

Chapter 9 Testing Hypotheses : One Sample ProceduresSection 9.1 Hypothesis – Testing Procedure in General

- The goal of testing statistical hypotheses is to determine whether a claim or conjecture about some feature of the population, a parameter, is strongly supported by the information obtained from the sample data

Steps in the Classical Hypothesis – Testing ProcedureStep 1 State the null and alternative hypotheses

- Alternative hypothesis (H_1) – the statement to be proved
- Null hypothesis (H_0 or H_a) – negation of H_1

Ask : Based on a given sample

- Is there strong evidence for rejecting H_0 ?
- Is there strong evidence to support H_1 ?

Once H_0 and H_1 are formulated, our goal is to analyze the sample data to choose between them.

Eg. H_0 : person is guilty

H_1 : person is not guilty

H_0 : Adam weights 130 lbs.

H_1 : Adam weights less than 130 lbs.

H_0 : $\mu = 35$

H_1 : $\mu \neq 35$ two – tailed test

Or $\mu < 35$ left – tailed test } One – tailed test

Or $\mu > 35$ right – tailed test }

Step 2 Select a level of significance (α)

	H_0 is True	H_0 is False
H_0 is rejected	Type I error	Correct decision
H_0 is not rejected	Correct decision	Type II error

$P(\text{Type I error}) = P(\text{Reject } H_0 \text{ when } H_0 \text{ is true}) = \alpha = \text{level of significance}$

$P(\text{Type II error}) = P(\text{Fail to reject } H_0 \text{ when } H_0 \text{ is false}) = \beta$

Step 3 Determine the test statistic

- Choose between z and t

When σ is known

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

When σ is unknown and $n < 30$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

When σ is unknown and $n \geq 30$

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Define the rejection or critical region

The curve is divided into two parts

- 1) Rejection region has area under the curve in the tail equal to α for one-tailed test or $\frac{\alpha}{2}$ for two-tailed test
- 2) Acceptance region is the rest of the area

Step 4 Computation

Computation z or t

Step 5 Decision

- Is your computed value in rejection region ?

If yes $\Rightarrow$ reject H_0

If no $\Rightarrow$ fail to reject H_0

- state the conclusion in English

Step 6 Calculate p-value

- For $H_1 : \mu < \mu_0$

$$\text{p-value} = P(Z \leq z_{obs})$$

- For $H_1 : \mu > \mu_0$

$$\text{p-value} = P(Z \geq z_{obs})$$

- For $H_1 : \mu \neq \mu_0$

$$\begin{aligned}\text{p-value} &= P(Z \leq -z_{obs} \text{ or } Z \geq z_{obs}) \\ &= P(Z \leq -z_{obs}) + P(Z \geq z_{obs}) \\ &= 2P(Z \geq z_{obs})\end{aligned}$$

If $\text{p-value} < \alpha$, reject H_0

If $\text{p-value} \not< \alpha$, fail to reject H_0