# EMPIRICAL APPLICATIONS OF SOLOW MODEL: SOLOW MODEL WITH HUMAN CAPITAL

EE 462 Development Macroeconomics

Semester 1/2019

Reference: Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, *107*(2), 407-437.

# MRW (1992)

- MRW examines whether the "textbook" Solow model is consistent with the empirical evidence.
- It augments the Solow model by including *human capital accumulation* and capital accumulation, and tests this augmented Solow model.
- It also tests for both unconditional and conditional convergence in per capita income.
- Findings:
  - The prediction of the Solow model are consistent with the data.
  - The cross-country variation in per capita income can best be explained by an augmented Solow model.
  - There is convergence once the differences in saving and population growth rates are controlled for.

# "Textbook" Solow Model

- Production function: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$, $0 < \alpha < 1$,

where A = Technology.

- *L* and *A* are assumed to grow exogenously at rates *n* and *g*:

$$L(t) = L(0)e^{nt} \qquad \text{and} \qquad A(t) = A(0)e^{gt}$$

- Define $k = \frac{K}{AL}$ and $y = \frac{Y}{AL}$, where AL is effective units of labor.
- The change in *k* is given by:

$$\dot{k} = sy(t) - (n + g + \delta)k(t)$$

- Hence, the steady-state value $k^*$ is:

$$k^* = [s/(n+g+\delta)]^{1/(1-\alpha)}$$

# "Textbook" Solow Model (cont'd)

- Substituting $k^*$in the production function and taking log yields:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = lnA(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta)$$

- Assume that *g* and $\delta$ are constant across countries. (In this paper, it's assume that $\text{g} + \delta = 0.05.$)

- Also, assume that $lnA(0) = \text{a} + \epsilon$, where *a* is a constant, and $\epsilon$ is a country-specific shock.

- Hence, log income per capita at a given point time (say t=0) is:

$$\ln\left(\frac{Y}{L}\right) = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) \qquad \text{-- (*)}$$

# Data and Samples

- Data are from the Real National Accounts constructed by Summers and Heston (1988).
- Data are annual and cover the period 1960-1985.
- *n* measures the average growth rate of the working-age (15-64) population.
- s is the average share of real investment in real GDP.
- *Y/L* is real GDP in 1985 divided by the working-age pop.
- There are three samples:
  - 98 non-oil producing countries
  - 75 countries – exclude those with <1m pop or those with data problems
  - 22 OECD countries with pop > 1m

TABLE I

ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

| Sample: | Non-oil | Intermediate | OECD |
|---|---|---|---|
| Observations: | 98 | 75 | 22 |
| CONSTANT | 5.48 | 5.36 | 7.97 |
| | (1.59) | (1.55) | (2.48) |
| ln(I/GDP) $\propto/(1-\alpha)$ | 1.42 | 1.31 | 0.50 |
| | (0.14) | (0.17) | (0.43) |
| $\ln(n+g+\delta)$ $-\propto/(1-\alpha)$ | −1.97 | −2.01 | −0.76 |
| | (0.56) | (0.53) | (0.84) |
| $\overline{R}^2$ | 0.59 | 0.59 | 0.01 |
| *s.e.e.* | 0.69 | 0.61 | 0.38 |
| Restricted regression: | | | |
| CONSTANT | 6.87 | 7.10 | 8.62 |
| | (0.12) | (0.15) | (0.53) |
| $\ln(I/GDP) - \ln(n+g+\delta)$ | 1.48 | 1.43 | 0.56 |
| | (0.12) | (0.14) | (0.36) |
| $\overline{R}^2$ | 0.59 | 0.59 | 0.06 |
| *s.e.e.* | 0.69 | 0.61 | 0.37 |
| Test of restriction: | | | |
| *p*-value | 0.38 | 0.26 | 0.79 |
| Implied α | 0.60 | 0.59 | 0.36 |
| A priori value of α =1/3 | (0.02) | (0.02) | (0.15) |

*Note.* Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g+\delta)$ is assumed to be 0.05.

