Assignment 5

DUE DATE: Thursday 29^{th}, April 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Full name Gaatchasorn Atcharanukul **Student ID.** 6004640477

All data are downloadable from BE moodle

There are four questions.

Question1.

Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 13, 2017.You may download the data using quantmod. Let r_t be the log returns, which can be obtained via

```
rt <- diff(log(as.numeric(CAT[,6])))
```

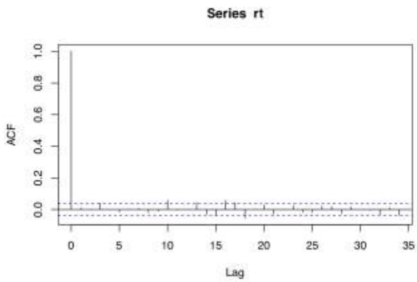
(a) Are there any serial correlations in the log return series r_t ? Why?

$H_0: \rho_1 = \rho_2 = \rho_3 = \ldots = \rho_m = 0$

$H_a: \exists \rho_i \neq 0$

Since the p-value is more than 0.05 which is the critical region. We can not reject the null hypothesis that there is no serial correlation in the log return series of r_t.

Also, the ACF plot represents no serial correlation. Hence, there is no linear dependence in r_t.




```r
Box.test(rt,lag=10,type='Ljung')
```

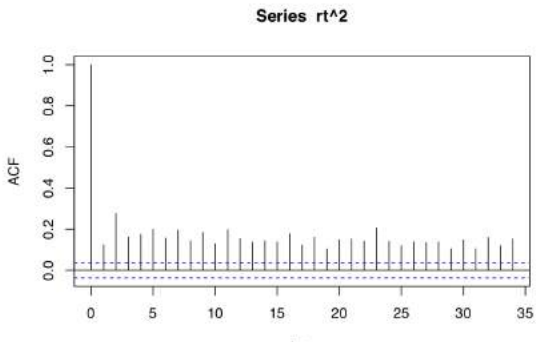
```
##
##  Box-Ljung test
##
## data:  rt
## X-squared = 16.291, df = 10, p-value = 0.09159
```

(b) Are there any ARCH effects in the log return series r_t (the linear depedence of squared returns)? Why?

$H_0: \rho_1 = \rho_2 = \dots \rho_m = 0$: no arch effect
$H_a: \exists \rho_i \neq 0$ ARCH effect

Since the p-value is less than 0.05, which is the critical value. Then, we can reject the null hypothesis that there is no ARCH effect in the log return series of r_t.

∴ There is ARCH effect at 95% confidence interval.




```r
Box.test(rt^2,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt^2
## X-squared = 917.21, df = 10, p-value < 2.2e-16
```

(c) Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

H_0: there is no ARCH effect in $\tilde{a}_t$
H_a: there is ARCH effect in $\tilde{a}_t$.

According to the Ljung box test, H_0 is not rejected at 0.05 level of significance. Making the mean-equation adequate. Also, there is no linear dependency in $\tilde{a}_t^2$ so the volatility model is also adequate.

The fitted model is

mean equation: $\hat{r}_t = \underset{(3.075e-04)}{4.830e-04}(1-1.687e-02) + \underset{(2.006e-02)}{0.01687} r_{t-1}$

volatility equation: $\sigma_t^2 = \underset{(1.278e-06)}{4.478e-06} + \underset{(8.191e-03)}{4.972e-02} a_{t-1}^2 + \underset{(1.031e-02)}{0.9387} \sigma_{t-1}^2$

```r
m1=garchFit(~arma(1,0)+garch(1,1),data=rt,trace=F)
```

