FN 201
BUSINESS FINANCE

Lecture 9

Hedging and Bond Risk Management

- Agenda
  - 1) Bond Price through times
  - 2) What is Hedging?
  - 3) Risks of bond holder
  - 3) Durations and Modified Duration
  - 4) Convexity

Bond Pricing

Bond Price = PV of coupons + PV of par value

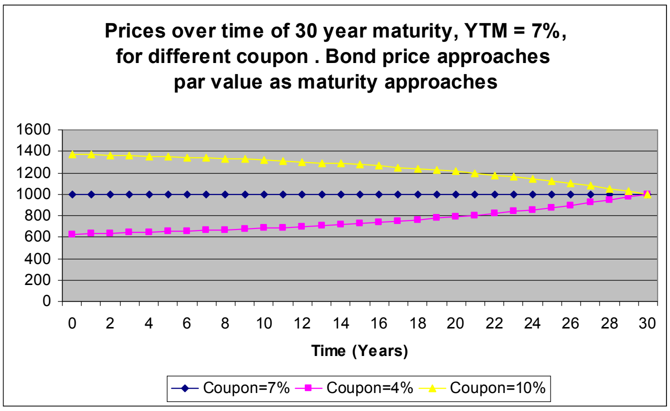
Bond Price = C*Annuity factor(y,T) + Par*discount factor(y,T)

Bond Price Through Time

- Find price of bond with coupon C (THB 70) paid annually, 30 yrs maturity, yield to maturity is 4%

- At time 0: Bond Price = C*Annuity factor(4%,30) + Par*discount factor(4%,30)
- At time 1: Bond Price = C*Annuity factor(4%,29) + Par*discount factor(4%,29)
- At time 2: Bond Price = C*Annuity factor(4%,28) + Par*discount factor(4%,28)
- ....
- At time 30: Bond Price = C*Annuity factor(4%,0) + Par*discount factor(4%,0) = face value

Price determined by YTM



Face	1000	Coupon=7%			Coupon=4 %			Coupon=10 %		
Coupon		7%			4%			10%		
		YTM=7%			YTM=7%			YTM=7%		
Time	TTM (Yrs)	Formula 7%	Hold and Sell	Return	Formula 7%	Hold and Sell	Return	Formula 7%	Hold and Sell	Return
0	30	1000.00			627.73			1372.27		
1	29	1000.00	1070	7%	631.67	672	7%	1368.33	1468	7%
2	28	1000.00	1070	7%	635.89	676	7%	1364.11	1464	7%
3	27	1000.00	1070	7%	640.40	680	7%	1359.60	1460	7%
4	26	1000.00	1070	7%	645.23	685	7%	1354.77	1455	7%
5	25	1000.00	1070	7%	650.39	690	7%	1349.61	1450	7%
6	24	1000.00	1070	7%	655.92	696	7%	1344.08	1444	7%
7	23	1000.00	1070	7%	661.83	702	7%	1338.17	1438	7%
8	22	1000.00	1070	7%	668.16	708	7%	1331.84	1432	7%
9	21	1000.00	1070	7%	674.93	715	7%	1325.07	1425	7%
10	20	1000.00	1070	7%	682.18	722	7%	1317.82	1418	7%
11	19	1000.00	1070	7%	689.93	730	7%	1310.07	1410	7%
12	18	1000.00	1070	7%	698.23	738	7%	1301.77	1402	7%
13	17	1000.00	1070	7%	707.10	747	7%	1292.90	1393	7%
14	16	1000.00	1070	7%	716.60	757	7%	1283.40	1383	7%
15	15	1000.00	1070	7%	726.76	767	7%	1273.24	1373	7%
16	14	1000.00	1070	7%	737.64	778	7%	1262.36	1362	7%
17	13	1000.00	1070	7%	749.27	789	7%	1250.73	1351	7%
18	12	1000.00	1070	7%	761.72	802	7%	1238.28	1338	7%
19	11	1000.00	1070	7%	775.04	815	7%	1224.96	1325	7%
20	10	1000.00	1070	7%	789.29	829	7%	1210.71	1311	7%
21	9	1000.00	1070	7%	804.54	845	7%	1195.46	1295	7%
22	8	1000.00	1070	7%	820.86	861	7%	1179.14	1279	7%
23	7	1000.00	1070	7%	838.32	878	7%	1161.68	1262	7%
24	6	1000.00	1070	7%	857.00	897	7%	1143.00	1243	7%
25	5	1000.00	1070	7%	876.99	917	7%	1123.01	1223	7%
26	4	1000.00	1070	7%	898.38	938	7%	1101.62	1202	7%
27	3	1000.00	1070	7%	921.27	961	7%	1078.73	1179	7%
28	2	1000.00	1070	7%	945.76	986	7%	1054.24	1154	7%
29	1	1000.00	1070	7%	971.96	1012	7%	1028.04	1128	7%
30	0	1000.00	1070	7%	1000.00	1040	7%	1000.00	1100	7%

Actual return might be higher / lower if sell before maturity

- If sell it before maturity for extra capital gain / loss
- YTM implies return in case of holding until maturity (as the name suggests)

Hedging and Risk Management

What is Hedging?

