EE 325 Answer HW 2 (Additional Questions highlighted in blue)

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0,\sigma^2)$ Use OLS to find the estimator of $\beta_0 and \beta_1$. Interpret the regression.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 3.2125 - 0.0341(77.625) = 0.5681$$

Interpretation:

When total microeconomics exam point is equal to zero, student's GPA is 0.5681. When total microeconomics exam point increase 1 point, student's GPA increases approximately 0.0341.

1.2 Find $\hat{Y}_i$ *and* $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

Construct the table that contain $\hat{Y}_i$ *and* $\hat{u}_i$, you will get $\sum_{i=1}^{n} \hat{u}_i = 8.327 \times 10^{-16}$

1.3 Find $\text{var}(\hat{u}_i), \text{var}(\hat{\beta}_0), and \ \text{var}(\hat{\beta}_1)$

$$\text{var}(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = \frac{0.4347253}{8-2} = 0.0725$$

We use $\text{var}(\hat{u}_i)$ as the estimator of σ^2

$$\text{var}(\hat{\beta}_0) = \frac{\sum_{i=1}^{n} X_i^2}{n\sum_{i=1}^{n} x_i^2}\hat{\sigma}^2 = \frac{48717(0.0725)}{8(511.875)} = 0.862; (x_i = X_i - \bar{X}, y_i = Y_i - \bar{Y})$$

$$\text{var}\left(\hat{\beta}_1\right) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} x_i^2} = \frac{0.0725}{511.875} = 0.00014163$$

1.4 Test the hypothesis that total microeconomics exam point has no influence on GPA at $\alpha = 5\%$

$H_0 : \beta_1 = 0,\ H_1 : \beta_1 \neq 0$

$$t = \frac{0.0341 - 0}{\sqrt{0.00014163}} = 2.863324$$

$$t_{\frac{0.05}{2}, 8-2} = 2.447$$

$$t = 2.863324 > t_{\frac{0.05}{2}, 8-2} = 2.447$$

Reject the null hypothesis. Total microeconomics exam point has influence on GPA at $\alpha = 5\%$

1.5 What percentage of the total variation in Y explained by the regression model?

$$R^2 = \frac{ESS}{TSS} = \frac{\sum_{i=1}^{8} \hat{y}_i^2}{\sum_{i=1}^{8} y_i^2} = \frac{0.59402}{1.02875} = 0.5774$$

The total variation in Y explained by the regression model is 57.74%.

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$ Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = 0.8955$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_2 \bar{X} = -8.8091$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$\bar{X} = 20,\ \bar{Y} = 9.1$

2.4 If $X_i = 16$, what is the predicted Y?

$Y_i = -8.8091 + 0.8955(16) = 5.5189$

2.5 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_0), \text{var}(\hat{\beta}_1)$

$$\text{var}(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = 1.7614$$

$$\text{var}(\hat{\beta}_0) = \frac{\sum_{i=1}^{n} X_i^2}{n \sum_{i=1}^{n} x_i^2} \hat{\sigma}^2 = 1.7774$$

$$\text{var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} x_i^2} = 0.0040$$

3. Given Y is wages per hour ($), X is year of schooling (years) from a sample of 528 observations

$$\hat{Y}_t = 0.7437 + 0.6416 X_t$$
$$se = (0.8355)(0.0664)$$
$$r^2 = 0.8944, \hat{\sigma}^2 = 0.8040$$
$$\bar{X} = 12 \quad \sum x_i^2 = 2054$$

3.1 Test the hypothesis that year of schooling has a positive influence on wages per hour at $\alpha = 1\%$

$H_0 : \beta_1 \leq 0$

$H_a : \beta_1 > 0$

$$t = \frac{0.6416 - 0}{0.0664} = 9.6625$$

Critical t value is $t_{0.05, df=526} = 2.326$

$$t = \frac{0.6416 - 0}{0.0664} = 9.6625 > t_{0.05, df=526} = 2.326$$

Reject the null hypothesis. Year of schooling has a positive influence on wages per hour at $\alpha = 1\%$

3.2 Interpret the regression.

$\beta_2 = 0.6416$

When year of schooling increases 1 year, wages per hour ($) will increase approximately $ 0.6416.

3.3 If Miss Lily has 8 years of schooling, what is the predicted average on wages per hour ($)?

$E(Y \mid X = 8) = 5.8765$ USD/hr.