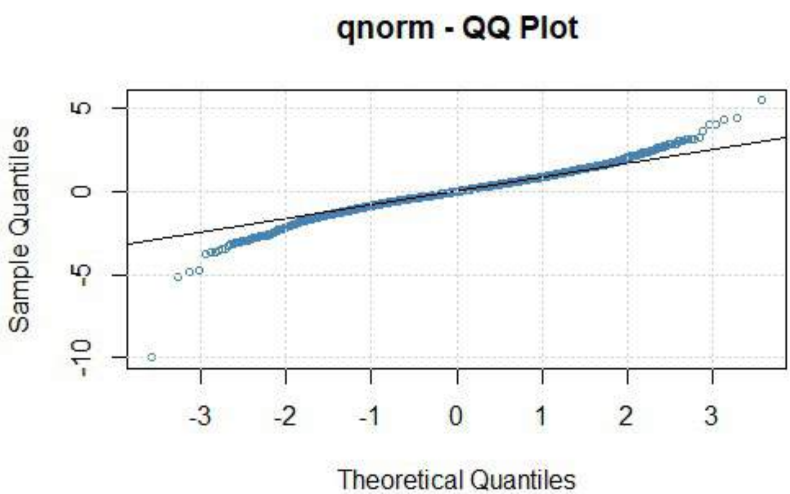
```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)


## <environment: 0x000000001f901ee8>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1
## 4.8298e-04  1.6866e-02  4.4779e-06  4.9720e-02  9.3866e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.830e-04   3.075e-04    1.571 0.116297
## ar1    1.687e-02   2.006e-02    0.841 0.400353
## omega  4.478e-06   1.278e-06    3.503 0.000461 ***
## alpha1 4.972e-02   8.191e-03    6.070 1.28e-09 ***
## beta1  9.387e-01   1.031e-02   91.048  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Description:
##  Tue Apr 27 12:20:52 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3298.441  0
##  Shapiro-Wilk Test  R    W      0.9663735 0
##  Ljung-Box Test     R    Q(10)  12.37554  0.2607088
##  Ljung-Box Test     R    Q(15)  14.79514  0.4662719
##  Ljung-Box Test     R    Q(20)  19.20107  0.5087928
##  Ljung-Box Test     R^2  Q(10)  0.980939  0.9998424
##  Ljung-Box Test     R^2  Q(15)  3.682825  0.9986048
##  Ljung-Box Test     R^2  Q(20)  6.9285    0.996913
##  LM Arch Test       R    TR^2   2.723165  0.9972029
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.196308 -5.185823 -5.196314 -5.192526
```

```
#plot(m1)
#13
#0
```



(d) Build a GARCH(1,1) model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

According to the Ljung box test, Ho is not rejected at 0.05 level of significance. So there is no linear dependence in the mean and volatility equation. So the equations are adequate.

Ho: there is no ARCH effect

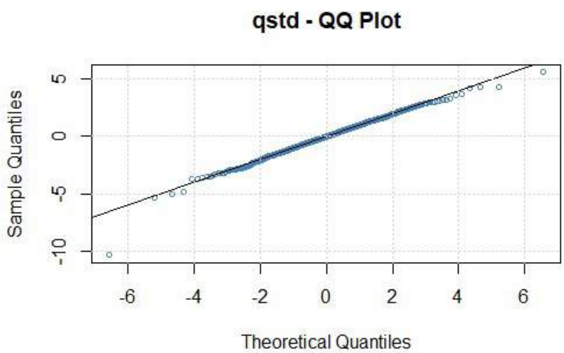
Ha: there is ARCH effect

```r
m2=garchFit(~garch(1,1),data=rt,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001f2e5440>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9780e-04  4.2035e-06  7.2376e-02  9.2033e-01  5.0958e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.978e-04   2.702e-04    2.212  0.02695 *
## omega  4.203e-06   1.571e-06    2.675  0.00747 **
## alpha1 7.238e-02   1.374e-02    5.267 1.39e-07 ***
## beta1  9.203e-01   1.472e-02   62.503  < 2e-16 ***
## shape  5.096e+00   4.825e-01   10.561  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7507.106    normalized:  2.64521
##
## Description:
##  Tue Apr 27 12:20:53 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4056.502  0
##  Shapiro-Wilk Test  R    W      0.9639091 0
##  Ljung-Box Test     R    Q(10)  14.77968  0.140303
##  Ljung-Box Test     R    Q(15)  16.74279  0.3344718
##  Ljung-Box Test     R    Q(20)  20.39783  0.433304
##  Ljung-Box Test     R^2  Q(10)  2.953085  0.9825066
##  Ljung-Box Test     R^2  Q(15)  5.482428  0.9871938
##  Ljung-Box Test     R^2  Q(20)  9.458146  0.9769677
##  LM Arch Test       R    TR^2   4.273688  0.977976
```



(e) Write down the fitted model.

