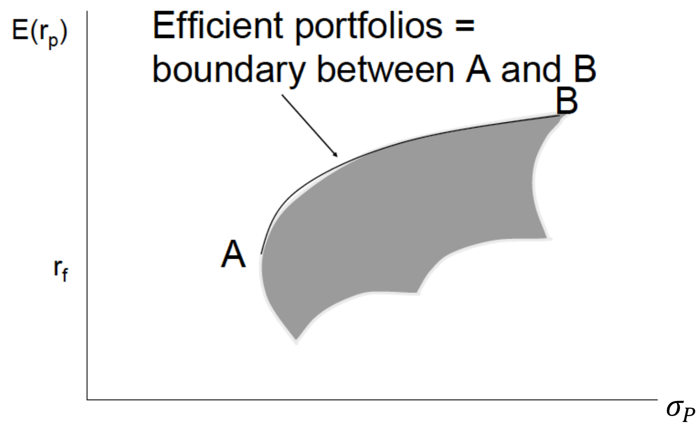
Practice Final
Suggested Solutions
FN 312

Question 1

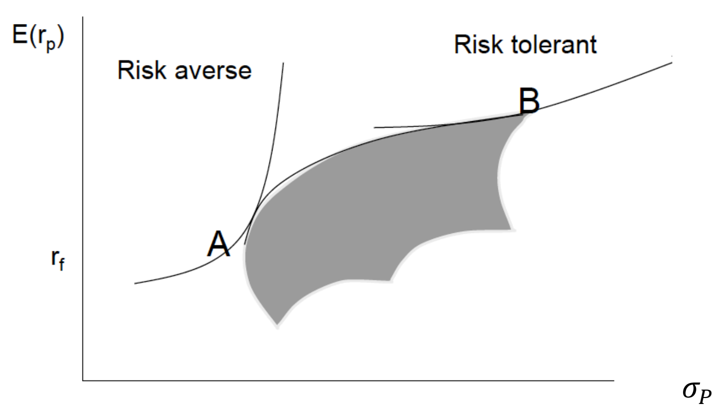
a)

Set of Capital Market Risky Portfolios



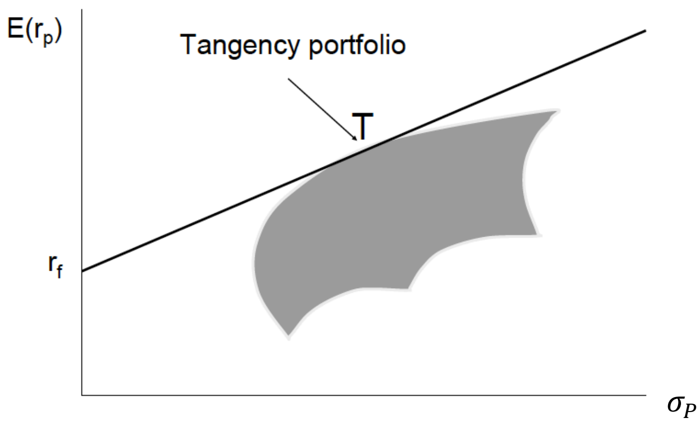
b)

Set of Capital Market Risky Portfolios



c)

Set of Capital Market Risky Portfolios



d) The mutual fund seperation theorm says that an investement problem can be broken down into two parts (1) choosing the optimal risky portfolio or the tangency portfolio (portfolio of risky assets with the highest Sharpe's ratio and (2) choosing an efficient portfolio that is a combination of the risk free asset and the optimal risky portfolio. The choice of (1) does not depend on the investor's level of risk aversion while the choice of (2) does.

e)

Since efficient portfolios are simple combinations of T-bills and the S&P 500, we know that

$$E[R_p] = r_f + x_{sp500}(E[R_{sp500}] - r_f)$$
$$\Rightarrow x_{sp500} = \frac{E[R_p] - r_f}{E[R_{sp500}] - r_f}$$

