B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Quiz 4 (b)

1. (3 point) Find the total derivative of y w.r.t. x when

$y = f(u, v, x) = u^2 - 3v^{1/3} - 2x^2$ where $u = g(x)$ and $v = h(x)$.

$$\frac{dy}{dx} = \frac{\partial f}{\partial u}\cdot\frac{du}{dx} + \frac{\partial f}{\partial v}\cdot\frac{dv}{dx} + \frac{\partial f}{\partial x}$$

$$= 2u\cdot g'(x) - (3)\left(\frac{1}{3}\right)v^{-2/3}\cdot h'(x) - 4x$$

$$= 2g(x)\cdot g'(x) - \frac{h'(x)}{[h(x)]^{2/3}} - 4x$$

2. (3 points) Find total differential of $y = (x_1 + 2x_2)(3x_2)(x_3^2 - 2)$.

Let $u = x_1 + 2x_2$, $v = 3x_2$, $w = x_3^2 - 2$

$\Rightarrow du = dx_1 + 2dx_2$, $dv = 3dx_2$, $dw = 2x_3 dx_3$.

$$dy = uv\,dw + uw\,dv + vw\,du$$

$$= (x_1+2x_2)(3x_2)(2x_3dx_3) + (x_1+2x_2)(x_3^2-2)(3dx_2) + (3x_2)(x_3^2-2)(dx_1+2dx_2)$$

$$= [(3x_2)(x_3^2-2)]dx_1 + [(3x_1+6x_2+6x_2)(x_3^2-2)]dx_2 + [(6x_2x_3)(x_1+2x_2)]dx_3$$

$$= \underbrace{[(3x_2)(x_3^2-2)]}_{\frac{\partial y}{\partial x_1}}dx_1 + \underbrace{[(3x_1+12x_2)(x_3^2-2)]}_{\frac{\partial y}{\partial x_2}}dx_2 + \underbrace{[(6x_2x_3)(x_1+2x_2)]}_{\frac{\partial y}{\partial x_3}}dx$$

3. (4 points) Given $F(z, x, y) = 2x^2 + y^2 + z^2 - 2xz + 5xy$, use the implicit function rule to determine $\frac{\partial z}{\partial y}$ and $\frac{\partial x}{\partial y}$.

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{2y + 5x}{2z - 2x}$$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_x} = -\frac{2y + 5x}{4x - 2z + 5y}$$