HW 2 Sec 02 Semester 2/2019 Nichanun Subsomboon 6104640716

1.

Table 1

Student	Y_i	X_i	X_iY_i	X_i^2	$x_i = X_i - \bar{X}$	$y_i = Y_i - \bar{Y}$	x_iy_i	x_i^2
1	2.8	63	176.4	3,969	-14.625	-0.4125	6.0328125	213.890625
2	3.4	72	244.8	5,184	-5.625	0.1875	-1.0546875	31.640625
3	3.0	78	234	6,084	0.375	-0.2125	-0.0796875	0.140625
4	3.5	81	283.5	6,561	3.375	0.2875	0.9703125	11.390625
5	3.6	87	313.2	7,569	9.375	0.3875	3.6328125	87.890625
6	3.0	75	225	5,625	-2.625	-0.2125	0.5578125	6.890625
7	2.7	75	202.5	5,625	-2.625	-0.5125	1.3453125	6.890625
8	3.7	90	333	8,100	12.375	0.4875	6.0328125	153.140625
Σ	25.7	621	2,012.4	48,717		0	17.4375	511.875
μ	3.2125	77.625						

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}, \quad \hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$

$$\therefore \hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{17.4375}{511.875} = 0.034$$

$$\hat{\beta}_1 = 3.2125 - (0.034)(77.625) = 0.57325$$

Interpretation :

$\hat{\beta}_1 = 0.57325$: If the student has no point, the lowest GPA they can get is 0.57

$\hat{\beta}_2 = 0.034$: On average, if the student's score increase by 1, their GPA is increasing 0.034, holding all other factors that might effect the GPA constant.

1.2) Find $\hat{y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

Sol$^{\underline{n}}$ $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

Student	y_i	X_i	$\hat{\beta}_1 + \hat{\beta}_2 X_i = \hat{y}_i$	$\hat{u}_i = y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$	$(\hat{u}_i)^2$
1	2.8	63	2.71525	0.08475	0.0071825625
2	3.4	72	3.02125	0.37875	0.1434515625
3	3.0	78	3.22525	-0.22525	0.0507375625
4	3.5	81	3.32725	0.17275	0.0298425625
5	3.6	87	3.53125	0.06875	0.0047265625
6	3.0	75	3.12325	-0.12325	0.0151905625
7	2.7	75	3.12325	-0.42325	0.1791405625
8	3.7	90	3.63325	0.06675	0.0044555625

$\therefore \Sigma \hat{u}_i = 0$ $\Sigma \hat{u}_i^2 = 0.4347275$

$y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$

$\therefore \hat{u}_i = y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$

1.3) $Var(\hat{u}_i)$

$$Var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.434727S}{8-2} = 0.07245458333$$

$$Var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2 = \frac{48,717}{8(511.875)} (0.07245458333)$$

$$= 0.86197068$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} = \frac{0\ 07245458333}{511.875} = 0.0001415474$$

$n = 10$

$\boxed{2.}$

	X_i	y_i	$x_i = X_i - \bar{X}$	$y_i = y_i - \bar{y}$	$x_i y_i$	x_i^2	X_i^2
	10	0	-10	-9.1	91	100	100
	12	2	-8	-7.1	56.8	64	144
	14	5	-6	-4.1	24.6	36	196
	16	6	-4	-3.1	12.4	16	256
	18	7	-2	-2.1	4.2	4	324
	22	10	2	0.9	1.8	4	484
	24	10	4	0.9	3.6	16	576
	26	15	6	5.9	35.4	36	676
	28	16	8	6.9	55.2	64	784
	30	20	10	10.9	109	100	900
Σ	200	91			394	440	4,440
μ	20	9.1					

2.1) Find $\hat{\beta}_1, \hat{\beta}_2$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X}$$

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$

$$\therefore \hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{394}{440} = 0.8954545\ldots$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X} = 9.1 - \left(\frac{394}{440}\right) 20 = 9.1 - 17.90909 = -8.80909\ldots$$

