Name [redacted] **Surname** [redacted] **Student ID.** [redacted]
DUE DATE : Thursday 3^{rd}, March 2016.

Assignment 3: (65 marks)

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Student Signature: [redacted]

1**(20 Marks) Midterm: Fall-2015**
(a). What is the Gauss-Markov Theorem?

(b). Show that the OLS intercept coefficient estimator $\hat{\beta}_1$ is a linear function of the sample observations Y_i and X_i.

(c). Stating explicitly all required assumptions, prove that the OLS intercept coefficient estimator $\hat{\beta}_1$ is an unbiased estimator of the true parameter β_1.

4. **(45 Marks) Midterm: Fall-2015**
A researcher is using data for a sample of 16 B.E students to investigate the relationship between the GPA (grade point average) Y_i where the grade point average is based on a four-point scale and the SAT scores X_i. Preliminary analysis of the sample data produces the following sample information:

$$\sum Y_i = 50.90 \qquad \sum X_i = 388.00 \qquad \sum Y_i^2 = 164.51$$

$$\sum X_i^2 = 9620.00 \qquad \sum X_i Y_i = 1254.90 \qquad \sum x_i y_i = 20.58$$

$$\sum y_i^2 = 2.5844 \qquad \sum x_i^2 = 211.00 \qquad \sum \hat{y_i}^2 = 2.0063$$

where $x_i = X_i - \bar{X}$,$y_i = Y_i - \bar{Y}$, and $\hat{y_i} = \hat{Y_i} - \bar{Y}$
Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations**

(a) **(5 Marks)** Use the above information to compute OLS estimates of the intercept coefficient β_1 and that of the slope coefficient β_2.

(b)**(5 Marks)** Interpret the slope coefficient estimate you calculated in part(a)–i.e.,explain in words what the numeric value you calculated for $\hat{\beta}_2$ means.

(c) **(10 Marks)** Calculate an estimate of σ^2.

(d) **(5 Marks)** Compute the value of r^2. Briefly explain what the calculated value of r^2 means.

(e) **(5 Marks)** Compute the estimated variance of $\hat{\beta}_1$ and the estimated variance of $\hat{\beta}_2$.

(f)**(5 Marks)** Compute the two-sided 90% confidence interval for the intercept coefficient β_1. Briefly explain what the two-sided 90% confidence interval means.

(g)**(5 Marks)** Perform a test of the null hypothesis $H_0 : \beta_1 = 0$ against the alternative hypothesis $H_1 : \beta_1 \neq 0$ at the 1 % significance level (i.e.,for significance level $\alpha = 0.01$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

(h) **(5 Marks)** Perform a test of the null hypothesis $H_0 : \sigma^2 \geq 0.025$ against the alternative hypothesis $H_1 : \sigma^2 < 0.025$ at the 5 % significance level (i.e.,for significance level $\alpha = 0.05$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.