

Name \_\_\_\_\_ Surname \_\_\_\_\_ Student ID. \_\_\_\_\_

DUE DATE : Thursday 3<sup>rd</sup>, March 2016.

### Assignment 3: (65 marks)

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Student Signature: \_\_\_\_\_

1(20 Marks) Midterm: Fall-2015

(a).What is the Gauss-Markov Theorem?

(b).Show that the OLS intercept coefficient estimator  $\hat{\beta}_1$  is a linear function of the sample observations  $Y_i$  and  $X_i$ .

(c). Stating explicitly all required assumptions, prove that the OLS intercept coefficient estimator  $\hat{\beta}_1$  is an unbiased estimator of the true parameter  $\beta_1$ .

4. (45 Marks) Midterm: Fall-2015

A researcher is using data for a sample of 16 B.E students to investigate the relationship between the GPA (grade point average)  $Y_i$  where the grade point average is based on a four-point scale and the SAT scores  $X_i$ . Preliminary analysis of the sample data produces the following sample information:

$$\sum Y_i = 50.90 \quad \sum X_i = 388.00 \quad \sum Y_i^2 = 164.51$$

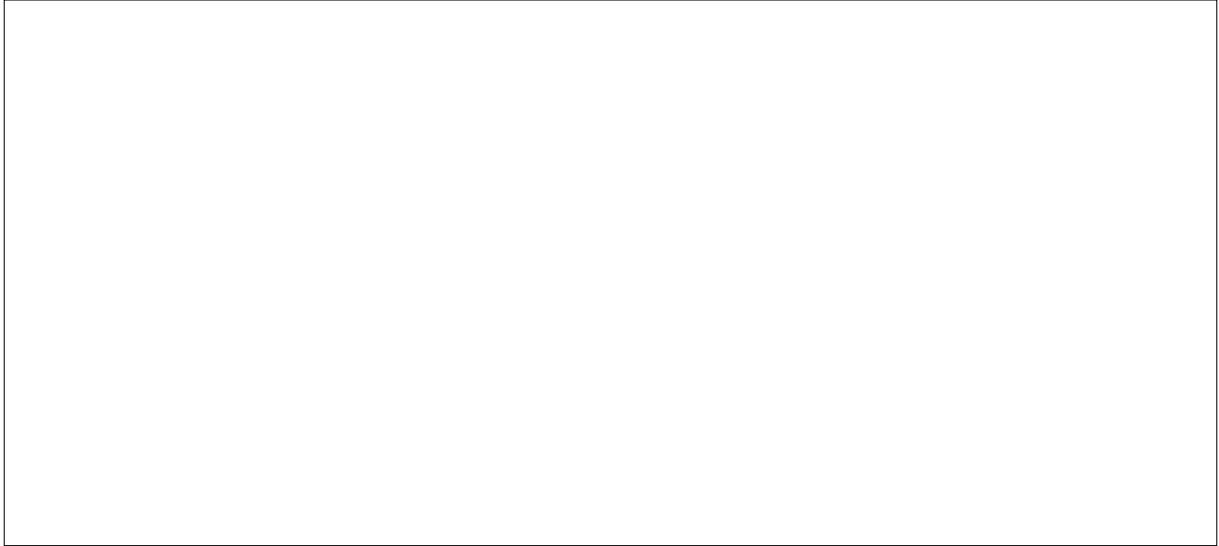
$$\sum X_i^2 = 9620.00 \quad \sum X_i Y_i = 1254.90 \quad \sum x_i y_i = 20.58$$

$$\sum y_i^2 = 2.5844 \quad \sum x_i^2 = 211.00 \quad \sum \hat{y}_i^2 = 2.0063$$

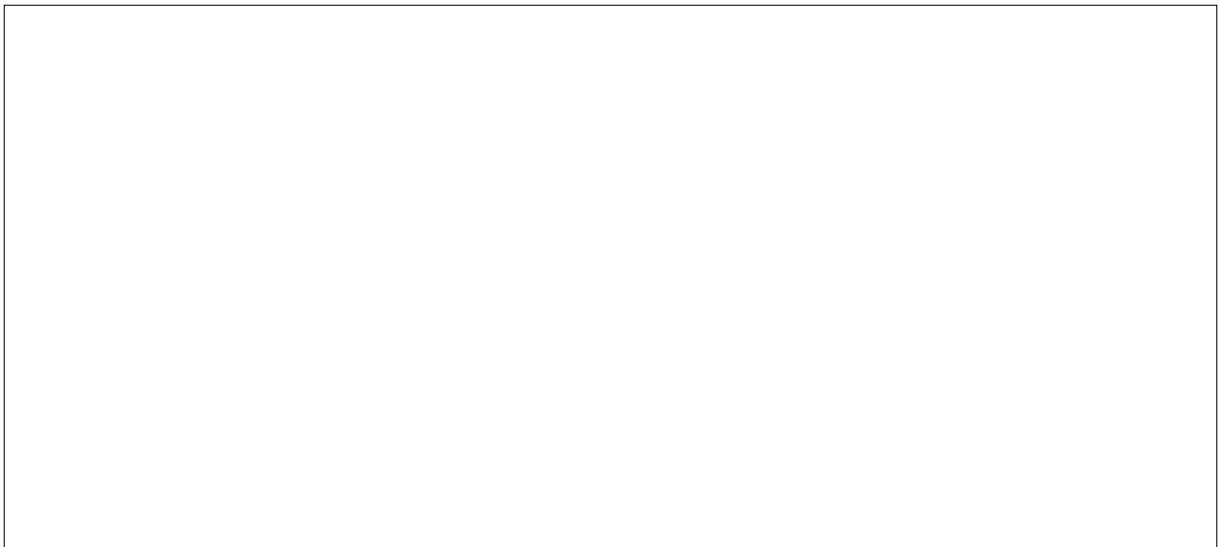
where  $x_i = X_i - \bar{X}$ ,  $y_i = Y_i - \bar{Y}$ , and  $\hat{y}_i = \hat{Y}_i - \bar{Y}$

Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations**

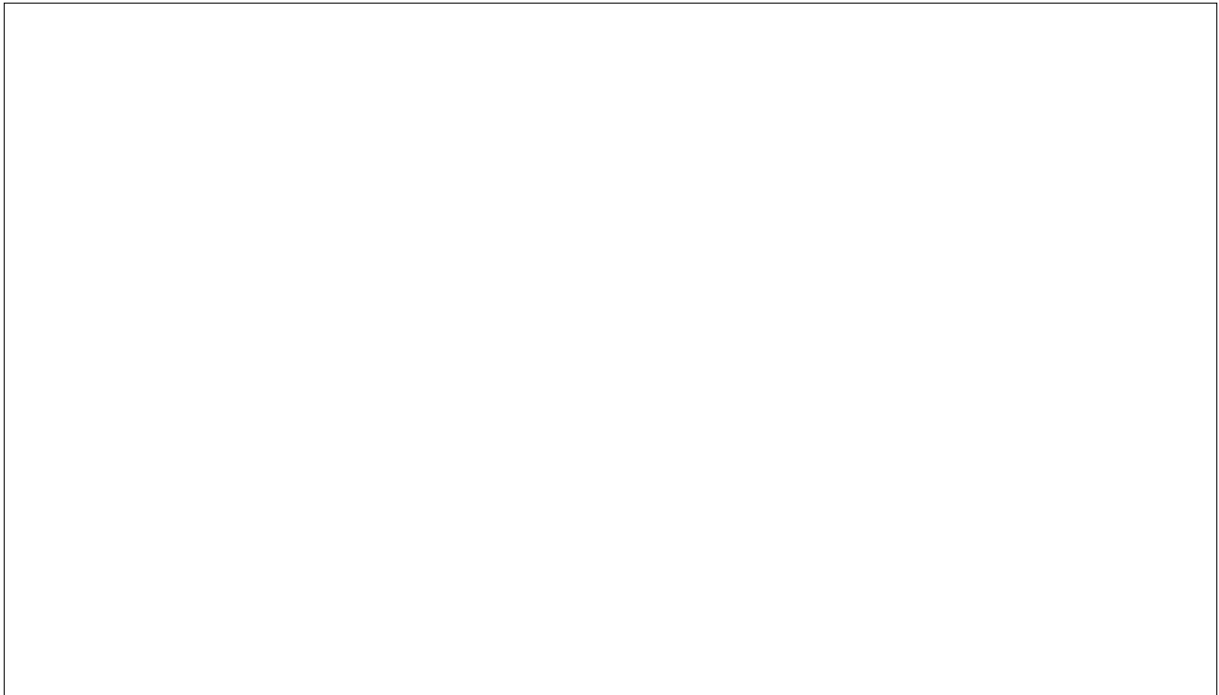
(a) **(5 Marks)** Use the above information to compute OLS estimates of the intercept coefficient  $\beta_1$  and that of the slope coefficient  $\beta_2$ .



