(1) *Regression through the origin*

There are some occasions that we assume or ***'force'*** our estimation model without an intercept as

- $Y_i = \hat{\beta}_2 X_i + \hat{u}_i$

Obtaining the estimator from OLS,

- $\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}$
- $\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}$
- $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-1}$

Note that the d.f. is one unit less due to the disappearance of $\hat{\beta}_1$.

(1) *Regression through the origin*

Differences that should also be noted are

- $\sum \hat{u}_i X_i = 0$ but $\sum \hat{u}_i$ need not be zero

› r^2 can be negative, so we need another coefficient of determination, defined as ***raw* $\mathbf{r^2}$**

- $raw\ r^2 = \frac{(\sum X_i Y_i)^2}{\sum X_i^2 \sum Y_i^2}$

Unless there is ***very strong a priori expectation***, we should avoid zero intercept regression model and stick to the conventional intercept-present model, because it may lead to ***specification error***.

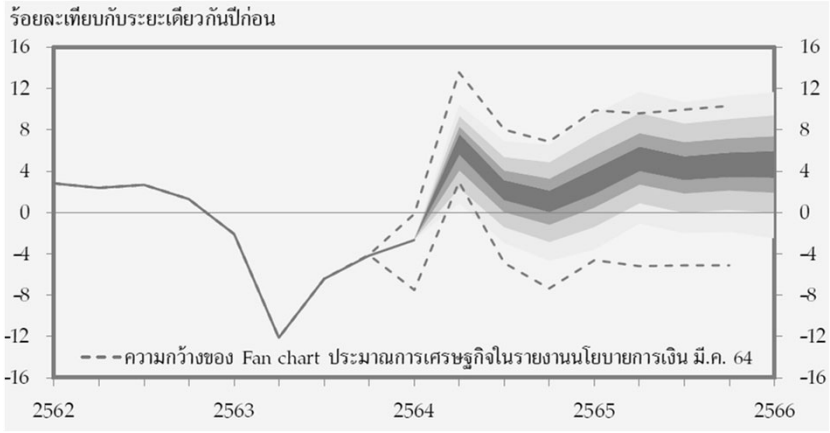
However, if $\hat{\beta}_1$ turns out to be statistically insignificant (from being zero), $\hat{\beta}_2$ is ***a lot more precise*** when estimated by the regression through the origin model.

(2) *Out-of-sample prediction*

Prediction is a very popular manner but note that the regression done so far is based on ***historical data***. Prediction is most of the time inaccurate, especially looking forward into the future since there are many unknown uncertainties lying ahead.

For this topic, we will use the regression result to 'predict' Y value from an out-of-sample X_0 value of interest.

- $\hat{Y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$



ร้อยละ	2564			2565				2566
	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
> 18	0	0	0	0	0	0	0	0
16.0-18.0	0	0	0	0	0	0	0	1
14.0-16.0	0	0	0	0	1	1	1	1
12.0-14.0	1	0	0	1	3	2	2	3
10.0-12.0	7	0	0	3	6	4	5	6
8.0-10.0	23	2	2	6	12	9	10	11
6.0-8.0	28	6	5	12	19	16	17	16
4.0-6.0	16	17	12	18	21	21	21	19
2.0-4.0	14	28	20	21	18	21	19	18
0.0-2.0	9	22	24	18	11	14	13	13
(-2.0)-0.0	2	15	19	12	5	8	6	7
(-4.0)-(-2.0)	0	7	11	6	2	3	3	3
(-6.0)-(-4.0)	0	2	5	2	1	1	1	2
< -6	0	0	3	2	0	1	1	1

Source: Financial Report, June 2021, BOT

(2) *Out-of-sample prediction*

There are two types of prediction that we can make once we retrieved the estimators. The methods are different due to its assumption on the variance.

