

Natthaphan
6109 641086



EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

Normal

1.2 Find \hat{Y}_i and \hat{u}_i and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

distribution

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of \hat{Y}_i and \hat{u}_i . Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass (\bar{X}, \bar{Y}) ?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

Table 1

Student	Y_i	X_i	$X_i Y_i$	X_i^2	$X_i - \bar{X} = \delta_i$	δ_i^2
1	2.8	63	176.4	3969	-14.625	213.8906
2	3.4	72	244.8	5184	-5.625	31.6406
3	3.0	78	279	6084	0.975	0.1406
4	3.5	81	283.5	6561	3.975	15.8006
5	3.6	87	313.2	7569	4.975	24.7506
6	3.0	75	225	5625	-2.625	6.8906
7	2.7	75	202.5	5625	-2.625	6.8906
8	3.7	90	333	8100	12.375	153.1406
Sum	25.7	621	2012.4	48717		511.9706
Mean	2.2125	77.625	251.55	6089.625		

$$1.1) \hat{\beta}_2 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$= \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389776 - 385601}$$

$$= \frac{139.5}{4095}$$

$$\hat{\beta}_2 = 0.0741$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 3.2125 - (0.0741)(77.625)$$

$$= 3.2125 - 2.6345$$

$$\hat{\beta}_1 = 0.5655$$

∴ Regression equation is, $\hat{Y}_i = 0.5655 + 0.0741 X_i$.

If BE student get 0 in test exam their GPA likely to be 0.5655 and every 1 score increase on the test exam by average, GPA will increase by 0.0741.

1.2)

X_i	$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$	$Y_i - \hat{Y}_i = \hat{u}_i$
63	$0.5655 + 0.0741(63) = 2.7138$	$2.8 - 2.7138 = 0.0862$
72	$0.5655 + 0.0741(72) = 3.0207$	$3.4 - 3.0207 = 0.3793$
78	$0.5655 + 0.0741(78) = 3.2253$	$3.0 - 3.2253 = -0.2253$
81	$0.5655 + 0.0741(81) = 3.7276$	$3.5 - 3.7276 = -0.1724$
87	$0.5655 + 0.0741(87) = 3.5222$	$3.6 - 3.5222 = 0.0778$
75	$0.5655 + 0.0741(75) = 3.123$	$3.0 - 3.123 = -0.123$
75	$0.5655 + 0.0741(75) = 3.123$	$2.7 - 3.123 = -0.423$
90	$0.5655 + 0.0741(90) = 3.6245$	$3.7 - 3.6245 = 0.0655$

$$\sum_{i=1}^8 \hat{u}_i = 0.0862 + 0.3793 + (-0.2253) + 0.1724 + 0.0778 + (-0.123) + (-0.423) + 0.0655 = 0.0001 \approx 0$$

1.3.) Since $\text{Var}(U_i | X_1, X_2, \dots, X_n) = \sigma^2$ but σ^2 is unknown, so we will find $\hat{\sigma}^2$ instead.

$$\hat{\sigma}^2 = \frac{\sum U_i^2}{n-2} = \frac{(0.0862)^2 + (0.3797)^2 + (-0.2257)^2 + (0.1724)^2 + (0.0678)^2 + (-0.127)^2 + (-0.927)^2 + (0.0655)^2}{8-2}$$

$$= \frac{0.0074 + 0.1479 + 0.0504 + 0.0297 + 0.0046 + 0.0157 + 0.789 + 0.0043}{6}$$

$$= \frac{0.9747}{6}$$

$$\text{Var}(\hat{\mu}_1) = 0.0725 = \hat{\sigma}^2$$

$$\text{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum X_i^2} = \frac{0.0725}{511.8748}$$

$$= 0.0001$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum \delta_i^2} \cdot \hat{\sigma}^2$$

$$= \frac{48717}{8(511.8748)} \cdot 0.0725$$

$$= \frac{3531.9835}{4094.9984}$$

$$= 0.8625$$

2.)

i	X_i	Y_i	$X_i \cdot Y_i$	X_i^2	$X_i - \bar{X} = \delta_i$	δ_i^2
1	10	0	0	100	10 - 20 = -10	100
2	12	2	24	144	12 - 20 = -8	64
3	14	5	70	196	14 - 20 = -6	36
4	16	6	96	256	16 - 20 = -4	16
5	18	7	126	324	18 - 20 = -2	4
6	22	10	220	484	22 - 20 = 2	4
7	24	10	240	576	24 - 20 = 4	16
8	26	15	390	676	26 - 20 = 6	36
9	28	16	448	784	28 - 20 = 8	64
10	30	20	600	900	30 - 20 = 10	100
sum	200	91	2314	4440	0	440
mean	20	9.1	231.4	444	0	44

$$2.1) \hat{\beta}_2 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$= \frac{10(2214) - (200)(91)}{10(4440) - (200)^2}$$

$$= \frac{22140 - 18200}{44400 - 40000}$$

$$= \frac{3940}{4400}$$

$$\hat{\beta}_2 = 0.8955$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 9.1 - 0.8955(20)$$

$$\hat{\beta}_1 = -8.81$$

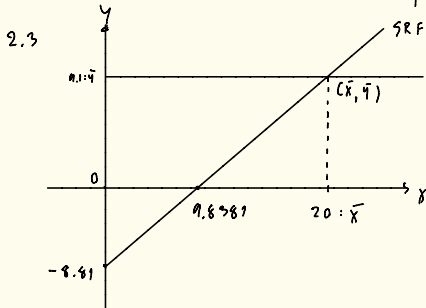
The fitted model is $\hat{Y}_i = -8.81 + 0.8955X_i$
 \therefore If $X_i = 0$ then, $\hat{Y}_i = -8.81$
 and If X_i increase 1 unit in average, \hat{Y}_i will increase by 0.8955 unit.