# Augmented Solow Model

- Let the production function be:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}, 0 < \alpha + \beta < 1,$$

Where H =stock of human capital.

- Let $s_k$ and $s_h$ be the fractions of income invested in physical capital and human capital, respectively.

- The evolution of the economy is determined by:

$$\dot{k} = s_k y(t) - (n + g + \delta)k(t)$$

$$\dot{h} = s_h y(t) - (n + g + \delta)h(t)$$

Where *k = K/AL*, *h = H/AL*, and *y= Y/AL*.

# Augmented Solow Model (cont'd)

- Hence, the steady-state levels of K and H are:

$$k^*=\left[\frac{s_k^{1-\beta}s_h^{\beta}}{n+g+\delta}\right]^{1/(1-\alpha-\beta)} \text{ and } h^*=\left[\frac{s_k^{\alpha}s_h^{1-\alpha}}{n+g+\delta}\right]^{1/(1-\alpha-\beta)}$$

- Substituting $k^*$ and $h^*$ in the production function yields:

$$\boxed{\ln\left(\frac{Y}{L}\right) = lnA(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h)}$$

Where $s_h$= secondary school enrollment ratio x proportion of labor force of secondary school age.

- Alternatively, the above equation can be rewritten as:

$$\ln\left(\frac{Y}{L}\right) = lnA(0) + gt + \frac{\alpha}{1-\alpha}\ln(s_k) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \frac{\beta}{1-\alpha}\ln(h^*)$$

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

| Sample: | | Non-oil | Intermediate | OECD |
|---|---|---|---|---|
| Observations: | | 98 | 75 | 22 |
| CONSTANT | | 6.89 | 7.81 | 8.63 |
| | | (1.17) | (1.19) | (2.19) |
| ln(I/GDP) | $\frac{\alpha}{1-\alpha-\beta}$ | 0.69 | 0.70 | 0.28 |
| | | (0.13) | (0.15) | (0.39) |
| $\ln(n + g + \delta)$ | $-\frac{\alpha+\beta}{1-\alpha-\beta}$ | −1.73 | −1.50 | −1.07 |
| | | (0.41) | (0.40) | (0.75) |
| ln(SCHOOL) | $\frac{\beta}{1-\alpha-\beta}$ | 0.66 | 0.73 | 0.76 |
| | | (0.07) | (0.10) | (0.29) |
| $\bar{R}^2$ | | 0.78 | 0.77 | 0.24 |
| *s.e.e.* | | 0.51 | 0.45 | 0.33 |

# TABLE II (Cont'd)

## Estimation of the Augmented Solow Model

| Dependent variable: log GDP per working-age person in 1985 | | | |
|---|---|---|---|
| Sample: | Non-oil | Intermediate | OECD |
| Observations: | 98 | 75 | 22 |
| Restricted regression: | | | |
| CONSTANT | 7.86 | 7.97 | 8.71 |
| | (0.14) | (0.15) | (0.47) |
| $\ln(I/GDP) - \ln(n + g + \delta)$ | 0.73 | 0.71 | 0.29 |
| | (0.12) | (0.14) | (0.33) |
| $\ln(SCHOOL) - \ln(n + g + \delta)$ | 0.67 | 0.74 | 0.76 |
| | (0.07) | (0.09) | (0.28) |
| $\bar{R}^2$ | 0.78 | 0.77 | 0.28 |
| *s.e.e.* | 0.51 | 0.45 | 0.32 |
| Test of restriction: | | | |
| *p*-value | 0.41 | 0.89 | 0.97 |
| Implied $\alpha$ | 0.31 | 0.29 | 0.14 |
| A priori value of α =1/3 | (0.04) | (0.05) | (0.15) |
| Implied $\beta$ | 0.28 | 0.30 | 0.37 |
| A priori value of β =1/3 | (0.03) | (0.04) | (0.12) |

*Note.* Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

# Endogenous Growth and Convergence

- Goal: reexamine the evidence on convergence of Y/L.
- Previous model assumes that countries in 1985 were in their steady states (or that the deviations from steady state were random). → relax this assumption!
- Let *y** be the steady-state income per capita and *y(t)* be the actual value at time *t*.
- The **speed of convergence** is given by:

$$\frac{d\ln(y(t))}{dt} = \lambda[\ln(y^*) - \ln(y(t))],$$

Where $\lambda = (n + g + \delta)(1 - \alpha - \beta)$ is the convergence rate.

