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HW2

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1.1) $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$
 $\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$

$\bar{y} = \frac{2.4 + 3.4 + 3 + 3.5 + 3.6 + 3 + 2.7 + 3.7}{9} = 3.2125$
 $\bar{x} = \frac{63 + 72 + 78 + 91 + 97 + 75 + 75 + 90}{9} = 77.625$
 $x_i = x_i - \bar{x}, y_i = y_i - \bar{y}$

$x_1 = -14.625$	$y_1 = -0.4125$	$x_1 y_1 = 6.0318$
$x_2 = -5.625$	$y_2 = 0.1875$	$x_2 y_2 = -1.0547$
$x_3 = -0.375$	$y_3 = -0.2125$	$x_3 y_3 = 0.0797$
$x_4 = 3.375$	$y_4 = 0.2875$	$x_4 y_4 = 0.9703$
$x_5 = 9.375$	$y_5 = 0.3875$	$x_5 y_5 = 3.6323$
$x_6 = -2.625$	$y_6 = -0.2125$	$x_6 y_6 = 0.5578$
$x_7 = -2.625$	$y_7 = -0.5125$	$x_7 y_7 = 1.3453$
$x_8 = 12.375$	$y_8 = 0.4875$	$x_8 y_8 = 6.0328$
$\sum x_i^2 = 511.9748$		$\sum x_i y_i = 17.4374$

$\hat{\beta}_2 = \frac{17.4374}{511.9748} = 0.0341$
 $\hat{\beta}_1 = 3.2125 - (0.0341)(77.625) = 0.5655$

$x_1^2 = 213.906$
$x_2^2 = 31.6406$
$x_3^2 = 0.1406$
$x_4^2 = 11.3906$
$x_5^2 = 97.9906$
$x_6^2 = 6.8906$
$x_7^2 = 6.8906$
$x_8^2 = 153.1406$

1.2 $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$
 $\hat{y}_1 = 0.5655 + 0.0341 x_1$
 $\hat{u}_i = y_i - \hat{y}_i$

$\hat{y}_1 = 0.5655 + 0.0341(63) = 2.7138$
$\hat{y}_2 = 0.5655 + 0.0341(72) = 3.0207$
$\hat{y}_3 = 0.5655 + 0.0341(78) = 3.2253$
$\hat{y}_4 = 0.5655 + 0.0341(91) = 3.3276$
$\hat{y}_5 = 0.5655 + 0.0341(97) = 3.5322$
$\hat{y}_6 = 0.5655 + 0.0341(75) = 3.123$
$\hat{y}_7 = 0.5655 + 0.0341(75) = 3.123$
$\hat{y}_8 = 0.5655 + 0.0341(90) = 3.3345$

$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$	$\hat{u}_1^2 = 0.0074$
$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$	$\hat{u}_2^2 = 0.1439$
$\hat{u}_3 = 3 - 3.2253 = -0.2253$	$\hat{u}_3^2 = 0.0508$
$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$	$\hat{u}_4^2 = 0.0297$
$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$	$\hat{u}_5^2 = 0.0046$
$\hat{u}_6 = 3 - 3.123 = -0.123$	$\hat{u}_6^2 = 0.0151$
$\hat{u}_7 = 2.7 - 3.123 = -0.423$	$\hat{u}_7^2 = 0.1789$
$\hat{u}_8 = 3.7 - 3.3345 = 0.3655$	$\hat{u}_8^2 = 0.0043$

$\sum \hat{u}_i = 0.00001 \approx 0$ $\sum \hat{u}_i^2 = 0.4347$

1.3 $\text{var}(\hat{\beta}_2) = \frac{\sum x_i^2}{n \sum x_i^2} \hat{\sigma}^2 = \frac{[(63)^2 + (72)^2 + (78)^2 + (91)^2 + (97)^2 + (75)^2 + (75)^2 + (90)^2](0.0725)}{(8)(511.9748)} = 0.8625$

$\text{var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{0.0725}{511.9748} = 1.416 \times 10^{-4}$