(1) Mean prediction: Providing that mean estimation follows this equation

- $\hat{Y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$

when $\hat{Y}_0$ is an estimator of $E(Y|X_0)$ while X_0 represents a value of interest. Let's consider an easier example here, given that

- $\hat{Y}_i = -0.0144 + 0.7240 X_i$

If we are interested in out-of-sample $X_0 = 20$, then

- $\hat{Y}_0 = -0.0144 + 0.7240(20) = 14.4656$

(2) *Out-of-sample prediction*

Given that the variance of $\hat{Y}_0$ is, (no proof provided here)

- $\text{var}(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(x_i - \bar{X})^2}\right]$

Again, we do not have the true value of σ^2, so $\hat{\sigma}^2$ is replaced. We can then find the CI at the specific point of X_0 by

- $\Pr\left[\hat{Y}_0 - \left(t_{\frac{\alpha}{2}} \cdot \text{se}_{\hat{Y}_0}\right) \leq Y_0 \leq \hat{Y}_0 + \left(t_{\frac{\alpha}{2}} \cdot \text{se}_{\hat{Y}_0}\right)\right] = 1 - \alpha$

(2) *Out-of-sample prediction*

Example: Given that

- $\hat{Y}_i = -0.0144 + 0.7240X_i$ and $X_0 = 20$, $\hat{Y}_0 = 14.4656$
- $n = 13, \bar{X} = 12, \sum(x_i - \bar{X})^2 = 182, \hat{\sigma}^2 = 0.8936$

Step 1: Find the $\text{var}(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$

(2) *Out-of-sample prediction*

Step 2: find the $\text{se}_{\hat{Y}_0}$

Step 3: find the 95% CI for $E(Y|X_0 = 20)$

Interpretation: the CI covers the mean value or 95 out of 100 times that the CI will cover true value $E(Y|X_0)$.

(2) *Out-of-sample prediction*

(2) Individual prediction: Contrast to the mean prediction, which estimates the variance around Y_0, individual prediction focuses on forecasting error (fe), defined as

- $fe = \hat{Y}_0 - Y_0$

Therefore, we define the variance of this fe as

- $\text{var}(fe) = \text{var}\left(\hat{Y}_0 - Y_0\right) = \sigma^2 \left[1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right]$

Similarly, replacing the unknown σ^2 with the unbiased estimator $\hat{\sigma}^2$, we can derive the CI for Y_0 corresponding to X_0

- $\Pr\left[\hat{Y}_0 - \left(t_{\frac{\alpha}{2}} \cdot \text{se}_{fe}\right) \leq Y_0 \leq \hat{Y}_0 + \left(t_{\frac{\alpha}{2}} \cdot \text{se}_{fe}\right)\right] = 1 - \alpha$

(2) *Out-of-sample prediction*

Example: Given that

- $\hat{Y}_i = -0.0144 + 0.7240X_i$ and $X_0 = 20$, $\hat{Y}_0 = 14.4656$
- $n = 13, \bar{X} = 12, \sum(x_i - \bar{X})^2 = 182, \hat{\sigma}^2 = 0.8936$

Step 1: Find the $\text{var}(fe) = \sigma^2 \left[1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$