- Wikipedia

In Simple Words

- To hedge means you want Loss from one instrument to be off-setted by Gain from another instrument based on same underlying / risk factor

- Ex: how can you hedge against rise in petrol price?
  - For THB 0.50 rise in petrol price, your fuel cost per month will increase by 1,000 THB
  - And if price of PTT usually increases by THB 0.25 for every THB 0.50 increase in petrol price
  - How can you hedge this risk?

In Simple Words (2)

- In this case -> we have exposure on petrol price
- If I buy 4000 units of PTT stocks, what would be my net payoff when petrol price increases by THB 0.50?
  - Loss from exposure = -1,000 THB
  - Gain from hedging = For THB 0.50 rise in petrol -> PTT stock rises by 0.25 THB -> Gain equals to 4000 * THB0.25 = +1,000 THB
  - Thus, net loss would be 0
- What if petrol price drop by THB 0.50
  - Gain from exposure = +1,000 THB
  - Loss from hedging = For THB 0.50 drop in petrol -> PTT stock drops by 0.25 THB -> Loss equals to 4000 * THB0.25 = -1,000 THB
  - Thus, net gain would be 0

In Simple Words (3)

- Hedging means limiting possible losses
- It also means limiting possible gains as well
- What are the risk of bond?

Bond Risk Management

- The price of long-term bonds fluctuates more than the price of short-term bonds for a given change in interest rates

- In other words, long-term bonds carry more (interest rate) risk than short-term bonds

- But… with same maturity can still carry different levels of interest rate risk, depends on its payment structure also (think about zero coupon bonds)

Payment Structure

- Fixed coupon bond
- Zero coupon bond
- Annuities bond
- Perpetual bond or consols
- Floating-coupon
- Structured note
- Inflation-protected

What are the risks of bond holder?

- Interest rate or Yield Curve risk
- Inflation (expectation) risk
- Default risk
- Liquidity risk
- Optionality risk

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First Rule of Risk Management

- We can only “manage” what we can “measure”
- Risk of bond caused by changes in interest rate -> lead to changes in price
- Can we measure it?
- We need some types of “risk measurement” that indicates the “risk” of bond, taking into account both maturity and cash flow structure

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(Macaulay) Duration / Modified Duration

- Duration is a popular measurement of interest rate risk
  - Elasticity of bond price with respect to the interest rate
- Duration offers a better measure of interest rate risk than maturity because it considers the timing of cash flows and PV of cash flows
  - It combines both maturity and payment structure into one measurement
  - Enable to compare across different bond
- It can be thought of as average time arrival of the CF

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(Macaulay) Duration / Modified Duration (2)

$$D = \frac{\sum_{t=1}^{T} t \cdot \frac{C_t}{(1+y)^t}}{\sum_{t=1}^{T} \frac{C_t}{(1+y)^t}} = \frac{\sum_{t=1}^{T} t \cdot \frac{C_t}{(1+y)^t}}{P}$$

- D = Macaulay Duration
- D is the PV of bond cash flows weighted by their time of arriving, divided by bond price
- D is an indicator of bond sensitivity to interest rate changes
- t = time period of cash flow (in years).
- For 6 months, t=0.5
- Ct=cash flows in year t
- y = YTM

$$MD = \frac{D}{1+y}$$

- MD = Modified Duration
- MD provide approximate change in bond price due to interest rate changes
- Also known as bond volatility

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$$P = \sum_{t=1}^{N} \frac{C_t}{(1+y)^t}$$

$$\frac{\partial P}{\partial y} = -\sum_{t=1}^{N}\left(t \cdot \frac{C_t}{(1+y)^{t+1}}\right)$$

$$\frac{\partial P}{\partial y} = \frac{-1}{1+y}\sum_{t=1}^{N}\left(t \cdot \frac{C_t}{(1+y)^t}\right)$$

$$But: \sum_{t=1}^{N}\left(t \cdot \frac{C_t}{(1+y)^t}\right) = D * P$$

$$\frac{\partial P}{\partial y} = \frac{-D}{1+y} \cdot P$$

$$\frac{1}{P}\frac{\partial P}{\partial y} = \frac{-D}{1+y} = -D_m^*$$

Another way to think about duration

Using MD to estimate changes in bond prices

- MD is quoted per 1 percentage point or 100 basis point changes
- **Bond A has MD of 5% means if interest rate change by 1 percentage point or 100 basis points -> price of bond will change by 5%**
- If Bond A has current price of THB 107, if interest rate increase by 1 percentage point or 100 basis points -> price of bond will decrease approximately by 5% or from 107 to 101.65 (=0.95*107) THB
- If interest rate change by 0.5% or 50 bps, what should be the approximated price?

