EE425: Homework2

1. Given the indifference curve is $X_i Y_i = \beta_1 + \beta_2 X_i + u_i$. To estimate the parameters, divide the equation by X_i. So, $Y_i = \beta_2 + \frac{\beta_1}{X_i} + \frac{u_i}{X_i}$. Estimation result of the OLS is

$$Y_i = 1.1009 + 3.2827 \frac{1}{X_i}$$

Dependent Variable: Y
Method: Least Squares
Date: 09/17/11 Time: 21:20
Sample: 1 5
Included observations: 5

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.100884	0.681665	1.614994	0.2047
1/X	3.282735	1.259920	2.605511	0.0800

R-squared	0.693524	Mean dependent var	2.600000
Adjusted R-squared	0.591365	S.D. dependent var	1.278671
S.E. of regression	0.817385	Akaike info criterion	2.723761
Sum squared resid	2.004355	Schwarz criterion	2.567536
Log likelihood	-4.809403	F-statistic	6.788687
Durbin-Watson stat	1.294360	Prob(F-statistic)	0.079994

Therefore, the marginal rate of substitution is $\frac{dY}{dX} = -3.2827\left(\frac{1}{X^2}\right)$. The mean of X is 3.

Therefore, MRS at the mean value is $\frac{dY}{dX} = -3.2827\left(\frac{1}{3^2}\right) = -0.3647$.

2. Dependent Variable: Y
Method: Least Squares
Date: 09/17/11 Time: 21:42
Sample: 1 36
Included observations: 36

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-38.11125	39.78857	-0.957844	0.3449
X	1.454896	0.039747	36.60379	0.0000

R-squared	0.975252	Mean dependent var	345.6300
Adjusted R-squared	0.974524	S.D. dependent var	1442.843
S.E. of regression	230.2955	Akaike info criterion	13.77056
Sum squared resid	1803225.	Schwarz criterion	13.85853
Log likelihood	-245.8700	F-statistic	1339.838
Durbin-Watson stat	2.041119	Prob(F-statistic)	0.000000

- To test if $\beta_1 = 0$, the hypothesis is

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

According to the estimation output, the p-value of the intercept term is larger than any reasonable significance level. Therefore, we do not reject H_0. That means β_1 is practically zero as suggested by PPP.

- To test if $\beta_2 = 1$, the hypothesis is

$$H_0 : \beta_2 = 1$$
$$H_a : \beta_2 \neq 1$$

The computed t-value is $t = \frac{1.454896 - 1}{0.039747} = 11.4448$. Applying the 0.05 level of significance, the critical value of t-statistic is $t_{\frac{0.05}{2},34} \approx 2.042$. Since computed t-value is greater than the critical value, we reject H_0 at 0.05 level of significance, or $\beta_2 \neq 1$.

So from the estimated equation of 36 countries in 2011, PPP does not hold. This may happen because the PPP is expected to hold in the long run. This estimation covers only 36 countries and one year period only.

3. (i) $Y_t = \alpha_1 + \alpha_2 X_t + u_t$

(ii) $Y_t = \beta X_t + u_t$

a) Given Y is consumption and X is GDP, α_2 and β tell us how much the consumption is expected to change given 1 unit increase in GDP. However, model (i) and (ii) differ regarding the intercept term. Model (i) assumes that even though people have no income at all, they still have to consume. α_1 can be regarded as subsistence level or autonomous consumption. Model (ii), on the other hand, assumes the model through the origin which means people will consume only when they have income, representing the long run consumption function.

b) To judge which model is preferred, firstly, we need the estimation outputs of these 2 models.

- Model (i) $Y_t = \alpha_1 + \alpha_2 X_t + u_t$

Dependent Variable: RCONS
Method: Least Squares
Date: 09/17/11 Time: 22:24
Sample: 1988 2009
Included observations: 22

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	117229.3	21830.86	5.369891	0.0000
RGDP	0.504787	0.006948	72.65245	0.0000

R-squared	0.996225	Mean dependent var	1650790.
Adjusted R-squared	0.996037	S.D. dependent var	415025.4
S.E. of regression	26128.40	Akaike info criterion	23.26594
Sum squared resid	1.37E+10	Schwarz criterion	23.36513
Log likelihood	-253.9254	F-statistic	5278.379
Durbin-Watson stat	0.772316	Prob(F-statistic)	0.000000

- Model (ii) $Y_t = \beta X_t + u_t$

Dependent Variable: RCONS
Method: Least Squares
Date: 09/17/11 Time: 22:26
Sample: 1988 2009
Included observations: 22

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RGDP	0.540862	0.002704	200.0500	0.0000

R-squared	0.990783	Mean dependent var	1650790.
Adjusted R-squared	0.990783	S.D. dependent var	415025.4
S.E. of regression	39844.84	Akaike info criterion	24.06776
Sum squared resid	3.33E+10	Schwarz criterion	24.11736
Log likelihood	-263.7454	Durbin-Watson stat	0.342659

Model (i) is preferred to model (ii) because, according to the estimation output of model (i), both intercept and the slope coefficients have the expected sign and are statistically different from zero. The explanatory power of model(i) is slightly better.