- mean equation: $\hat{r}_t = 5.003e\text{-}04$
  $(3.075e\text{-}04)$
- volatility equation: $\sigma_t^2 = 4.419e\text{-}06 + 4.947e\text{-}02\, a_{t-1}^2 + 9.397e\text{-}01\, \sigma_{t-1}^2$
  $(1.258e\text{-}06) \quad (8.092e\text{-}03) \quad (1.076e\text{-}02)$

(f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted GARCH(1,1) model with standardized Student-t innovations.

```
##   meanForecast  meanError standardDeviation
## 1 0.0005977987 0.01515760        0.01515760
## 2 0.0005977987 0.01524072        0.01524072
## 3 0.0005977987 0.01532280        0.01532280
## 4 0.0005977987 0.01540384        0.01540384
## 5 0.0005977987 0.01548387        0.01548387
```

(g) Compute the 95 % 1-step to 5-step interval predictions of the log return series using standardized student-t innovations.

	upper	Lower
• 1 step:	1.0108855	-1.107854
• 2 step:	1.01088	-1.009879
• 3 step:	1.010918	-1.009917
• 4 step:	1.010955	-1.009954
• 5 step:	1.010992	-1.009991

Question2.

Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file m-kovw-5116.txt. Obtain the log return series of KO stock.

(a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

$H_0: E(r_t) = 0$
$H_a: E(r_t) \neq 0$ } t-test. As the p-value is less than 0.05, we can reject the null hypothesis at 95% confidence interval.

∴ The expected value of KO log return is not zero.

$H_0: \rho_1 = \rho_2 = \ldots = \rho_m = 0$
$H_a: \exists \rho_i \neq 0$ } Box Ljung test: As the p-value is more than 0.05, therefore we cannot reject the null hypothesis at 95% confidence interval

∴ There is no serial correlation

- Ljung box test (ARCH effect) → As the p-value is less than 0.05, we can reject the null hypothesis at 95% CI.

∴ There is ARCH effect.

```
rt2=log(da$ko+1)
t.test(rt2)
```

```
##
##  One Sample t-test
##
## data:  rt2
## t = 4.9853, df = 779, p-value = 7.628e-07
```

```
Box.test(rt2,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt2
## X-squared = 5.9201, df = 10, p-value = 0.8219
```

```
Box.test(rt2^2,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt2^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16
```

(b) Build a AR(1)-GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

- Model checking: Using Ljung box test, the p-value is more than 0.05 so the mean and volatility equation is adequate as there are no linear dependence both in the mean equation and in $\hat{a}_t^2$.
- The fitted model: mean equation: $\hat{r}_t = \underset{(1.897e-03)}{1.125e-02} - \underset{(3.887e-02)}{2.634e-02}\, r_{t-1}$

  volatility equation: $\sigma_t^2 = \underset{(5.852e-05)}{1.877e-04} + \underset{(1.915e-02)}{9.535e-02}\, a_{t-1}^2 + \underset{(2.766e-02)}{8.486e-01}\, \sigma_{t-1}^2$

```
m3=garchFit(~arma(1,0)+garch(1,1),data=rt2,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt2, trace = F)
##
```

```
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001c91f428>
##  [data = rt2]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Tue Apr 27 12:20:53 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

(c) Fit a AR(1)-GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

- model checking → using Ljung box test, the p-value is more than 0.05, so there is no linear dependence in the mean equation and in a_t^2.

→ mean equation: $\hat{r}_t = \underset{(1.870e-03)}{1.124e-02}(1-(-1.888e-02)) - \underset{(3.697e-02)}{1.888e-02}\, r_{t-1}$

→ volatility equation: $\sigma_t^2 = \underset{(6.596e-05)}{1.739e-04} + \underset{(1.338e-02)}{9.643e-02}\, a_{t-1}^2 + \underset{(3.267e-02)}{8.504e-01}\, \sigma_{t-1}^2$

```r
m4=garchFit(~arma(1,0)+garch(1,1),data=rt2,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt2, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000002023df68>
##  [data = rt2]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1        shape
##  0.01124020  -0.01887601   0.00017395   0.09642927   0.85044151   7.47877780
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1    -1.888e-02   3.691e-02   -0.511  0.60904
## omega   1.739e-04   6.596e-05    2.637  0.00836 **
## alpha1  9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1   8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape   7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
##  Tue Apr 27 12:20:53 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816987
##  LM Arch Test       R    TR^2   10.48089  0.5738501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```

