Solution: Quiz 2 (Part 2)

1. Let $B = \{1,2,3\}$ and $C = \{-1,0,1\}$. Let the set $D = \{-8,-6,-3,-1,0,1,3,6,8\}$ be the domain for the predicate $P(y)$ which is defined as

$$P(y): \ \sim\Big([\forall x \in B,\ xy \ > \ 6] \ \vee \ [\exists z \in C,\ y+z \ \leq \ 0]\Big).$$

Find the truth set of the predicate $P(y)$.

Solution: First, we can simplify $P(y)$ as shown below by using De Morgan's law.

$$\begin{aligned} P(y): \quad & \quad \sim\Big([\forall x \in B,\ xy \ > \ 6] \ \vee \ [\exists z \in C,\ y+z \ \leq \ 0]\Big) \\ \equiv & \quad \sim[\forall x \in B,\ xy \ > \ 6] \vee \sim[\exists z \in C,\ y+z \ \leq \ 0] \\ \equiv & \quad [\exists x \in B,\ xy \ \leq \ 6] \ \wedge \ [\forall z \in C,\ y+z \ > \ 0] \end{aligned}$$

To find the truth set T_p of $P(y)$, we must have both
(1) $\exists x \in B,\ xy \ \leq \ 6$ and (2) $\forall z \in C,\ y+z \ > \ 0$ true.

(1) Consider $\exists x \in B,\ xy \ \leq \ 6$.
Consider each element of $B = \{1,2,3\}$.
- (i) For $x = 1$, $(1)y \leq 6$ is true when $y \leq 6$.
- (ii) For $x = 2$, $(2)y \leq 6$ is true when $y \leq 3$.
- (iii) For $x = 3$, $(3)y \leq 6$ is true when $y \leq 2$.
That is, since (1) is an existential statement, it is true when at least one case in (i) -(iii) is true, i.e. $y \leq 6$, or

$$y \in (-\infty, 6] \cup (-\infty, 3] \cup (-\infty, 2] = (-\infty, 6]$$

(2) Consider $\forall z \in C,\ x+y \ > \ 0$.
Consider each element of $B = \{-1,0,1\}$.
- (i) For $z = -1$, $-1+y > 0$ is true when $y > 1$.
- (ii) For $z = 0$, $0+y > 0$ is true when $y > 0$.
- (iii) For $z = 1$, $1+y > 0$ is true when $y > -1$.
That is, since (2) is a universal statement, it is true when all cases in (i) -(iii) are true, i.e. $y > 1$, or

$$y \in (1,\infty) \cap (0,\infty) \cap (-1,\infty) = (1,\infty)$$

Since the truth set T_p of $P(y)$ comes from $y \in D$ that makes both (1) and (2) true, then we must use

$$y \in \{(-\infty,6] \cap (1,\infty)\} \ \cap \ D = (1,6] \ \cap \ \{-8,-6,-3,-1,0,1,3,6,8\} = \{3,6\}$$

and the truth set is

$$T_p = \{3,6\}.$$

$\blacksquare$