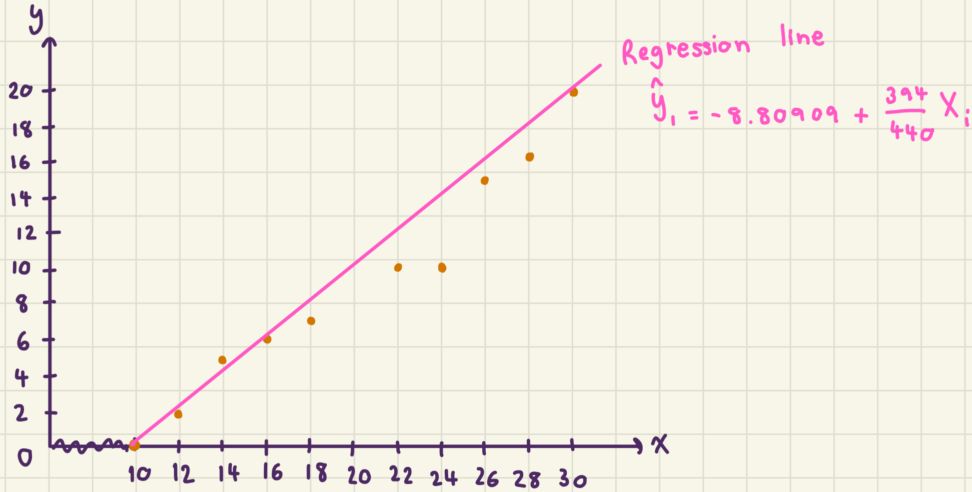
2.2) Find $\hat{y}_i$ and $\hat{u}_i$, Show that $\Sigma\mu_i \approx 0$

X_i	y_i	$\hat{y}_1 = \hat{\beta}_1 + \hat{\beta}_2 X_i$	$\mu_i = y_1 - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$	$\hat{\mu}_i^2$
10	0	0.14545	-0.14545	0.0211557
12	2	1.93636	0.06364	0.00405
14	5	3.72727	1.27273	1.619842
16	6	5.51818	0.48182	0.23215
18	7	7.30909	-0.30909	0.09554
22	10	10.890909	-0.890909	0.79372
24	10	12.681818	-2.681818	7.19214
26	15	14.472727	0.527273	0.27802
28	16	16 263636	-0.263636	0.069504
30	20	18.054545	1.945455	3.784795
Σ 200	91		$\Sigma\hat{\mu}_i = 0.000015$	$\Sigma\hat{\mu}_i^2 = 14.0909$
μ 20	9.1		$\therefore \Sigma\hat{\mu}_i \approx 0$	

$$y_i = \hat{y} + \hat{\mu}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\mu}_i$$

$$\therefore \hat{\mu}_i = y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$$

2.3) Plot graph and draw a regression line. Does the line pass $(\bar{x}, \bar{y})$?



2.4) If $X_i = 18$, $\hat{y}_i = ?$

$$\hat{y}_i = \beta_1 + \beta_2 X_i$$

$$= -8.80909 + \left(\frac{394}{440} \times 18\right)$$

$$\therefore \hat{y}_i = 7.30909$$

2 5) $$Var(\hat{u}_i) = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{14.0909}{10-2} = 1.76136$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2 = \frac{4,440}{10 \times 440} \times 1.76136 = 1.76136$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} = \frac{1.76136}{440} = 0.004003$$

$\boxed{3.}$

$$y_i = \beta_1 + \beta_2 X_i + \mu_i \quad , \; u_i \sim NIID(0, \sigma^2)$$

Assumption #1 : Linear regression model : $y_i = \beta_1 + \beta_2 X_i + \mu_i$

$$\hat{\beta}_1 = \frac{\Sigma X_i^2 \Sigma y_i - \Sigma X_i \Sigma X_i y_i}{n \Sigma X_i^2 - (\Sigma X_i)^2}$$

$$= \bar{y} - \hat{\beta}_2 \bar{X}$$ ← Assumption #4 : Homoscedasticity,

Assumption # 8: Variabilities in x Value

Proof that this is an unbiased estimator.

$$E(\hat{\beta}_1) = E(\bar{y} - \hat{\beta}_2 \bar{X})$$

Assumption #9: The regression model is correctly specified

$$= E(\bar{y}) - \hat{\beta}_2 E(\bar{X})$$

$$= \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2$$

$$\therefore E(\hat{\beta}_1) = \beta_1$$