(d) Build a GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

- model checking: Ljung box test → The p-value is more than 0.05 so there is no linear dependence in both the mean equation and a_t^2. So the model is adequate.
- mean equation: $\hat{r}_t = \underset{(1.846e-03)}{1.098e-02}$
- volatility equation: $\sigma_t^2 = \underset{(5.899e-05)}{1.850e-04} + \underset{(1.912e-02)}{9.480e-02}\, a_{t-1}^2 + \underset{(2.787e-02)}{8.478e-01}\, \sigma_{t-1}^2$

```
m5=garchFit(~garch(1,1),data=rt2,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt2, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001cbbba08>
##  [data = rt2]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Tue Apr 27 12:20:53 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
```

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```

(e) Fit a GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

- model checking: Ljung box test → The p-value is more than 0.05 so there is no linear dependence in both the mean equation and a_t^2. So the model is adequate.
- mean equation: $\underset{(1.757e03)}{\hat{r}_t = 1.105e\text{-}02}$
- volatility equation: $\sigma_t^2 = \underset{(6.627e-05)}{1.753e\text{-}04} + \underset{(2.337e-02)}{9.633e\text{-}02}\, a_{t-1}^2 + \underset{(3.277e-02)}{8.501e\text{-}01}\, \sigma_{t-1}^2$

```r
m6=garchFit(~garch(1,1),data=rt2,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt2, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001fa400e8>
##  [data = rt2]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Tue Apr 27 12:20:54 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
```

```
## Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
## LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

(f) Compare the model (b)-(e) which model you select.

The model of e) which is GARCH(1,1) with student t innovations to the log return series is the best model with the lowest AIC value.

Question3.

Consider the daily returns of the stock *S&P500* from January 2, 2005 to March 31, 2021. Let r_t be the percentage log returns.

(a) Is the expected value of r_t zero? Why? Are there any serial correlations in r_t? Why?

(b) Fit a Gaussian ARMA-GARCH model to the r_t series. Obtain the normal QQ- plot of the standardized residuals, and write down the fitted model. Is the model adequate? Why?

a) T-test: Ho is not rejected at 0.05 level of significance. So the expected value of r_t is equal to 0.

Ljung box test: Ho is rejected at 0.05 level of significance. So there is serial correlation in r_t.

```
## [1] "^GSPC"
```

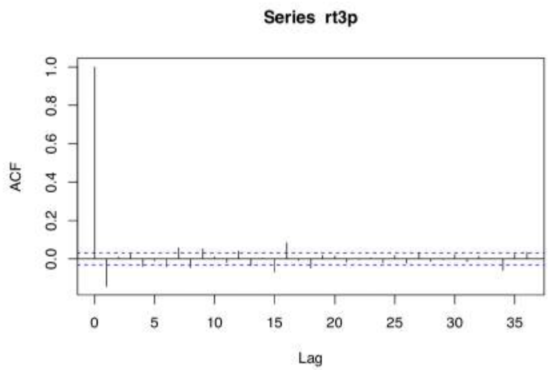
```
rt3 = diff(log(as.numeric(GSPC[,6])))
rt3p = rt3*100

t.test(rt3p)
```

```
##
##  One Sample t-test
##
## data:  rt3p
## t = 1.4961, df = 4086, p-value = 0.1347
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.009051939  0.067374643
## sample estimates:
##  mean of x
## 0.02916135
```

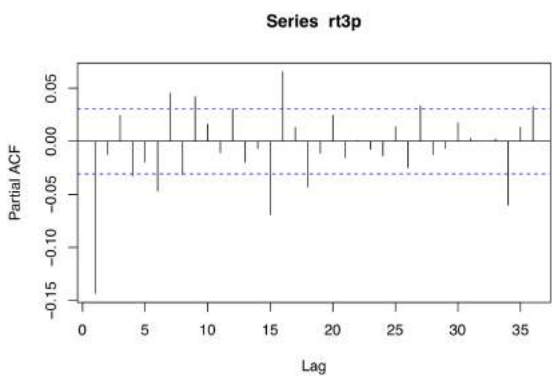
```r
Box.test(rt3p,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt3p
## X-squared = 131.85, df = 10, p-value < 2.2e-16
```