- Example: If $\alpha = \beta = \frac{1}{3}$ and $n + g + \delta = 0.06$, then $\lambda = 0.02$.

# Convergence Rate

- The above equation implies that:

$$\ln\big(y(t)\big) = \big(1 - e^{-\lambda t}\big)ln(y^*) + e^{-\lambda t}\ln(y(0)),$$

Where y(0) is income per effective worker at an initial date.

- Subtracting *ln(y(0))* and substituting *y** gives:

$$\ln\big(y(t)\big) - \ln\big(y(0)\big)$$

$$= \big(1 - e^{-\lambda t}\big)\frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \big(1 - e^{-\lambda t}\big)\frac{\beta}{1-\alpha-\beta}\ln(s_h)$$

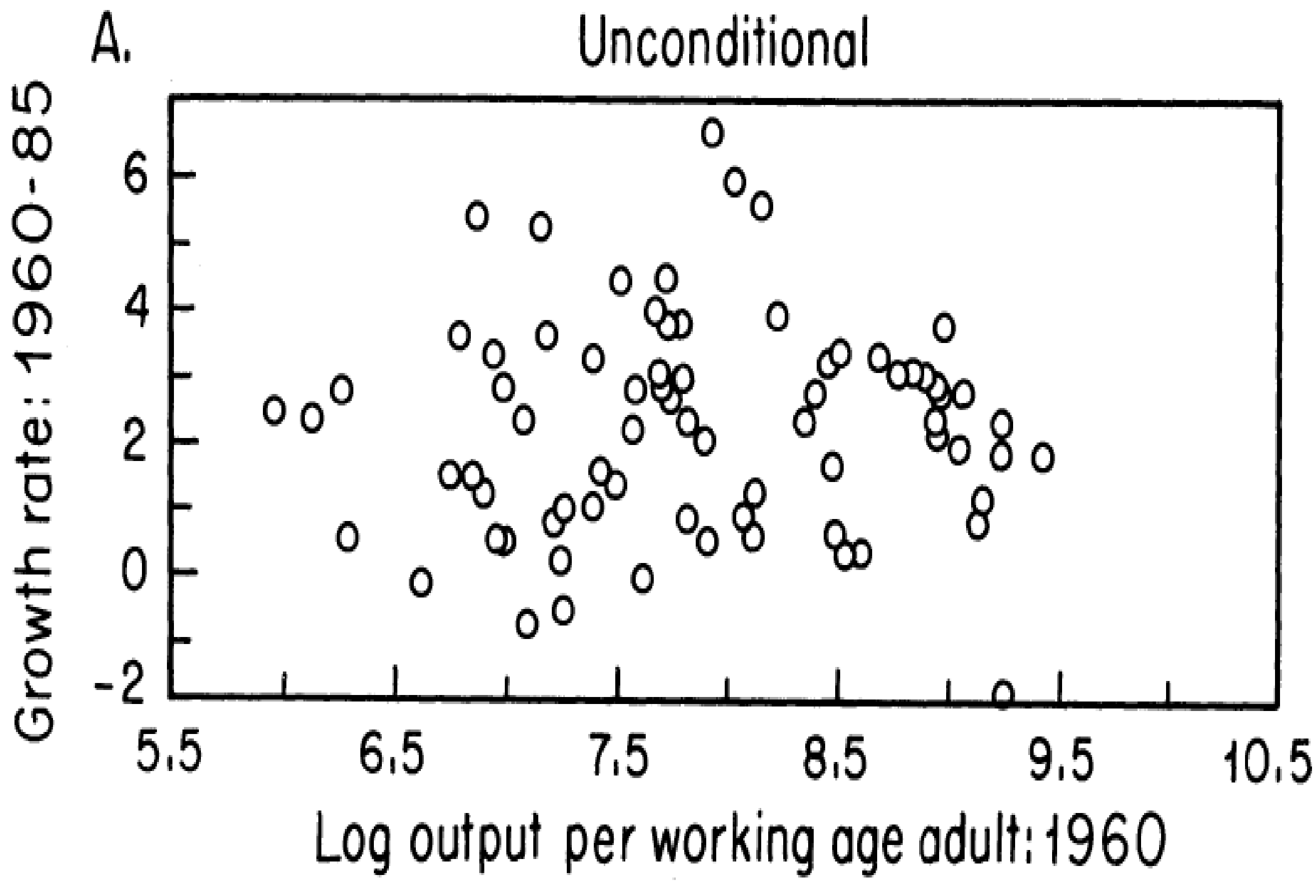
$$- \big(1 - e^{-\lambda t}\big)\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) - (1 - e^{-\lambda t})\ln(y(0))$$

