

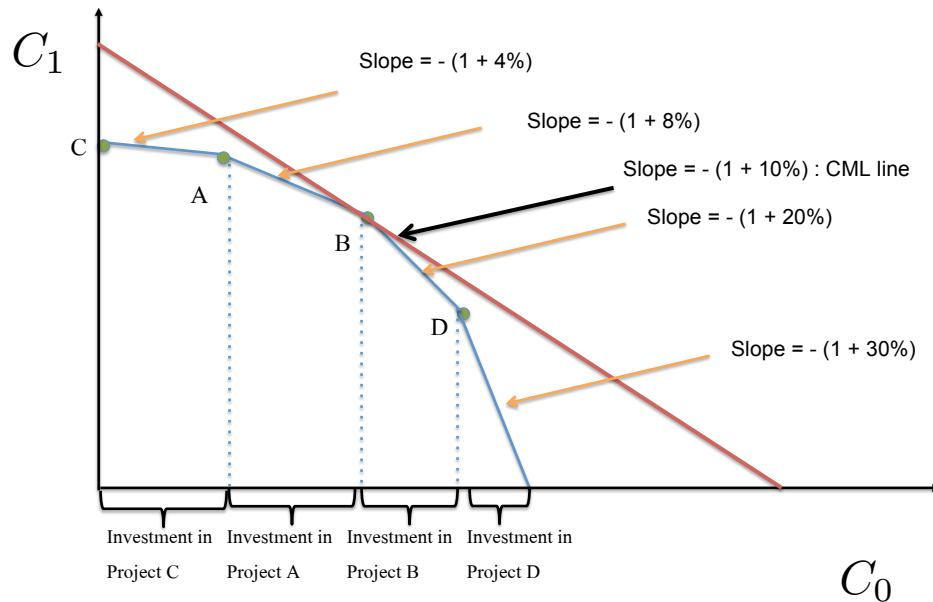
EE431/438 Economics of Financial Markets and Institutions

Exercise 1: Intertemporal Choice

1. Suppose your production opportunity set in a world of perfect certainty consists of the following possibilities

Project	Investment Outlay	Rate of return
A	1,000,000	8%
B	1,000,000	20%
C	2,000,000	4%
D	3,000,000	30%

- (a) Graph the production opportunity set in a C_0, C_1 framework.
 (b) If the market rate of return is 10%, draw the capital market line for the optimal investment decision

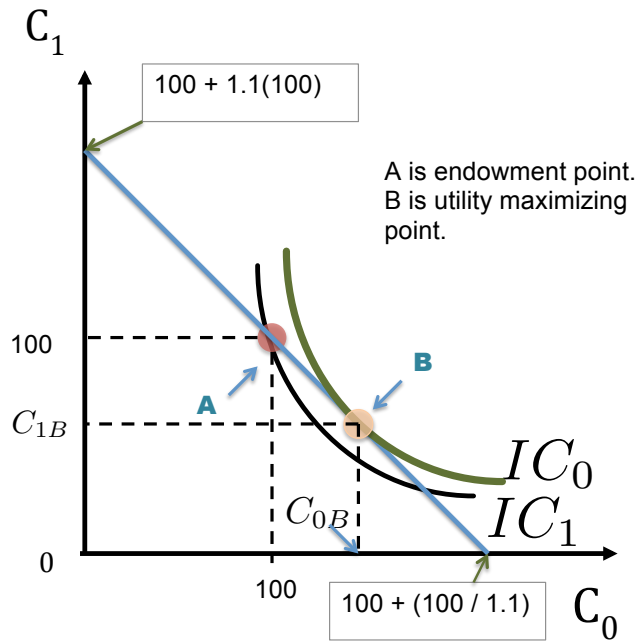


- The consumer will invest the first 3,000,000 dollars in project D. The next 1,000,000 dollars will be invested in project B. Then, 1,000,000 dollars for project A and then 2,000,000 dollars for project C.
- The slope of PPC indicates the rate of returns on investment projects, how much C_1 the consumer will get if she gives up one unit of C_0 to invest in the investment project.
- Thus, the PPC graph is not smooth. It has kinks due to the nature of investment opportunities.