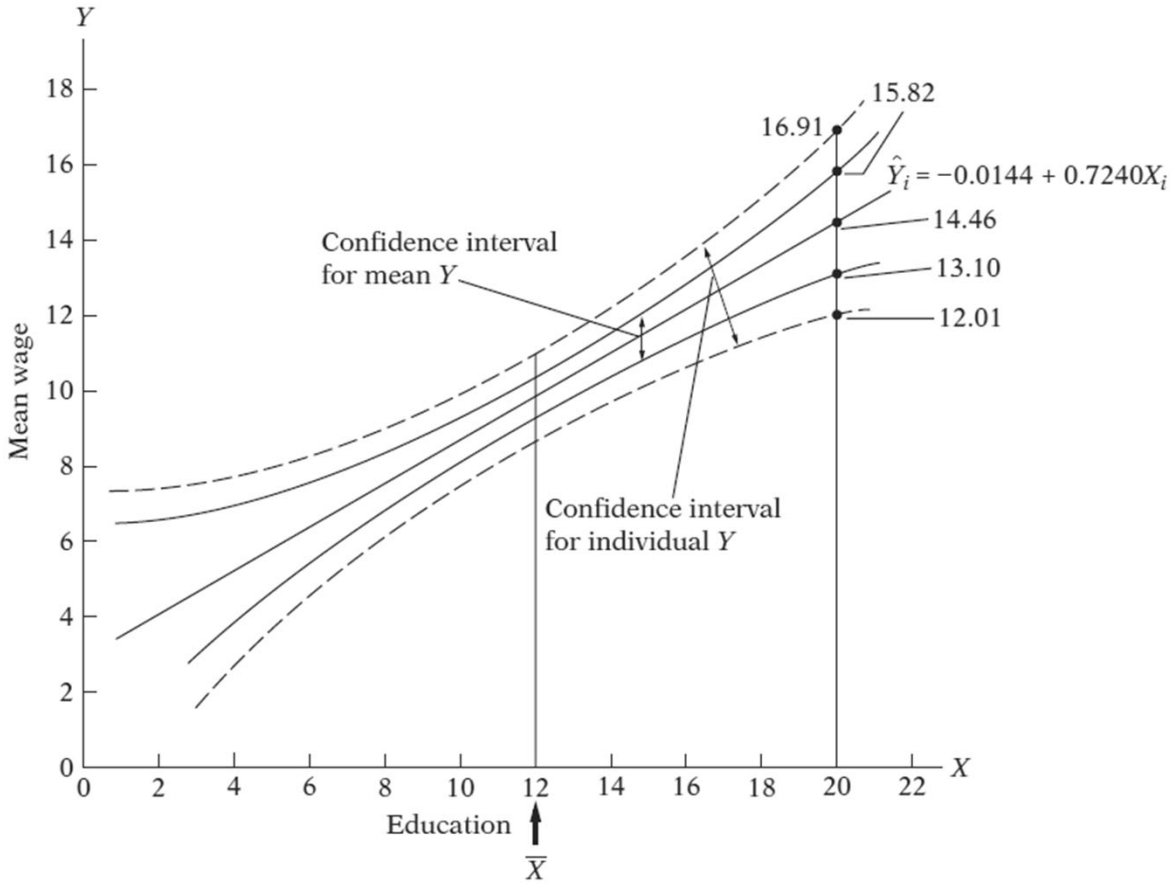
(2) *Out-of-sample prediction*

Step 2: Find the se_{fe}

Step 3: Find the 95% CI for Y_0 corresponding to $X_0 = 20$

(2) *Out-of-sample prediction*

Comparing CI band of mean and individual prediction.



(3) *Data scaling*

Sometimes when the result is reported, scaling can be difficult to make sense of. For example, if $\hat{\beta}_2 = 3.054e^{-15}$ which is not very effective for communication. Thus, data scaling can fix this ***without affecting the result or the interpretation***. See the examples below.

Original dataset

Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

Transformed dataset

Observation	X_i	Y_i
1	200	85
2	100	60
3	180	80
4	60	40
5	100	30
6	500	100
7	.	.
8	.	.
.	.	.

(3) *Data scaling*

1. Scale both sides

From the table in the previous page, we transform both X_i and Y_i down by ***dividing*** all the values with 1,000. The unit of the data becomes ***thousands*** Baht. See the result of the regression below.