$\hat{\sigma}^2$ estimate by $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{9-2} = 0.0725$

② $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$

1.1) $\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$

$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$
 $\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$
 $x_i = x_i - \bar{x}, y_i = y_i - \bar{y}$

$x_1 = -10$	$x_1^2 = 100$
$x_2 = -8$	$x_2^2 = 64$
$x_3 = -6$	$x_3^2 = 36$
$x_4 = -4$	$x_4^2 = 16$
$x_5 = -2$	$x_5^2 = 4$
$x_6 = 2$	$x_6^2 = 4$
$x_7 = 4$	$x_7^2 = 16$
$x_8 = 6$	$x_8^2 = 36$
$x_9 = 8$	$x_9^2 = 64$
$x_{10} = 10$	$x_{10}^2 = 100$
	$\sum x_i^2 = 440$

$\hat{\beta}_2 = \frac{394}{440} = 0.8954$

$\hat{\beta}_1 = 9.1 - 0.8954(20) = -3.808$

$y_1 = -9.1$	$x_1 y_1 = 91$
$y_2 = -7.1$	$x_2 y_2 = 56.8$
$y_3 = -4.1$	$x_3 y_3 = 24.6$
$y_4 = -3.1$	$x_4 y_4 = 12.4$
$y_5 = -2.1$	$x_5 y_5 = 4.2$
$y_6 = 0.9$	$x_6 y_6 = 1.8$
$y_7 = 0.9$	$x_7 y_7 = 3.6$
$y_8 = 5.9$	$x_8 y_8 = 35.4$
$y_9 = 6.9$	$x_9 y_9 = 55.2$
$y_{10} = 10.9$	$x_{10} y_{10} = 109$

$\sum x_i y_i = 394$

$$\hat{y}_1 = \hat{\beta}_0 + \hat{\beta}_1 x_1$$

$$\hat{y}_1 = -8.808 + 0.8954 x_1$$

$$\hat{u}_1 = y_1 - \hat{y}_1$$

2.2)