c) $Y_t^* = \alpha_1 + \alpha_2 X_t^* + u_t$

Dependent Variable: SRCONS
Method: Least Squares
Date: 09/17/11 Time: 22:34
Sample: 1988 2009
Included observations: 22

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.00E-18	0.013422	2.24E-16	1.0000
SRGDP	0.998111	0.013738	72.65245	0.0000

R-squared	0.996225	Mean dependent var	1.61E-16
Adjusted R-squared	0.996037	S.D. dependent var	1.000000
S.E. of regression	0.062956	Akaike info criterion	-2.606249
Sum squared resid	0.079270	Schwarz criterion	-2.507063
Log likelihood	30.66874	F-statistic	5278.379
Durbin-Watson stat	0.772316	Prob(F-statistic)	0.000000

$\alpha_2 = 0.9981$: when standardized GDP increases by 1, the standardized consumption is expected to increase by 0.9981.

α_1 is practically zero. That means when the standardized GDP is zero, the standardized consumption is also expected to be zero.

d) Divide real GDP and real consumption by 1000,

Dependent Variable: RCONS_1000
Method: Least Squares
Date: 09/17/11 Time: 22:42
Sample: 1988 2009
Included observations: 22

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	117.2293	21.83086	5.369891	0.0000
RGDP_1000	0.504787	0.006948	72.65245	0.0000

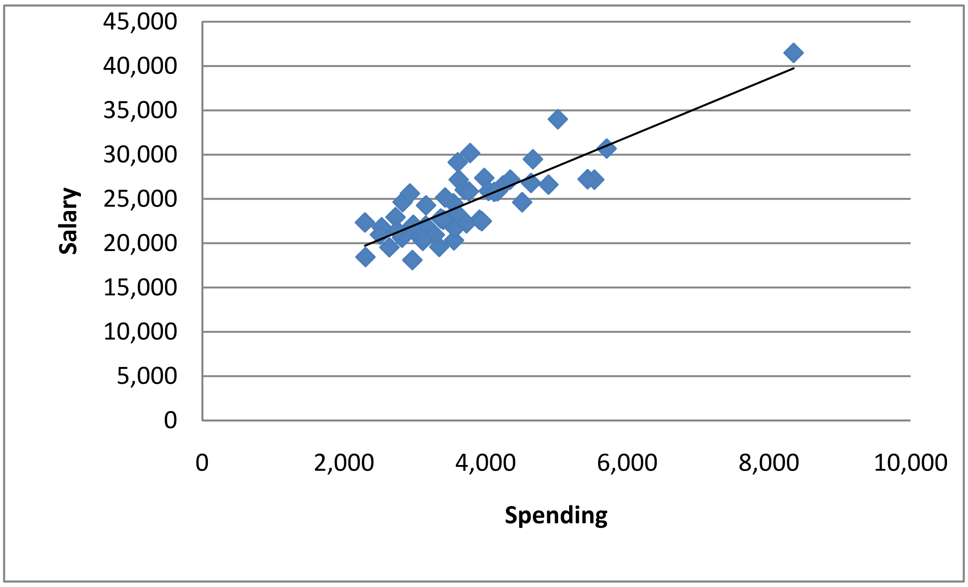
R-squared	0.996225	Mean dependent var	1650.790
Adjusted R-squared	0.996037	S.D. dependent var	415.0254
S.E. of regression	26.12840	Akaike info criterion	9.450431
Sum squared resid	13653.87	Schwarz criterion	9.549617
Log likelihood	-101.9547	F-statistic	5278.379
Durbin-Watson stat	0.772316	Prob(F-statistic)	0.000000

Comparing the estimation output in (d) with the original model (i), one can see that the coefficient of real GDP remains the same. On the other hand, the intercept term in (d) is 1000 times less than the intercept term in (i).

$$\alpha_2^* = \alpha_2$$
$$\alpha_1^* = w_1\alpha_1;\; w_1 = \frac{1}{1000}$$

according to the relationship as derived.

4. a)



b)

Dependent Variable: SALARY
Method: Least Squares
Date: 09/18/11 Time: 21:49
Sample: 1 51
Included observations: 51

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	12129.37	1197.351	10.13017	0.0000
SPEND	3.307585	0.311704	10.61129	0.0000

R-squared	0.696781	Mean dependent var	24356.22
Adjusted R-squared	0.690593	S.D. dependent var	4179.426
S.E. of regression	2324.779	Akaike info criterion	18.37906
Sum squared resid	2.65E+08	Schwarz criterion	18.45482
Log likelihood	-466.6661	F-statistic	112.5995
Durbin-Watson stat	1.254380	Prob(F-statistic)	0.000000

The estimates of the parameters, their standard errors, R^2 and RSS can be viewed from the estimation output. In estimation output, RSS is Sum squared resid (=2.65x10^8). From the calculation ESS = 608,555,015. In order to find ESS, you may have to calculate TSS from $R^2 = 1 - \frac{RSS}{TSS}$ first. Then calculate ESS by $R^2 = \frac{ESS}{TSS}$.

c) From estimation, $\beta_2 = 3.3076$, if the per pupil expenditure increases by 1 dollar, the expected salary will increase by 3.3076 dollars. In addition, if there is no spending on public schools per pupil, the salary will be 12129.37 dollars per year.