```r
acf(rt3p)
```




```r
pacf(rt3p)
```



a) T-test: Ho is not rejected at 0.05 level of significance. So the expected value of r_t is equal to 0.

Ljung box test: Ho is rejected at 0.05 level of significance. So there is serial correlation in r_t.

b) According to the ACF of r_t, there is no exponential decay, so the MA model should be used with optimal lag = 1. (using the PACF)

→ Test on residuals to check ARCH effect using Ljung box test.

∴ There is an ARCH effect as the p-value is less than 0.05 so we reject the null hypothesis.

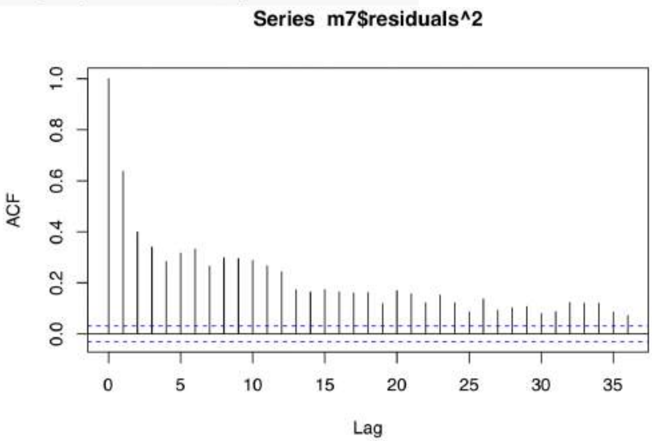
→ The fitted model:

mean equation: $0.068986 - 0.077582 a_{t-1}$
$(0.010800) \quad (0.017503)$

volatility equation: $\sigma_t^2 = 0.02717 + 0.141874 a_{t-1}^2 + 0.837406 \sigma_{t-1}^2$
$(0.003504) \quad (0.012017) \quad (0.011951)$

∴ The model is adequate: Using the Ljung box, Ho is not rejected at 0.05 level of significance → There are no linear dependence in the mean and volatility equation.

```
m7=arima(rt3p,order=c(0,1,0))
acf(m7$residuals^2)
```



```r
Box.test(m7$residuals^2,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  m7$residuals^2
## X-squared = 5312, df = 10, p-value < 2.2e-16
```

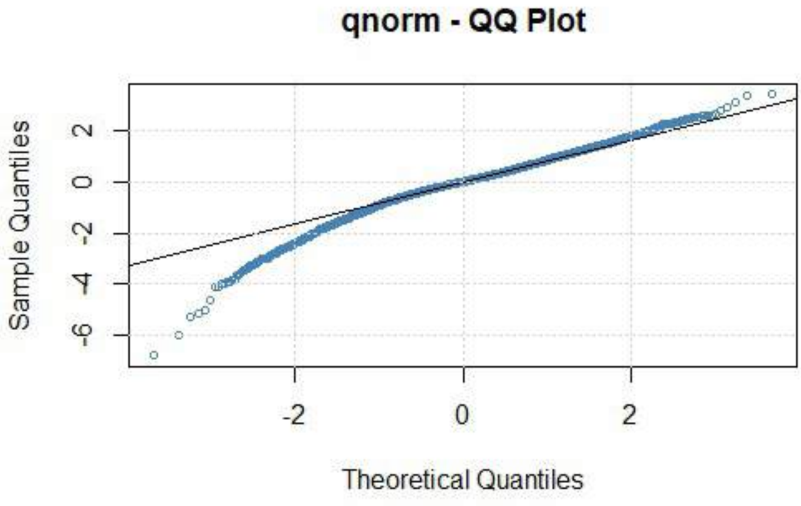
```r
m9=garchFit(~arma(0,1)+garch(1,1),data=rt3p,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m9)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rt3p, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x000000001d9cfaf0>
##  [data = rt3p]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1      omega     alpha1      beta1
##  0.068986  -0.077582   0.027170   0.141814   0.837406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.068986    0.010800    6.387 1.69e-10 ***
## ma1    -0.077582    0.017503   -4.433 9.31e-06 ***
## omega   0.027170    0.003504    7.754 8.88e-15 ***
## alpha1  0.141814    0.012017   11.801  < 2e-16 ***
## beta1   0.837406    0.011956   70.039  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5392.874    normalized:  -1.319519
##
## Description:
##  Tue Apr 27 12:20:55 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1134.026  0
##  Shapiro-Wilk Test  R    W      0.9717603 0
##  Ljung-Box Test     R    Q(10)  17.7885   0.05863789
##  Ljung-Box Test     R    Q(15)  26.08443  0.03714455
##  Ljung-Box Test     R    Q(20)  31.72122  0.04636094
##  Ljung-Box Test     R^2  Q(10)  16.02024  0.09905437
##  Ljung-Box Test     R^2  Q(15)  18.27301  0.2485849
##  Ljung-Box Test     R^2  Q(20)  19.66181  0.4792555
##  LM Arch Test       R    TR^2   16.80097  0.1572388
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.641485 2.649211 2.641482 2.644220
```