## TABLE III

### Tests for Unconditional Convergence

| Dependent variable: log difference GDP per working-age person 1960–1985 | | | |
|---|---|---|---|
| Sample: | Non-oil | Intermediate | OECD |
| Observations: | 98 | 75 | 22 |
| CONSTANT | −0.266 | 0.587 | 3.69 |
| | (0.380) | (0.433) | (0.68) |
| ln(Y60) $-(1-e^{-\lambda t})$ | 0.0943 | −0.00423 | −0.341 |
| | (0.0496) | (0.05484) | (0.079) |
| $\overline{R}^2$ | 0.03 | −0.01 | 0.46 |
| *s.e.e.* | 0.44 | 0.41 | 0.18 |
| Implied λ | −0.00360 | 0.00017 | 0.0167 |
| | (0.00219) | (0.00218) | (0.0023) |

*Note.* Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.


A.
Unconditional
Growth rate: 1960-85
6
4
2
0
-2
5.5
6.5
7.5
8.5
9.5
10.5
Log output per working age adult: 1960

## TABLE IV
### TESTS FOR CONDITIONAL CONVERGENCE

| Dependent variable: log difference GDP per working-age person 1960–1985 | | | |
|---|---|---|---|
| Sample: | Non-oil | Intermediate | OECD |
| Observations: | 98 | 75 | 22 |
| CONSTANT | 1.93 | 2.23 | 2.19 |
| | (0.83) | (0.86) | (1.17) |
| ln(Y60) | −0.141 | −0.228 | −0.351 |
| | (0.052) | (0.057) | (0.066) |
| ln(I/GDP) | 0.647 | 0.644 | 0.392 |
| | (0.087) | (0.104) | (0.176) |
| $\ln(n + g + \delta)$ | −0.299 | −0.464 | −0.753 |
| | (0.304) | (0.307) | (0.341) |
| $\overline{R}^2$ | 0.38 | 0.35 | 0.62 |
| *s.e.e.* | 0.35 | 0.33 | 0.15 |
| Implied $\lambda$ | 0.00606 | 0.0104 | 0.0173 |
| | (0.00182) | (0.0019) | (0.0019) |