$$\hat{y}_1 = -8.808 + 0.8954(10) = 0.146$$

$$\hat{y}_2 = -8.808 + 0.8954(12) = 1.9368$$

$$\hat{y}_3 = -8.808 + 0.8954(14) = 3.7276$$

$$\hat{y}_4 = -8.808 + 0.8954(16) = 5.5184$$

$$\hat{y}_5 = -8.808 + 0.8954(18) = 7.3092$$

$$\hat{y}_6 = -8.808 + 0.8954(22) = 10.8908$$

$$\hat{y}_7 = -8.808 + 0.8954(24) = 12.6816$$

$$\hat{y}_8 = -8.808 + 0.8954(26) = 14.4724$$

$$\hat{y}_9 = -8.808 + 0.8954(28) = 16.2632$$

$$\hat{y}_{10} = -8.808 + 0.8954(30) = 18.054$$

$$\hat{u}_1 = 0 - 0.146 = -0.146$$

$$\hat{u}_2 = 2 - 1.9368 = 0.0632$$

$$\hat{u}_3 = 5 - 3.7276 = 1.2724$$

$$\hat{u}_4 = 6 - 5.5184 = 0.4816$$

$$\hat{u}_5 = 7 - 7.3092 = -0.3092$$

$$\hat{u}_6 = 10 - 10.8908 = -0.8908$$

$$\hat{u}_7 = 10 - 12.6816 = -2.6816$$

$$\hat{u}_8 = 15 - 14.4724 = 0.5276$$

$$\hat{u}_9 = 16 - 16.2632 = -0.2632$$

$$\hat{u}_{10} = 20 - 18.054 = 1.946$$

$$\hat{u}_1^2 = 0.0213$$

$$\hat{u}_2^2 = 0.0039$$

$$\hat{u}_3^2 = 1.6190$$

$$\hat{u}_4^2 = 0.2319$$

$$\hat{u}_5^2 = 0.0956$$

$$\hat{u}_6^2 = 0.7935$$

$$\hat{u}_7^2 = 7.1909$$

$$\hat{u}_8^2 = 0.2784$$

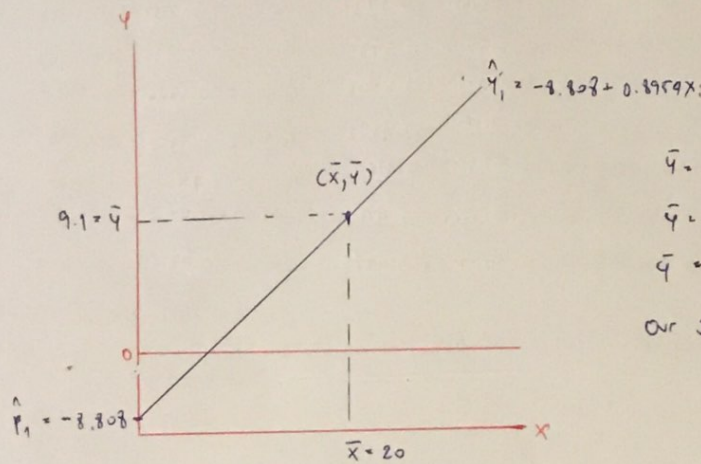
$$\hat{u}_9^2 = 0.0693$$

$$\hat{u}_{10}^2 = 3.7869$$

$$\sum \hat{u}_i = 0$$

$$\sum \hat{u}_i^2 = 14.0907$$

2.3)



$$\bar{y} = -8.808 + 0.8954(\bar{x})$$

$$\bar{y} = -8.808 + 0.8954(20)$$

$$\bar{y} = 9.1$$

Our SRP will pass through the (\bar{x}, \bar{y})

2.4) If $x_1 = 18$ the predicted of y is $-8.808 + 0.8954(18) = 7.3092$

2.5) $\text{Var}(\hat{u}_1), \text{Var}(\hat{\beta}_1), \text{Var}(\hat{\beta}_2)$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum x_i^2 \hat{u}_i^2}{\sum x_i^2} = \frac{(10^2 + 12^2 + 14^2 + 16^2 + 18^2 + 22^2 + 24^2 + 26^2 + 28^2 + 30^2)(1.7613)}{(10)(440)} = 1.7773$$

$$\text{Var}(\hat{\beta}_2) = \frac{\hat{u}_i^2}{\sum x_i^2} = \frac{1.7613}{440} = 4.0029 \times 10^{-3}$$

$$\hat{\sigma}^2 u \text{ estimated by } \hat{u}_i^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0907}{8} = 1.7613$$

3) As $\hat{\beta}_1$ is an estimator of β_1 , $E(\hat{\beta}_1)$ should be equal to β_1

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}, \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \text{ let } h = \frac{\bar{x}}{\sum x_i^2}$$

$$\hat{\beta}_1 = \bar{y} - \sum h y_i \bar{x} \text{ and } y_i = \beta_1 + \beta_2 x_i + u_i$$

$$\hat{\beta}_1 = \frac{\sum y_i}{n} - \sum h y_i \bar{x} = \sum \left(\frac{1}{n} - h \bar{x} \right) y_i$$

$$= \sum \left(\frac{1}{n} - h \bar{x} \right) (\beta_1 + \beta_2 x_i + u_i)$$

$$= \sum \left(\frac{\beta_1}{n} + \frac{\beta_2 x_i}{n} + \frac{u_i}{n} - h \bar{x} \beta_1 - h \bar{x} \beta_2 x_i - h \bar{x} u_i \right)$$

$$\hat{\beta}_1 = \beta_1 + \beta_2 \bar{x} + \frac{\sum u_i}{n} - \beta_2 \bar{x} - \bar{x} \sum h u_i$$

take EC $\rightarrow E(\hat{\beta}_1) = E(\beta_1) + \bar{x} E(\sum h u_i) \rightarrow$ assumption 3: zero mean value of disturbance treat x as given $E(u_i | x_i)$

as h is $f(x)$, h would be constant $\sum h u_i = \text{constant}$

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{x} E(\sum h u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$

$\hat{\beta}_1$ is unbiased estimator of β_1