```r
#plot(m9)
#13
```



(c) Build an ARMA-GARCH model with Student-t innovations for the log return series. Perform model checking and write down the fitted model.

(d) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with Student-t innovations.

c) • Model checking: Using Ljung box test, H_0 is not rejected at 0.05 level of significance. Hence, there are no linear dependence in the mean and volatility equation. Hence, the model is adequate.

• Fitted model:

$$r_t = \underset{(0.009733)}{0.085537} - \underset{(0.016039)}{0.073702}\, a_{t-1}$$

$$\sigma_t^2 = \underset{(0.003621)}{0.016925} + \underset{(0.012848)}{0.141240}\, a_{t-1}^2 + \underset{(0.012848)}{0.856667}\, \sigma_{t-1}^2$$

```
m10=garchFit(~arma(0,1)+garch(1,1),data=rt3p,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m10)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rt3p, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x000000001f400548>
##  [data = rt3p]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ma1      omega     alpha1      beta1      shape
##  0.085537  -0.073712   0.016925   0.141240   0.856667   5.075507
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.085537    0.009733    8.789  < 2e-16 ***
## ma1    -0.073712    0.016039   -4.596 4.31e-06 ***
## omega   0.016925    0.003621    4.673 2.96e-06 ***
## alpha1  0.141240    0.014598    9.675  < 2e-16 ***
## beta1   0.856667    0.012848   66.675  < 2e-16 ***
## shape   5.075507    0.427751   11.866  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5257.344    normalized:  -1.286358
##
## Description:
##  Tue Apr 27 12:20:56 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1431.666  0
##  Shapiro-Wilk Test  R    W      0.969611  0
##  Ljung-Box Test     R    Q(10)  17.95781  0.05567943
##  Ljung-Box Test     R    Q(15)  26.2946   0.0350378
##  Ljung-Box Test     R    Q(20)  31.68531  0.04676904
##  Ljung-Box Test     R^2  Q(10)  13.12031  0.217023
##  Ljung-Box Test     R^2  Q(15)  17.77068  0.2749137
##  Ljung-Box Test     R^2  Q(20)  20.51701  0.4260342
##  LM Arch Test       R    TR^2   14.78812  0.2532284
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.575652 2.584923 2.575647 2.578935
```

d)

```r
predict(m10,5)
```

```
##   meanForecast meanError standardDeviation
## 1   0.11545471 0.9040206         0.9040206
## 2   0.08553707 0.9148267         0.9123965
## 3   0.08553707 0.9231321         0.9206790
## 4   0.08553707 0.9313464         0.9288705
## 5   0.08553707 0.9394718         0.9369735
```

Question 4.

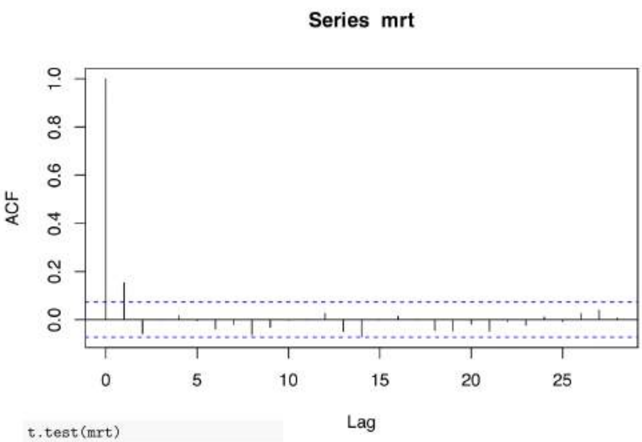
Consider the monthly returns log returns of the CRSP decile 9 portfolio from January 1951 to December 2010. The simple returns are in the file m-deciles.txt under the name CAP9RET.

(a) Is the expected value of the CRSP decile 9 portfolio log return zero? Why? Is there any serial correlation in the log returns? Why? If necessary, find an ARMA model to remove the serial correlations.

- using t-test, p-value is smaller than 0.05 level of significance, so we can reject Ho.
  ∴ The expected value of the CRSP decile 9 port log return $\neq 0$.

$H_0: E(r_t) = 0$
$H_a: E(r_t) \neq 0$

```
#4
da2=read.table("m-deciles.txt",header=T)
mrt=log(da2$CAP9RET+1)
acf(mrt)
```




```
t.test(mrt)
```

```
##
##  One Sample t-test
## data:  mrt
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054
```

- Using Ljung box test, the p-value is less than 0.05. So we can reject the Ho.
  ∴ There is serial correlation in the log returns.

$H_0: \rho_1 = \rho_2 = \ldots = \rho_m = 0$
$H_a: \exists \rho_i \neq 0$.