*Note.* Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

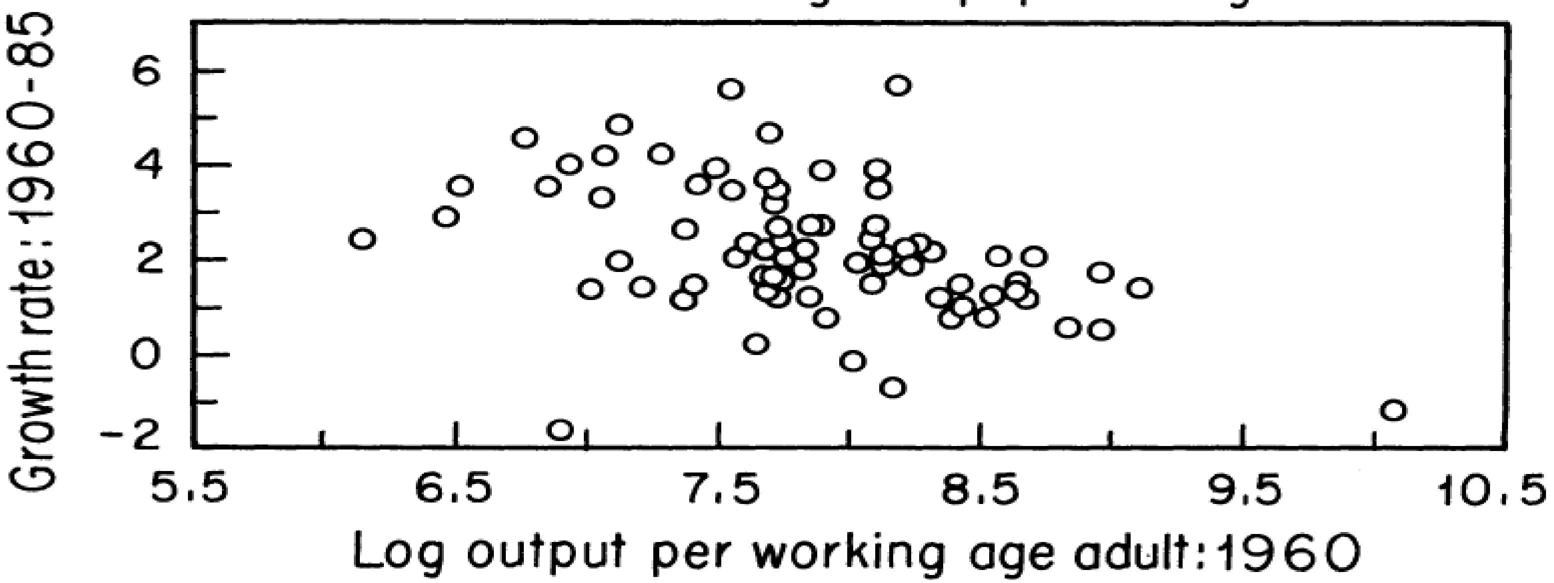
# TABLE V

## Tests for Conditional Convergence

| Dependent variable: log difference GDP per working-age person 1960–1985 | | | |
|---|---|---|---|
| Sample: | Non-oil | Intermediate | OECD |
| Observations: | 98 | 75 | 22 |
| CONSTANT | 3.04 | 3.69 | 2.81 |
| | (0.83) | (0.91) | (1.19) |
| ln(Y60) $-(1-e^{-\lambda t})$ | −0.289 | −0.366 | −0.398 |
| | (0.062) | (0.067) | (0.070) |
| ln(I/GDP) $-(1-e^{-\lambda t})\frac{\alpha}{1-\alpha-\beta}$ | 0.524 | 0.538 | 0.335 |
| | (0.087) | (0.102) | (0.174) |
| $\ln(n+g+\delta)$ $-(1-e^{-\lambda t})\frac{\beta}{1-\alpha-\beta}$ | −0.505 | −0.551 | −0.844 |
| | (0.288) | (0.288) | (0.334) |
| ln(SCHOOL) $-(1-e^{-\lambda t})\frac{\alpha+\beta}{1-\alpha-\beta}$ | 0.233 | 0.271 | 0.223 |
| | (0.060) | (0.081) | (0.144) |
| $\overline{R}^2$ | 0.46 | 0.43 | 0.65 |
| *s.e.e.* | 0.33 | 0.30 | 0.15 |
| Implied $\lambda$ | 0.0137 | 0.0182 | 0.0203 |
| | (0.0019) | (0.0020) | (0.0020) |

*Note.* Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g+\delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

B. Conditional on saving and population growth




C. Conditional on saving, population growth and human capital