Since our target portfolio expected return is 10% we solve

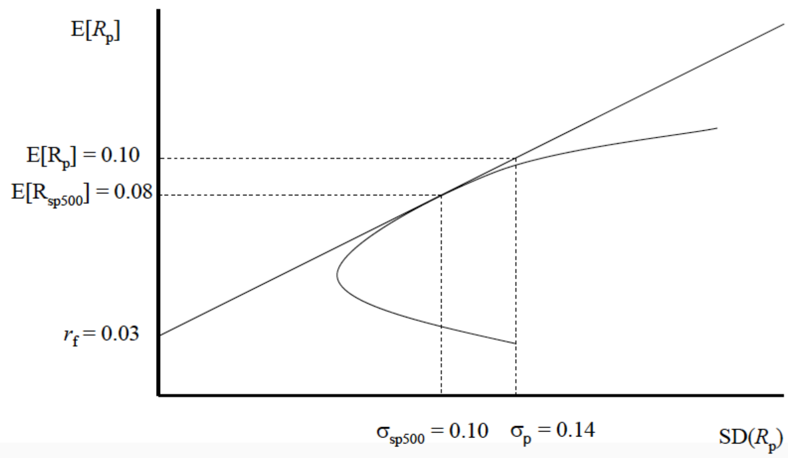
$$x_{sp500} = \frac{0.10 - 0.03}{0.08 - 0.03} = \frac{0.07}{0.05} = 1.4$$

The expected return on this portfolio is

$$E[R_p] = 0.03 + 1.4(0.08 - 0.03) = 0.10$$

The standard deviation of the return on this portfolio is

$$SD(R_p) = x_{sp500}SD(R_{sp500}) = 1.4(0.10) = 0.14$$



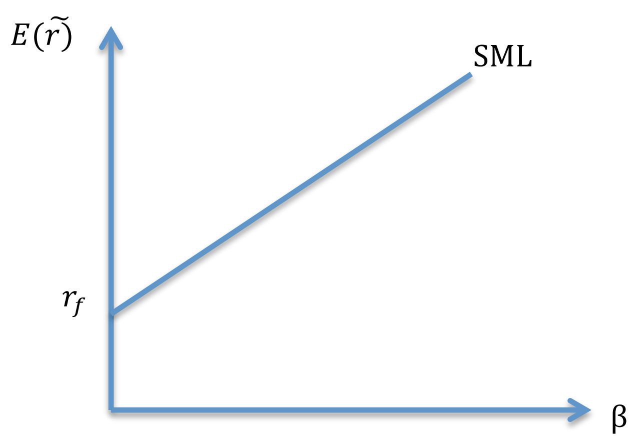
Question 2

a)

The SML gives the relationship $E[R]=r_f+\beta(E[R_M]-r_f)$. *For the given data, the SML equation is*

$$E[R]=0.04+\beta\times(0.12-0.04)=0.04+\beta\times(0.08)$$

Graphically, this relationship is shown below.



b)

$$E[R_M]-r_f=0.12-0.04=0.08$$
$$=\text{ slope of SML}$$

c)

$$E[R]=0.04+(1.5)\times(0.08)=0.16$$

d)

The required rate of return from the CAPM is

$$E[R] = 0.04 + 0.8(0.08) = 0.104$$

Since the project's expected rate of return of 0.098 is less than 0.104, using the IRR rule you should not do the project.

e)

$$0.112 = 0.04 + \beta(0.08) \Rightarrow \beta = \frac{0.112 - 0.04}{0.08} = 0.9$$

Since beta measures the part of return that is explained by market systematic risk, beta<1 implies a stock that is less affected by business cycles. These are typically stable, defensive stocks such as consumer products and utilities.