2.2)

i	X_i	\hat{Y}_i	$Y_i - \hat{Y}_i = \hat{u}_i$	\hat{u}_i^2
1	10	0.145	0 - 0.145 = -0.145	0.021
2	12	1.976	2 - 1.976 = 0.024	0.0041
3	14	3.787	5 - 3.787 = 1.213	1.4715
4	16	5.518	6 - 5.518 = 0.482	0.2322
5	18	7.209	7 - 7.209 = -0.209	0.0436
6	22	10.491	10 - 10.491 = -0.491	0.2410
7	24	12.682	10 - 12.682 = -2.682	7.1927
8	26	14.473	15 - 14.473 = 0.527	0.2777
9	28	16.264	16 - 16.264 = -0.264	0.0697
10	30	18.055	20 - 18.055 = 1.945	3.7830

$$\sum \hat{u}_i = 0$$

$$SRF; \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$



Given $X_i = 20$, $\hat{Y}_i = -8.81 + 0.8955(20)$
 $\hat{Y}_i = 9.1 = \bar{Y}$

$\therefore (\bar{X}, \bar{Y})$ is a member of SRF.

2.4.) Given $X_i: 18$

$$\hat{Y}_i = -8.41 + 0.4955(18)$$

$$\hat{Y}_i = 7.709$$

$$\begin{aligned} \text{2.5.) } \text{Var}(\hat{u}_i) &= \frac{\sum \hat{u}_i^2}{n-2} \\ &= \frac{14.0009}{10-2} \\ &= 1.7614 = \hat{\sigma}^2 \end{aligned}$$

$$\begin{aligned} \text{Var}(\hat{\beta}_2) &= \frac{\hat{\sigma}^2}{\sum X_i^2} \\ &= \frac{1.7614}{440} \\ &= 0.004 \end{aligned}$$

$$\begin{aligned} \text{Var}(\hat{\beta}_1) &= \frac{\sum X_i^2}{n \sum \delta_i^2} \cdot \hat{\sigma}^2 \\ &= \frac{440}{10(440)} \cdot 1.7614 \\ &= 1.7614 \end{aligned}$$

3.) We need to find $\hat{\beta}_2$ first
 $\hat{\beta}_2$ is an unbiased estimator
of β_2 when $E(\hat{\beta}_2) = \beta_2$

$$\begin{aligned} \hat{\beta}_2 &= \sum_{i=1}^n k_i Y_i \\ &= \sum_{i=1}^n k_i (\beta_1 + \beta_2 X_i + u_i) \\ &= \sum_{i=1}^n (k_i \beta_1 + k_i \beta_2 X_i + k_i u_i) \\ &= \beta_1 \sum_{i=1}^n k_i + \beta_2 \sum_{i=1}^n k_i X_i + \sum_{i=1}^n k_i u_i \end{aligned}$$

$$\hat{\beta}_2 = \beta_2 + \sum_{i=1}^n k_i u_i$$

$$E(\hat{\beta}_2) = E(\beta_2) + E\left(\sum_{i=1}^n k_i u_i\right) \rightarrow \text{Assumption } \rightarrow E(u_i | X_i) = 0$$

$$E(\hat{\beta}_2) = \beta_2$$

$\therefore \hat{\beta}_2$ is an unbiased estimator of β_2

And $\hat{\beta}_1$ is an unbiased estimator of β_1

$$\text{When } E(\hat{\beta}_1) = \beta_1$$

$$\text{since } \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\begin{aligned} E(\hat{\beta}_1) &= E(\bar{Y} - \hat{\beta}_2 \bar{X}) \\ &= E(\bar{Y}) - \bar{X} E(\hat{\beta}_2) \end{aligned}$$

$$\begin{aligned} \text{And } E(\bar{Y}) &= E\left[\frac{1}{n} \sum_{i=1}^n Y_i\right] \\ &= \frac{1}{n} \sum_{i=1}^n E(Y_i) \\ &= \frac{1}{n} \sum_{i=1}^n (\beta_1 + \beta_2 X_i) \\ &= \frac{n\beta_1}{n} + \frac{\beta_2 \sum_{i=1}^n X_i}{n} \\ E(\bar{Y}) &= \beta_1 + \beta_2 \bar{X} \end{aligned}$$

$$\text{From } E(\hat{\beta}_1) = E(\bar{Y}) - \bar{X} E(\hat{\beta}_2)$$

$$\text{If } E(\bar{Y}) = \beta_1 + \beta_2 \bar{X} \text{ and } \beta_2 = E(\hat{\beta}_2)$$

$$\text{Then } E(\hat{\beta}_1) = \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2$$

$$E(\hat{\beta}_1) = \beta_1$$

$\hat{\beta}_1$ is an unbiased estimator