B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 2/2013

Practice Problem 7

(Derivatives of More-Than-One Independent Variable Function)

1. Suppose that Mr. A's utility depends on two commodities: x_1 and x_2. His utility function is given by

$$U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2$$

(a) Determine the marginal utility function of each commodity.

(b) Find the marginal rate of substitution (MRS) between the two commodities when $x_1 = 2$ and $x_2 = 1$.

2. Write the Hessian matrix for each of the following functions:

(a) $U(x, y) = 6x^2 + 5xy + 9y^2$

(b) $z(x, y) = 3(11x - 6y)^2$

3. Find the total differentials for the following functions:

(a) $z = 4x^3 - 13xy - 6y^5$

(b) $z = (2x^2 - y)(3x - 4y^3)$

(c) $z = \frac{7y^3}{(5x-2y)}$

(d) $z = 8x^{\frac{1}{2}}y^{\frac{1}{4}}w^{\frac{1}{4}}$

4. Given the equation for the production isoquant

$$18[0.2K^{-0.4} + 0.8L^{-0.4}]^{-2.5} = 1936$$

Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of L for K.

5. Let the demand for a commodity be

$$Q_d = D(P, Y_0, t_0), \qquad \frac{\partial D}{\partial P} < 0; \frac{\partial D}{\partial Y_0} > 0; \frac{\partial D}{\partial t_0} > 0$$

$$Q_s = S(P, T_0), \qquad \frac{\partial S}{\partial P} > 0; \frac{\partial S}{\partial T_0} < 0$$

Where P is the price, Y_0 is the consumer's income, t_0 is the taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Find $\frac{\partial P^*}{\partial t_0}, \frac{\partial P^*}{\partial T_0}, \frac{\partial Q^*}{\partial Y_0}$, and $\frac{\partial Q^*}{\partial T_0}$. Discuss their economic implications.

*Questions 6-14 are from Sydsaeter and Hammond, 2008. Questions 15-16 are from Wainwright. * indicates more difficult problems.*

6. The demand for money, M, in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r-2)^{-0.84}, \qquad (r > 2)$$

where Y is the annual national income, and r is the interest rate measured in percent per year. Find $\partial M/\partial Y$ and $\partial M/\partial r$ and discuss their signs.

7. The demand for a product depends on the price p of the product and on the price q charged by a competing producer

$$D(p,q) = a - bpq^{-\alpha}$$

where a, b, and α are positive constants with $\alpha < 1$. Find $D'_p(p,q)$ and $D'_q(p,q)$, and comment on the signs of the partial derivatives.

8. Let x and y be the populations of two cities and d the distance between them. Suppose that the number of travelers T between the cities is given by

$$T = kxy/d^n \qquad \text{(k and n are positive constants.)}$$

Find $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial d$, and discuss their signs.

*9. Let *D(p, m)* indicate a typical consumer's demand for a particular commodity, as a function of its price *p* and the consumer's own income *m*. Show that the proportion *pD/m* of income spent on the commodity increases with income if $\varepsilon_m > 1$ (in which case the good is a "luxury" whereas it is a "necessity" if $\varepsilon_m < 1$).

10. The annual herring catch is given by the function $Y(K,S) = 0.06157K^{1.356}S^{0.562}$of the catching effort (K) and the herring stock (S).

(a) Find $\partial Y/\partial K$ and $\partial Y/\partial S$.

(b) If K and S are both doubled, what happens to the catch?

11. Find the marginal rate of substitution (MRS) between *y* and *x* when:

(a) $U(x,y) = 2x^{0.4}y^{0.6}$
(b) $U(x,y) = xy + y$
(c) $U(x,y) = 10(x^{-2} + y^{-2})^{-4}$

12. Suppose that a firm produces $Q = f(L) = \sqrt{L}$ units of commodity using L units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so f is strictly increasing and strictly concave. If the firm gets *P* baht per unit produced and pays *w* baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$.

*13. Suppose production *X* depends on the number of workers *N* according the formula

$$X = Ng(\frac{\varphi(N)}{N})$$

where g and φ are given differentiable functions. Find expressions for dX/dN and d^2X/dN^2.

*14. An equilibrium model of labor demand and output pricing leads to the following system of equations:

$$pF'(L) - w = 0$$
$$pF(L) - wL - B = 0 \qquad (*)$$

Suppose that F is twice differentiable with $F'(L) > 0$ and $F''(L) < 0$, and all the variables are positive. Treat w and B as exogenous, so that p and L are endogenous variables which are functions of w and B.

(a) Find expressions for $\partial p/\partial w$, $\partial p/\partial B$, $\partial L/\partial w$, and $\partial L/\partial B$ by implicit differentiation.

(b) What can be said about the signs of these partial derivatives? Show that $\partial L/\partial w < 0$.

15. If $z = x^3y^2 + x^2y^4 - 3xy$ and $x = r + 3s$ and $y = 2r - s$, then determine $\frac{\partial z}{\partial s}$ when r = 1 and s = 0.

16. If $x^2 + xy + yz + x^2 = 6$, then:

(a) Find $\frac{\partial z}{\partial y}$

(b) Evaluate $\frac{\partial z}{\partial y}$ at x =1, y=2, z =1.