The slope of the model makes sense. A district with high spending per pupil implies that district has more concern on the education of pupils. So, the teachers in high-spending-per-pupil district can be paid higher as well. On the other hand, the intercept term does not have much economic sense. If the school does not spend any money at all (spending on public schools per pupils = 0), then it means the school is not be able to pay the teachers.

d) The 95% confidence interval of β_2 is

$$\hat{\beta}_2 \pm t_{\frac{0.05}{2},49} se(\hat{\beta}_2) = [3.3076 - 2.010(0.3117), 3.3076 + 2.010(0.3117)] = [2.6811, 3.9341]$$

From the hypothesis,

$$H_0 : \beta_2 = 3$$
$$H_a : \beta_2 \neq 3$$

Since the 95% confidence interval of β_2 includes 3, therefore we do not to reject that the true slope coefficient is 3.

e) The ANOVA table is

Source	SS	df	MS
Model	608555015	1	608555015
Residual	264825250	49	5404596.94
Total	873380265	50	17467605.3

To test the following hypothesis,

$$H_0 : \beta_2 = 0$$
$$H_a : \beta_2 \neq 0$$

we may use the F-test, $\text{F}=\frac{ESS/(k-1)}{RSS/n-k} = \frac{608555015}{5404596.94} = 112.60$.

The critical value $F_{0.05,1,49} \approx 4.04$. Therefore, the computed F falls into the rejection region. We reject H_0 at 0.05 level of significant. In other words, spending has a significant impact on salary.

f) The mean forecast value of salary when per pupil spending equals 9000 is

$$E(salary | spending = 9000) = 12129.37 + 3.3076(9000) = 41897.77$$

$$\text{var}(sa\hat{l}ary) = \sigma^2 \left[\frac{1}{n} + \frac{(X_{n+1} - \bar{X})^2}{\sum x_i^2} \right] = 5404596.94 \left[\frac{1}{51} + \frac{(9000 - 3696.61)^2}{55625998} \right] = 2838676.70$$

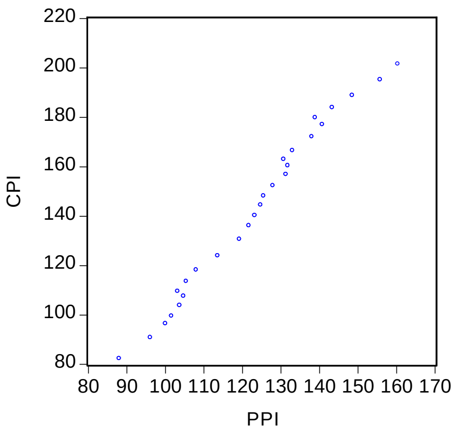
$$\therefore se(sa\hat{l}ary) = \sqrt{2838676.70} = 1684.84$$

The 95% confidence interval is

$$sa\hat{l}ary \pm t_{\frac{0.05}{2},49} se(sa\hat{l}ary) = [41897.77 - 2.010(1684.84), 41897.77 + 2.010(1684.84)]$$

$$= [38511.2416, 45284.2984]$$

5.



a) According to the scatter plot, CPI and PPI move together approximately in a linear fashion. Therefore, one may expect the linear relationship among these 2 variables.

b) CPI should be the dependent variable, while PPI is the independent variable. PPI is producers' price which is close to cost of production, and CPI is consumer price index for finished goods. Thus, CPI, the price of outputs at final sales, should depend on the producers'price.

c) $CPI_i = \beta_1 + \beta_2 PPI_i + \varepsilon_i$

Dependent Variable: CPI
Method: Least Squares
Date: 09/13/11 Time: 19:08
Sample: 1980 2006
Included observations: 27

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-81.01611	5.492246	-14.75100	0.0000
PPI	1.817620	0.044181	41.14020	0.0000

R-squared	0.985444	Mean dependent var	142.3963
Adjusted R-squared	0.984862	S.D. dependent var	34.67915
S.E. of regression	4.266824	Akaike info criterion	5.810804
Sum squared resid	455.1447	Schwarz criterion	5.906792
Log likelihood	-76.44585	F-statistic	1692.516
Durbin-Watson stat	0.601660	Prob(F-statistic)	0.000000

To test if these 2 variables has 1-to-1 relationship, it is the same as testing if these 2 indexes has a linear relationship. Therefore, there are 2 hypotheses to be tested

$$1.\ H_0 : \beta_1 = 0,\ H_a : \beta_1 \neq 0$$
$$2.\ H_0 : \beta_2 = 0,\ H_a : \beta_2 \neq 0$$

From the estimation output, p-values of both t-tests are practically zero. Therefore, we may conclude that there is a significant linear relationship between CPI and PPI. Since a liner relationship establishes a 1-to-1 relationship between them.