Question 3

a)

The spot rates are computed using

$$_0r_1 = \left(\frac{1000}{943.40}\right) - 1 = 0.0600$$

$$_0r_2 = \left(\frac{1000}{898.47}\right)^{1/2} - 1 = 0.0550$$

$$_0r_3 = \left(\frac{1000}{847.62}\right)^{1/3} - 1 = 0.0567$$

$$_0r_4 = \left(\frac{1000}{792.16}\right)^{1/4} - 1 = 0.0600$$

b)

The implied forward rates are computed using

$$_1f_2 = \frac{(1 +_0 r_2)^2}{1 +_0 r_1} - 1 = 0.0500$$

$$_2f_3 = \frac{(1 +_0 r_3)^3}{(1 +_0 r_2)^2} - 1 = 0.0600$$

$$_3f_4 = \frac{(1 +_0 r_4)^4}{(1 +_0 r_1)^3} - 1 = 0.0700$$

c) No. This means that you would be paying more today for \$ received in 5 years than you would for \$ received in 4 years. No rational person would do this.

d) The yield curve is upward sloping, so the implied forward rates are higher than today's 1 year spot rate. However, the yield curve flattens at about 20 years so the implied forward rate are constant after about 20 years.

e) When answering this question you should make an assumption about whether the liquidity premium is constant or increasing. For example you could have an upward sloping yield curve with falling expected short rates if the liquidity premium is increasing.

Question 4

a)

At expiration of the option (1 year) the values of the call and put will be

up state: $C^{up} = \max(0, 30-25) = 5; P^{up} = \max(0, 25-30) = 0$

down state: $C^{down} = \max(0, 20-25) = 0; P^{down} = \max(0, 25-20) = 5$

b) To determine the value of the call, we first compute the hedge ratio.

Hedge ratio = range of values for the call option/range of values for the stock
= 5/(30-20) = 0.5

According to the hedge ratio, a riskless portfolio can be constructed from buying 0.5 stock and shorting 1 call option:

	Up state	Down state
Stock Value (0.5 shares)	15	10
Call obligation (write 1 call)	-5	0
Total	10	10

As shown, the value of the portfolio is 10 no matter what the price of the stock is ie. The portfolio is perfectly hedged.

For the reason, the return on this portfolio must be equal to the risk free rate

0.5*25 – C = 10/0.03
C=2.791

c)

To determine the value of the put, use the put-call parity relationship:

$$C + PV(X) = S + P \Rightarrow P = C + PV(X) - S$$

$$\Rightarrow P = 2.791 + \frac{25}{1.03} - 25 = 2.063$$

d) Here, we can simply repeat the above calculations using the higher T-bill rate. Notice that the increase in the T-bill rate does not change the Hedge ratio.

$$C = 0.5(25) - \frac{10}{1.04} = 2.885$$

Here we see that the call increases in value. By put-call parity

$$P = 2.885 + \frac{25}{1.04} - 25 = 1.830$$

and we see that the put value declines.

This makes sense because as the interest rate increase, the present value of the exercise price of the option falls which causes call price to increase and the put price to decline.

Question 5

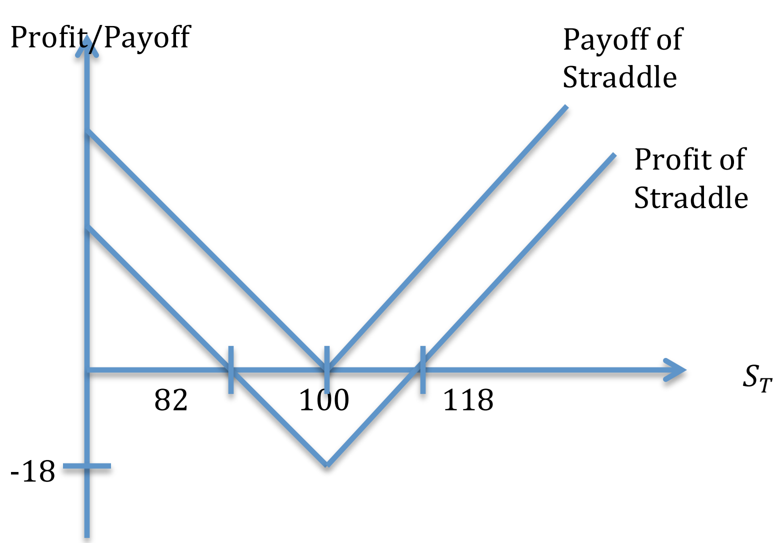
a)