Ex1: Duration Calculation

- 5-yrs bond with notional = 1000, 7% coupon paid annually, 9% yield

(1)	(2)	(3)	(4)	(5)	(6)
Year	Cash flow	DF	PV of CF	Weighted	Duration
1	70	0.9174	64.22	64.22	
2	70	0.8417	58.92	117.84	
3	70	0.7722	54.05	162.16	
4	70	0.7084	49.59	198.36	
5	1070	0.6499	695.43	3477.13	
			922.21	4019.71	4.36
(Macaulay) duration=		4.36			
Modified duration=		3.999			

Ex2: Zero Coupon Bond

- Recall, bond with notional = 100, 7yr maturity, zero coupon. If yield on comparable bond is 5%, what is the duration of this bond?

$$D = \frac{7 \cdot \frac{100}{(1+.05)^7}}{71.07} = \frac{7 \cdot 71.07}{71.07} = 7$$

Ex3: Perpetual Bond

- Bond A is a perpetual bond that pay coupon of C annually forever? What is the price and duration of this bond given YTM of y ?

Ex4:

- A bond with has face value of $1,000, coupon rate of 0%, yield to maturity of 9%, and ten years to maturity. This bond's duration is:
  a. 6.7 years b. 7.5 years c. 9.6 years d. 10.0 years

- A bond with duration of 10 years has yield to maturity of 10%. This bond's MD is:
  a. 9.09% b. 6.8% c. 14.6% d. 6.0%

- If a bond's volatility (MD) is 10% and the interest rate goes down by 0.75% (points) then the price of the bond:
  a. Decreases by 10% b. Decreases by 7.5% c. Increases by 7.5%
  d. Increases by 0.75%

Ex5: MD only provides approximated price changes

- Given that
  - Notional = 100, 7yr maturity, 8% annual coupon, Current market yield = 5%
  - Find 1) price, 2) duration, 3) modified duration 4) change in price given market yield rises to 6% using MD 5) Find exact new price 6) Find % error

How to hedge the risk?

- We need an instrument that will have higher value when interest rate goes up (because price of your bond will go down)
- Issue of sensitivity and hedging ratio
- How much risk do we want to eliminate?

Ex6 : True or False

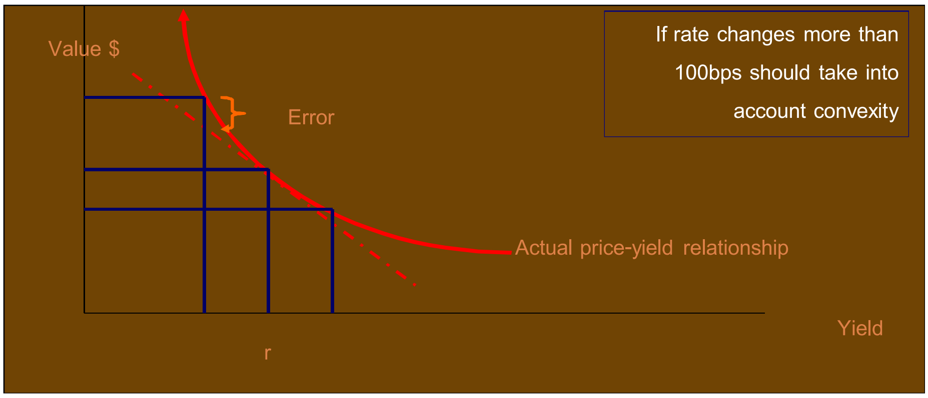
- 1) The duration of any bond is the same as its maturity.
- 2) The duration of a zero coupon bond is the same as its maturity.
- 3) The longer a bond's duration greater is its volatility.
- 4) The term structure of interest rate is the relationship between yield to maturity and maturity.
- 5) Short-term and long-term interest rates always move in parallel.
- 6) The expectations theory implies that the only reason for a declining term structure is that investors expect spot interest rates to fall.
- 7) Fixed coupon bond does not bear an interest rate risk

Convexity

- The actual relationship between bond price and yield is convex -> duration offer a linear approximation of price-yield relationship
- Duration is a first-order approximation of the sensitivity of the value change of a portfolio with respect to changes in the interest rates
- As for fixed rate bond, it will always underestimate the actual price change after an increase or decrease of the interest rates

Convexity (2)

- The actual relationship between bond price and yield is convex



Ex7: Bond Parameters and Duration

Factor Increase	Impact on Duration	Rational
Maturity ↑	?	
Coupon ↑	?	
Market Yield ↑	?	
Frequency of coupon ↑	?	

Ex8 Duration of the portfolio

- Duration is additive, that is duration of a portfolio is the weighted sum of the durations of the bonds or cash flows in the portfolio
- The weights are the market values of the bond positions

$$D_{portfolio} = \sum_{i=1}^{n} \frac{PV_i}{PV_{portfolio}} * D_i$$

- 3 bond in the portfolio; Calculate Duration of the portfolio:
- Bond A, price = 82.67, duration = 3.36 yrs
- Bond B, price = 101.54, duration = 4.32 yrs
- Bond C, price = 98.76, duration = 1.54 yrs

Key Points to take away

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Q & A