```r
Box.test(mrt,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  mrt
## X-squared = 24.257, df = 10, p-value = 0.006946
```

(b) Is there any ARCH effect in the log returns? Why?

Ho: there is no ARCH effect
Ha: there is ARCH effect.
Using the Ljung box test, we can not reject Ho at 0.05 level of significance. Hence, there is an ARCH effect.

```r
at=mrt-mean(mrt)
Box.test(mrt^2,lag=10,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  mrt^2
## X-squared = 19.824, df = 10, p-value = 0.03096
```

(c) Build a AR(1)-ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

Model checking - Using Ljung box test, Ho is not rejected at 0.05 level of significance. Hence, there are no linear dependence in the mean and volatility equation. Hence, the model is adequate.
Fitted model:

$$r_t = \underset{(0.0019275)}{0.0105300} + 0.1470670 r_{t-1}$$

$$\sigma_t^2 = \underset{(0.0001482)}{0.0020000} + \underset{(0.0151142)}{0.1815182} a_{t-1}^2$$

```r
m11 = garchFit(~arma(1,0)+garch(1,0),data=mrt,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = mrt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x00000000203cc0a8>
##  [data = mrt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu       ar1     omega    alpha1
```

```
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1     0.1470670   0.0424301    3.466 0.000528 ***
## omega   0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1  0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Tue Apr 27 12:20:56 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480
```

(d) Fit a AR(1)-ARCH(1) model with Standardized Student-t innovations to the log return series.

Perform model checking and write down the fitted model.

Model checking - Using Yung Box test, Ho is not rejected at 0.05 level of significance. Hence, there are no linear dependence in the mean and volatility equation. Hence, the model is adequate.

Fitted model:

$$r_t = \underset{(0.0017710)}{0.0116186} + \underset{(0.0403169)}{0.1076982}\, r_{t-1}$$

$$\sigma_t^2 = \underset{(0.0001818)}{0.0019203} + \underset{(0.0713692)}{0.1900830}\, a_{t-1}^2$$

```
m12 = garchFit(~arma(1,0)+garch(1,0),data=mrt,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = mrt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x0000000021747ef8>
##  [data = mrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1     0.1076982   0.0403169    2.671  0.00756 **
## omega   0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1  0.1900830   0.0713692    2.663  0.00774 **
## shape   6.4225253   1.3115905    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Tue Apr 27 12:20:57 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
```

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

(e) Build a ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

Model checking : Using Ljung Box test, H0 is not rejected at 0.05 level of significance. Hence, there are no linear dependence in the mean and volatility equation. Hence, the model is adequate.

Fitted model:

$$r_t = \underset{(0.001923)}{0.012346}$$

$$\sigma_t^2 = \underset{(0.000145)}{0.002016} + \underset{(0.062209)}{0.194126}\, a_{t-1}^2$$

```
m13 = garchFit(~garch(1,0),data=mrt,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m13)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = mrt, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x000000001fa421c8>
##  [data = mrt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Tue Apr 27 12:20:57 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
```

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

(f) Fit a ARCH(1) model with Standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

Model checking: Using Ljung Box test, Ho is not rejected at 0.05 level of significance. Hence, there are no linear dependence in the mean and volatility equation. Hence, the model is adequate.

Fitted model:

$$r_t = \underset{(0.001685)}{0.013356}$$

$$\sigma_t^2 = \underset{(0.000185)}{0.001928} + \underset{(0.072375)}{0.204163}\, a_{t-1}^2$$

```r
m14 = garchFit(~garch(1,0),data=mrt,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m14)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = mrt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x00000000205718d0>
##  [data = mrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Tue Apr 27 12:20:57 2021 by user: Orachai
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705
```

(g) Compare the model (c)-(f) which model you select.

The model in d) w/ model AR(1)- ARCH(1) and student t innovation.
is the model with lowest AIC.