Using Put-Call Parity: S= C=P+PV(X)=10-8+X/1.06 = 96.3392

b)

	S_T<100	S_T>100
Call	0	S_T-100
Put	100-S_T	0
Total	100-S_T	S_T-100

The breakeven values are 100-ST-18 =0 → ST=82 and ST-100-18=0 → ST=118

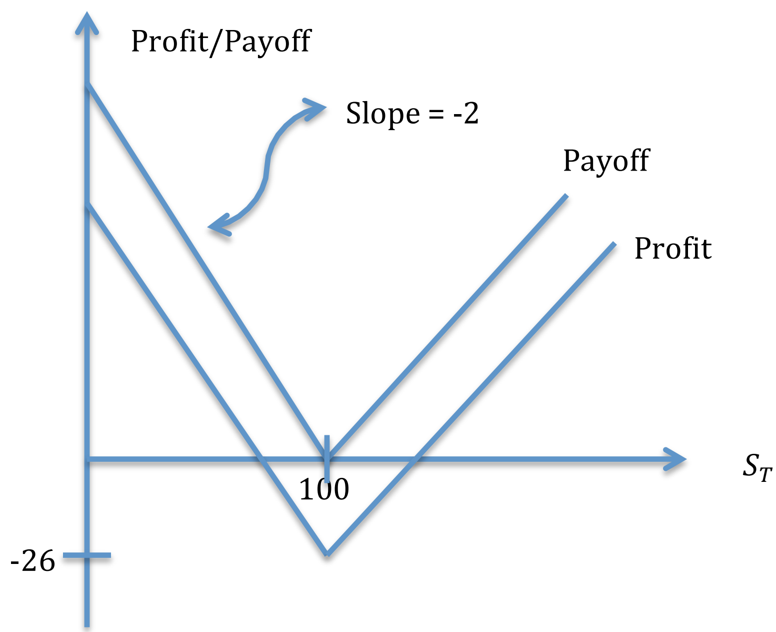


c) The investor is making a bet on variability. The straddle will pay off if the stock price moves up or down. Therefore the straddle will payoff if the volatility of the stock is high. Hence, the straddle is a bet on there being high stock return volatility. In this particular example, the investor will make a profit if the stock price is less than or greater than 82 and 118 respectively.

d) In this case, the investor would want to increase the profit on the downside by, for example, investing in 2 puts and 1 call which will double the profit if the stock price falls (bear market).

Payoff:

	S_T<100	S_T>100
Call	0	S_T-100
2 Puts	2(100-S_T)	0
Total	2(100-S_T)	S_T-100



Question 6)

a.

Action	Now	T_1	T_2
	Cash Flows		
Long futures with maturity T_1	0	$P_1 - F(T_1)$	0
Short futures with maturity T_2	0	0	$F(T_2) - P_2$
Buy asset at T_1, sell at T_2	0	$-P_1$	P_2
At T_1, borrow $F(T_1)$	0	$F(T_1)$	$-F(T_1) \times (1+r_f)^{(T_2-T_1)}$
Total	0	0	$F(T_2) - F(T_1) \times (1+r_f)^{(T_2-T_1)}$

b. Since the T_2 cash flow is riskless and the net investment was zero, then any profits represent an arbitrage opportunity.

c. The zero-profit no-arbitrage restriction implies that

$$F(T_2) = F(T_1) \times (1 + r_f)^{(T_2 - T_1)}$$

<u>Question 7</u>

See Determinants of Options Value in section 21.1 and intuition behind the Black Scholes formula in section 21.4. You answer should be about 2-3 paragraphs.