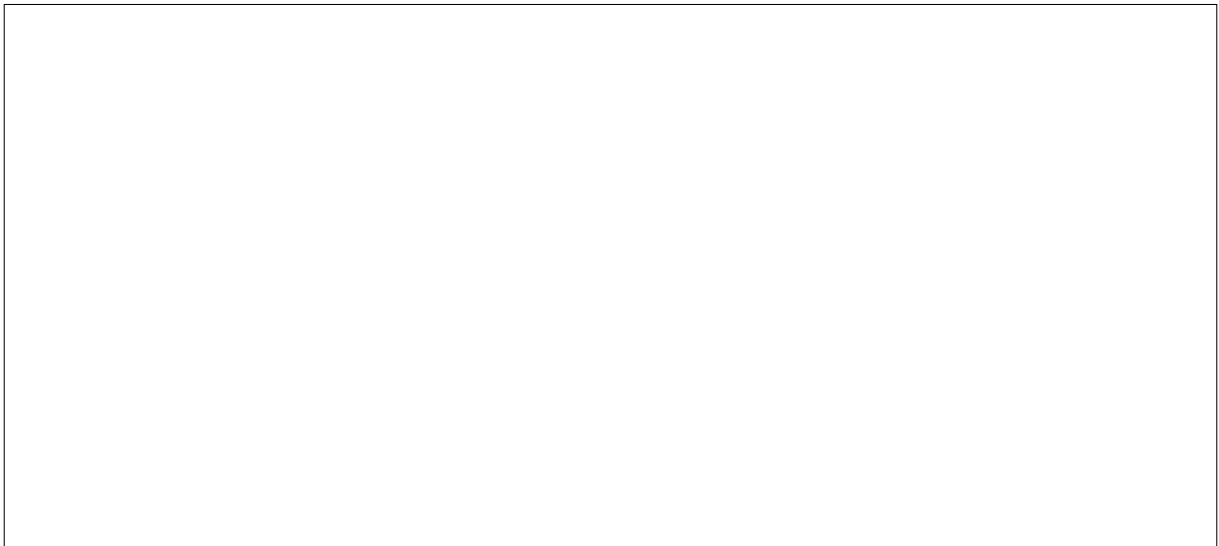
(b) **(5 Marks)** Interpret the slope coefficient estimate you calculated in part(a)—i.e., explain in words what the numeric value you calculated for  $\hat{\beta}_2$  means.



(c) **(10 Marks)** Calculate an estimate of  $\sigma^2$ .



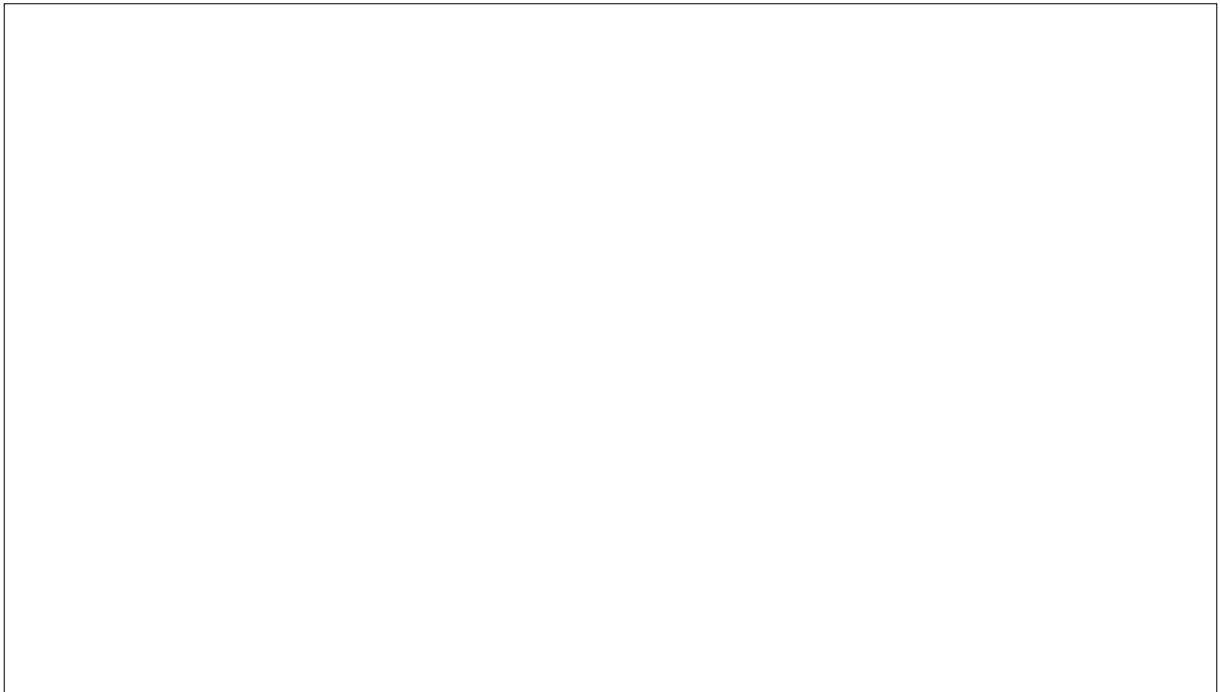
(d) **(5 Marks)** Compute the value of  $r^2$ . Briefly explain what the calculated value of  $r^2$  means.



(e) **(5 Marks)** Compute the estimated variance of  $\hat{\beta}_1$  and the estimated variance of  $\hat{\beta}_2$ .



(f) **(5 Marks)** Compute the two-sided 90% confidence interval for the intercept coefficient  $\beta_1$ . Briefly explain what the two-sided 90% confidence interval means.



(g)(5 Marks) Perform a test of the null hypothesis  $H_0 : \beta_1 = 0$  against the alternative hypothesis  $H_1 : \beta_1 \neq 0$  at the 1 % significance level (i.e., for significance level  $\alpha = 0.01$ ). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

(h) **(5 Marks)** Perform a test of the null hypothesis  $H_0 : \sigma^2 \geq 0.025$  against the alternative hypothesis  $H_1 : \sigma^2 < 0.025$  at the 5 % significance level (i.e., for significance level  $\alpha = 0.05$  ). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.