bar{X}$$

$\hat{\beta}_1$ is an unbiased estimator

$$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$$

$$= E(\bar{Y}) - \hat{\beta}_2 E(\bar{X})$$

$$= \beta_1 + \cancel{\beta_2 \bar{X}} - \cancel{\beta_2 \bar{X}}$$

$$E(\hat{\beta}_1) = \beta_1$$