- The CML (Capital Market Line) touches the PPC at point B. The market rate of returns (10%) lies between the rate of returns on project B (20%) and project A (8%).
 - The consumer will produce at point B where the CML touches the PPC. She will invest in project D and project B. The rates of returns on both projects (B and D) are higher than the market interest rate.
 - The consumer will not invest in project A and C because the rates of returns on investment are lower than the market interest rate.
2. Jane has £100 today and will receive another £100 tomorrow. She can borrow or lend at a fixed interest rate of 10%. In this situation she chooses to borrow.

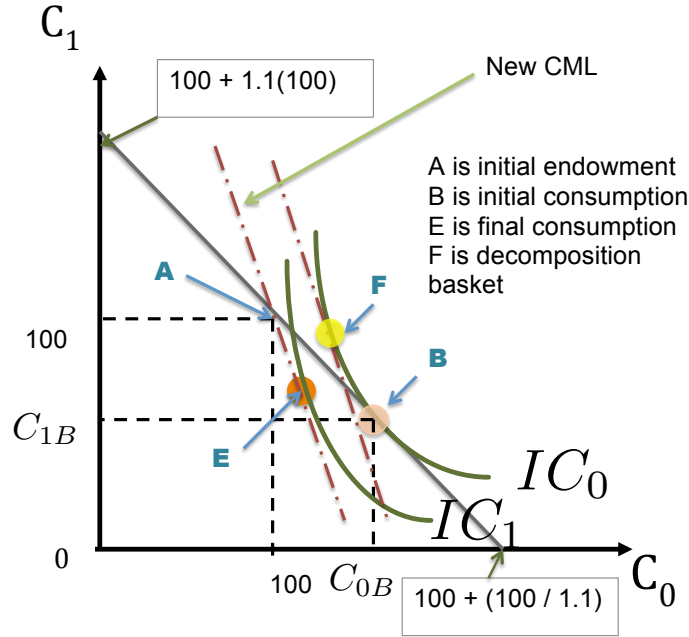
(a) Illustrate this choice using an appropriate diagram.

Figure 1 :



- (b) Now, suppose that the interest rate rises to 20%. By decomposing the effect of the rise in the interest rate into a substitution effect and an income effect, discuss whether Jane will now choose to borrow more or less than in the previous case. Is it possible that she will now choose to lend? Explain.

Figure 2:



- The substitution effect of the change in interest rate is represented by the move from B to F . The income effect is represented by the change from F to E . Substitution effect always reduces C_0 and increases C_1 (current consumption becomes relatively more expensive). Income effect for the borrower is negative (due to higher debt repayment). From the picture, this leads to less consumption for both periods. Jane is going to borrow less (See figure 3). Notice that one may consider a case where C_0 is an inferior good, which is also correct. In that case, the income effect may raise C_0 . Jane will borrow more (See figure 4).

Figure 3:

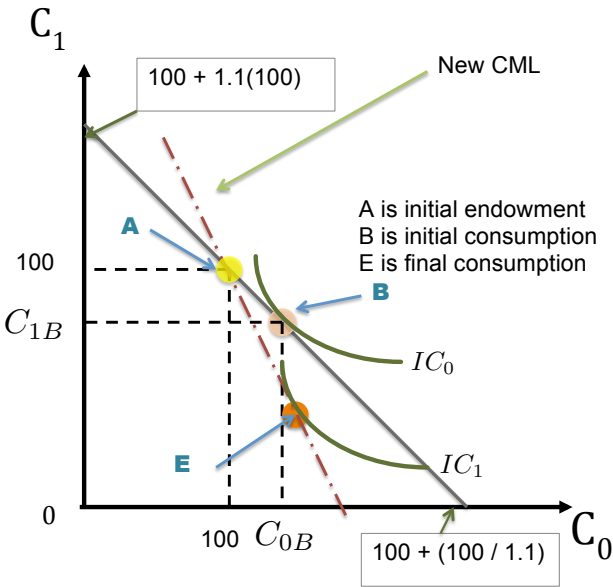
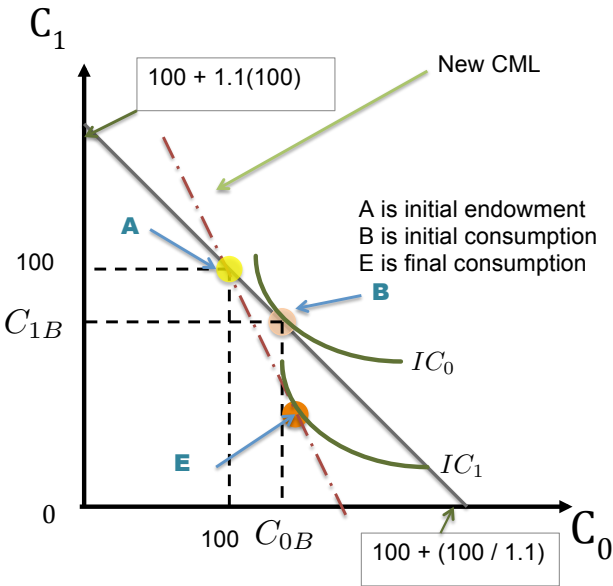
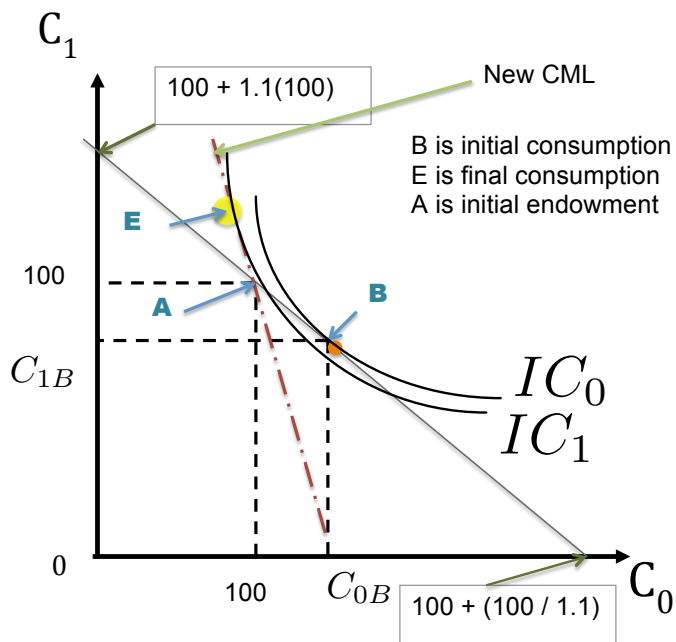


Figure 4:



- If the negative effect of interest rate on C_0 is sufficiently strong, Jane could conceivably lend at the new interest rate. (Notice that in this case, the current consumption (C_0) is lower while the future consumption (C_1) is higher. See figure 5.)

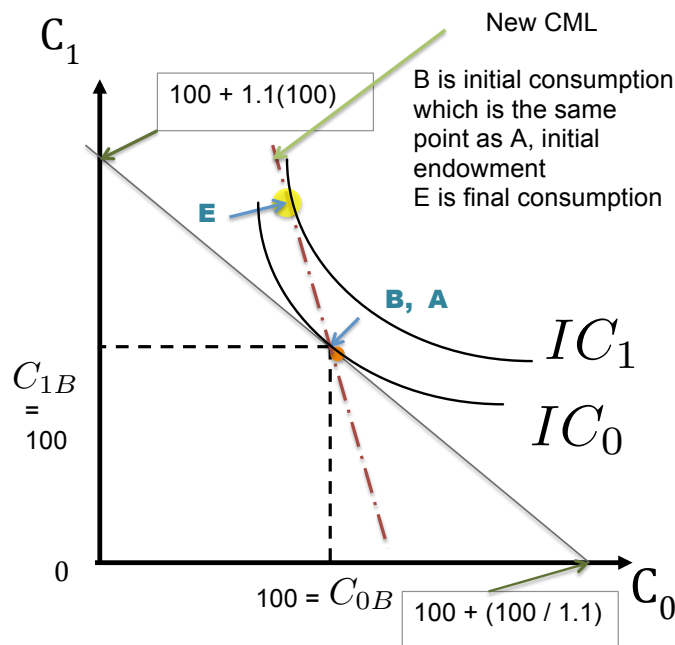
Figure 5:



- Note that the increase in interest rate steepens the budget constraint so that it intersects the horizontal axis at $(100 + 100/1.2)$ and intersects the vertical axis at $(100 + 100(1.2))$. Since the endowment point is still feasible, however, the new budget line goes through the endowment point as well.

(c) Jack also has £100 today and will receive another £100 tomorrow. When the interest rate is 10%, he chooses neither to lend nor to borrow. Explain and illustrate whether he will choose to borrow or to lend when the interest rate rises to 20%.

Figure 6:



Jack must lend when the interest rate rises, since the steeper budget line must now cut the indifference curve at the endowment point. This means that Jack can always increase utility by decreasing C_0 and increasing C_1 . (See figure 6.)

3. An individual cares about present consumption C_1 and future consumption C_2 (subscript denotes time indicator). Her tastes are represented by the following utility function:

$$U(C_1, C_2) = U(C_1) + \beta U(C_2),$$

where $0 < \beta < 1$, $U'(C) > 0$, $U''(C) < 0$. The individual receives an income of Y_1 at period 1 and Y_2 at period 2. She can borrow or lend unlimited amount at a market determined rate of interest, r .

- (a) Calculate marginal rate of substitution between present consumption and future consumption.

$$MRS_{C_1, C_2} = \frac{MU_{C_1}}{MU_{C_2}} = \frac{\frac{\partial U(C_1, C_2)}{\partial C_1}}{\frac{\partial U(C_1, C_2)}{\partial C_2}} = \frac{U'(C_1)}{\beta U'(C_2)}$$

- (b) Show the utility maximizing condition.

$C_2 = (1 + r)(Y_1 - C_1) + Y_2$; therefore, the slope of the budget line is equal to $-(1 + r)$.

The consumer will maximise her preference at the point where the slope of MRS equals to the slope of the budget line, that is

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r$$

rearrange,

$$\frac{U'(C_1)}{U'(C_2)} = (1+r)\beta$$

- (c) If $\beta = \frac{1}{1+r}$, $C_1 = C_2$. True or false? Explain.

If $\beta = \frac{1}{1+r}$, from the utility maximisation condition in (b) $U'(C_1) = U'(C_2)$. Since $U'(C) > 0$, $U''(C) < 0$ (the utility function is always increasing and it is strictly concave), $U'(C_1) = U'(C_2)$ implies $C_1 = C_2$. The statement is true.

- (d) If $\beta \neq \frac{1}{1+r}$, how does the individual allocate her consumption over time? What do you infer from a large value of β ? (e.g. the individual is patient or impatient.)

If $\beta > \frac{1}{1+r}$, $U'(C_1) > U'(C_2)$. Since $U'(C) > 0$, $U''(C) < 0$ (the utility function is always increasing and it is strictly concave), $U'(C_1) > U'(C_2)$ implies that $C_1 < C_2$. (See the graph.)

If $\beta < \frac{1}{1+r}$, $U'(C_1) < U'(C_2)$. Since $U'(C) > 0$, $U''(C) < 0$ (the utility function is always increasing and it is strictly concave), $U'(C_1) < U'(C_2)$ implies that $C_1 > C_2$.

Therefore, a large value of β implies that the individual is patient. (In fact, this is obvious from the utility function. As β is large, the individual give a large weight to the utility from her future consumption.)

