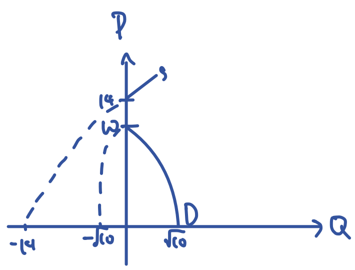
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



1.2) $P = 10 - Q^2$ ——— 1

$Q = -14 + P$

$P = Q + 14$ ——— 2

1 = 2 ; $10 - Q^2 = Q + 14$

$Q^2 + Q + 4 = 0$

1.3) supply curve will shift to the right

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$R'(Q) = \frac{1}{Q^2+1}(2Q) + \frac{3}{(Q+1)^2}$ $\left(\frac{Q}{Q+1}\right)' = \frac{(Q+1)(1)-(Q)(1)}{(Q+1)^2} = \frac{1}{(Q+1)^2}$

$; Q \geqslant 0, Q^2+1 \geqslant 1 > 0 \Rightarrow (Q+1)^2 > 0$ $\therefore R'(Q) = DNE\ \phi$

set $R'(Q) = 0$

$R'(Q) = 0$

$\frac{1}{Q^2+1}(2Q) + \frac{3}{(Q+1)^2} = 0$

$Q = 0$

when $Q \geq 0$ $R'(Q) > 0$

so $R(Q)$ is an increasing function

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

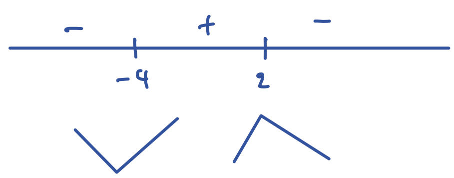
$\pi'(Q) = -Q^2 - 2Q + 8$

critical point $\Rightarrow \pi'(Q) = 0$

$-Q^2 - 2Q + 8 = 0$

$(Q+4)(Q-2) = 0$

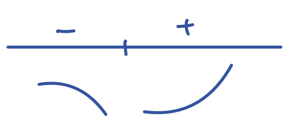
$Q = -4, 2$



profit maximizing when $Q = 2$

$\pi(2) = \frac{25}{3}$

<u>Second derivative</u> $\pi''(Q) = -2Q - 2$



point of infection $(-1, \pi(-1)) = (-1, -\frac{29}{3})$

$\therefore \pi''(-4) = 6 > 0 \Rightarrow$ Rel. min

$\pi''(2) = -6 > 0 \Rightarrow$ Rel. max

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B ; unable because of size

4.2 A*B $= \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$

4.3 det(A) $= |A| = \begin{vmatrix} 8 & 9 \\ 10 & 11 \end{vmatrix} = (8\times 11) - (10 \times 9) = -2$

4.4 det(B) ; unable because B is not a square matrix

4.5 det(C) $= \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \underset{R_2 - R_1}{\overset{R_3 - R_2}{=}} \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} \underset{R_2 - R_1}{\overset{R_3 - R_1}{=}} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = a x^{a-1}(y^b) + \frac{x+y}{x}\left[\frac{(x+y)(1)-(x)(1)}{(x+y)^2}\right]$$

$$= a x^{a-1} \cdot y^b + \frac{(x+y)\,y}{x(x+y)^2}$$

$$\frac{\partial U}{\partial y} = (x^a)\, b y^{b-1} + \frac{x+y}{y}\left[\frac{(x+y)(0)-(x)(1)}{(x+y)^2}\right]$$

$$= b x^a y^{b-1} - \frac{(x+y)(x)}{y(x+y)^2}$$