0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial z}{\partial x} = \frac{x^2y^2(3x^2) - [(x^3-y^3)\,2xy^2]}{(x^2y^2)^2}$$

$$= \frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{(x^2y^2)^2}$$

$$\frac{\partial z}{\partial x} = \frac{x^4y^2 + 2xy^5}{(x^2y^2)^2}$$

$$\frac{\partial z}{\partial y} = \frac{x^2y^2(-3y^2) - [(x^3-y^3)\,2x^2y]}{(x^2y^2)^2}$$

$$= \frac{-3x^2y^4 - 2x^5y + 2x^2y^4}{(x^2y^2)^2}$$

$$= \frac{-x^2y^4 - 2x^5y}{(x^2y^2)^2}$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -z$$

$$x\left[\frac{x^4y^2 + 2xy^5}{(x^2y^2)^2}\right] + y\left[\frac{-x^2y^4 - 2x^5y}{(x^2y^2)^2}\right] = \frac{x^5y^2 + 2x^2y^5 - x^2y^5 - 2x^5y^2}{(x^2y^2)^2}$$

$$= \frac{x^2y^5 - x^5y^2}{(x^2y^2)^2}$$

$$= \frac{\cancel{x^2y^2}(y^3 - x^3)}{(x^2y^2)^{\cancel{2}}}$$

$$-z = \frac{y^3 - x^3}{(x^2y^2)} \quad \#$$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

$$z < \begin{matrix} x \\ y \end{matrix}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$= \frac{x+y - (x-y)}{(x+y)^2}dx + \frac{x+y(-1) - (x-y)}{(x+y)^2}dy$$

$$= \frac{2y}{(x+y)^2}dx - \frac{2x}{(x+y)^2}dy$$

when $dx = 2$ $dy = -2$ $x = 1$ $y = 1$

so, $\frac{\cancel{2(1)}}{(\cancel{2})^{\cancel{2}}}(\cancel{2}) - \frac{\cancel{2(1)}}{\cancel{2^2}}(-\cancel{2}) = 1 + 1 = 2$

Hence, when x increase by 2 units and y decrease by 2 units z will increase by 2 units.

0.3) If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

$$z \begin{cases} x \begin{cases} r \\ s \end{cases} \\ y \begin{cases} r \\ s \end{cases} \end{cases}$$

(0.3a)

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$x = 1 \quad y = 2$

$$= (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= (4xy + 3y)(2r + 2s) + (2x^2 + 3x + 2y)(2)$$

(0.3b) when $r = 1$ and $s = 0$

$$\frac{\partial z}{\partial s} = (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$$

$$= (4(1)(2) + 3(2))(2(1)) + ((2)(1)^2 + 3(1) + 2(2))(-4)$$

$$= (8 + 6)(2) + ((2) + 3 + 4)(-4)$$

$$= 28 - 36 \quad = \quad -8$$

$$\frac{\partial z}{\partial r} = (4xy + 3y)(2r + 2s) + (2x^2 + 3x + 2y)(2)$$

$$= [4(1)(2) + 3(2)][2(1)] + [2(1) + 3 + 4](2)$$

$$= (8 + 6)(2) + 18$$

$$= 46 \#$$

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

$$F(x, y, z) = 2x^2 + 3y^2 + 2z^2 - 16$$

$$F_x = 4x$$

$$F_y = 6y$$

$$F_z = 4x$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{6y}{4z}$$

$$\frac{\partial z}{\partial y} = -\frac{3}{2} \frac{y}{z}$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{-1} = 3}$$

0.5) Given that $ln\,(x+y+z)+xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

$$\ln(x+y+z)+xyz-ze^{x+y+z}=0$$

$$\frac{\partial z}{\partial x}=\frac{-Fx}{Fz}=-\frac{\frac{1}{(x+y+z)}+yz-ze^{x+y+z}}{\frac{1}{(x+y+z)}+xy-e^{x+y+z}}$$

when $x=0,\ y=1,\ z=0$

$$-\frac{\frac{1}{(0+1+0)}+1(0)-(0)e^{0+1+0}}{\frac{1}{(0+1+0)}+(0)1-e^{0+1+0}}$$

$$-\frac{1+0-0}{1+0-2.7183}$$

$$\frac{1}{1.7183},\ \text{when } x\uparrow z\uparrow$$ ※

Question 2 Given the production function $Q=f(K,L)=A[K^n+L^n\,]$ where A is the level of technology, K is capital and L is labor. Suppose that $n>0$. Consider the following problem.

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

decreasing return to scale : $\alpha+\beta<1$
: $n<1$

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.

$$\frac{\partial Q}{\partial K}=MPK=AnK^{n-1}$$

$$\frac{\partial MPK}{\partial K}=An(n-1)K^{n-2}$$

when $n<1: n-1<0 \rightarrow MPK\downarrow\ K\uparrow$ = Law of Diminishing

$$\frac{\partial Q}{\partial L}=MPL=AnL^{n-1}$$

$$\frac{\partial MPL}{\partial L}=An(n-1)L^{n-2}$$

when $n<1: n-1<0 \rightarrow MPL\downarrow\ L\uparrow$ = Law of Diminishing

$Q = f(K, L) = A[K^n + L^n]$ $= AK^n + AL^n$ $n > 0$

2.3

$MP_K = \frac{\partial Q}{\partial K} = AK^n + AL^n = nAK^{n-1}$

$MP_L = \frac{\partial Q}{\partial L} = AK^n + AL^n = nAL^{n-1}$

$MRTS = \frac{MP_K}{MP_L} = \frac{\cancel{nA}K^{n-1}}{\cancel{nA}L^{n-1}} = \frac{K^{n-1}}{L^{n-1}}$ ✗

2.4 As MRTS is fraction $\frac{K}{L}$, when we fixed K and L is increasing, MRTS will decrease.

2.5 $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$

$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial K} \cdot \frac{dK}{dt} + \frac{\partial Q}{\partial L} \cdot \frac{dL}{dt}$

$\frac{dQ}{dt} = 0 + nAK^{n-1} \cdot (t+2) + nAL^{n-1} \cdot (e^t)$

$\frac{dQ}{dt} = tnAK^{n-1} + 2nAK^{n-1} + e^t nAL^{n-1}$ ✗

2.6 $\frac{\frac{dQ}{Q}}{dt} = \frac{tnAK^{n-1} + 2nAK^{n-1} + e^t nAL^{n-1}}{AK^n + AL^n}$ ✗

Question 4: Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Solve for L^* and calculate $\frac{\partial L^*}{\partial w}$ *and* $\frac{\partial L^*}{\partial P}$.

$$\pi = P \cdot Q - w \cdot L$$

$$= P \cdot (L^{1/2}) - w \cdot L$$

$$\frac{\partial \pi}{\partial L} = \frac{1}{2} P \cdot L^{-1/2} - W = 0$$

$$\frac{P}{2\sqrt{L}} = W$$

$$\sqrt{L} = \frac{P}{2W}$$

$$L^* = \frac{P^2}{4w^2} > 0$$

$P > 0$

$w > 0$

$$\frac{\partial L^*}{\partial w} = \frac{P^2}{4}(-2)w^{-3}$$

$$= \frac{-\cancel{2}P^2}{2 \cdot \cancel{4} w^3}$$

$$= -\frac{P^2}{2w^3}$$

$$\frac{\partial L^*}{\partial P} = \frac{2P}{4w^2}$$