Original dataset (Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Transformed dataset (thousands Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  106751.785         1  106751.785   Prob > F        =    0.0000
    Residual |  251567.523        50  5031.35046   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  358319.308        51  7025.86878   Root MSE        =    70.932

------------------------------------------------------------------------------
       thexp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       thinc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38.68786   16.22366     2.38   0.021      6.10168    71.27403
------------------------------------------------------------------------------
```

(3) *Data scaling*

1. Scale both sides

- For $\hat{\beta}_1$, when household income is zero, the intercept (autonomous consumption) is 38,687.86 Baht (38.68786 thousands Baht).

- For $\hat{\beta}_2$, when household income increases by 1 Baht (1 thousand Baht), on average household expenditure increases by 0.2797 Baht (0.2797 thousand Baht).

Notice that there is no difference when we interpret the coefficients.

On the other hand, if we multiply both sides with 1,000, how would it change the coefficient(s), standard error and the confidence interval.

(3) *Data scaling*

2. Scale Y_i

Now imagine we only scale the Y_i, dividing them with 1,000, the data table is displayed below.

Original dataset

Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

Transformed dataset

Observation	X_i	Y_i
1	200,000	85
2	100,000	60
3	180,000	80
4	60,000	40
5	100,000	30
6	500,000	100
7	.	.
8	.	.
.	.	.

(3) *Data scaling*

2. Scale Y_i

From the table in the previous page, the unit of expenditure becomes ***thousands*** Baht while the unit of income remains the same. See the result of the regression below.

Original dataset (Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Transformed dataset (Y_i thousands Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  106751.785         1  106751.785   Prob > F        =    0.0000
    Residual |  251567.523        50  5031.35046   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  358319.308        51  7025.86878   Root MSE        =    70.932

------------------------------------------------------------------------------
       thexp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .0002797   .0000607     4.61   0.000     .0001577    .0004017
       _cons |   38.68786   16.22366     2.38   0.021      6.10168    71.27403
------------------------------------------------------------------------------
```

(3) *Data scaling*

2. Scale Y_i

- For $\hat{\beta}_1$, when household income is zero, the intercept (autonomous consumption) is 38.68786 thousands Baht.

- For $\hat{\beta}_2$, when household income increases by 1 Baht, on average household expenditure increases by 0.0002797 thousand Baht.

Notice that, again, there is no difference when we interpret the coefficients.

On the other hand, if we multiply Y_i with 1,000, how would it change the coefficient(s), standard error and the confidence interval.

(3) *Data scaling*

3. Scale X_i

Now imagine we only scale the X_i, dividing them with 1,000, the data table is displayed below.

Original dataset

Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

Transformed dataset

Observation	X_i	Y_i
1	200	85,000
2	100	60,000
3	180	80,000
4	60	40,000
5	100	30,000
6	500	100,000
7	.	.
8	.	.
.	.	.

(3) *Data scaling*

3. Scale X_i

From the table in the previous page, the unit of income becomes ***thousands*** Baht while the unit of expenditure remains the same. See the result of the regression below.

Original dataset (Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Transformed dataset (X_i thousands Baht)

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       thinc |   279.7096   60.72423     4.61   0.000     157.7414    401.6778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

(3) *Data scaling*

3. Scale X_i

- For $\hat{\beta}_1$, when household income is zero, the intercept (autonomous consumption) is 38,687.86 Baht.

- For $\hat{\beta}_2$, when household income increases by 1 thousand Baht, on average household expenditure increases by 279.7096 Baht.

Notice that, again, there is no difference when we interpret the coefficients.

On the other hand, if we multiply X_i with 1,000, how would it change the coefficient(s), standard error and the confidence interval.

(3) *Data scaling*

Summary

Scaling	Factor	$\hat{\beta}_1, \text{se}_{\hat{\beta}_1}, \text{CI}$	$\hat{\beta}_2, \text{se}_{\hat{\beta}_2}, \text{CI}$
Both Y_i and X_i	w	w	-
	$\frac{1}{w}$	$\frac{1}{w}$	-
Y_i	w	w	w
	$\frac{1}{w}$	$\frac{1}{w}$	$\frac{1}{w}$
X_i	w	-	$\frac{1}{w}$
	$\frac{1}{w}$	-	w

(4) *Functional forms*

1. Log-log model

Sometimes called **double-log, or log-linear** models, a log-log model takes a form of

- $Y = \beta_1 X_i^{\beta_2} e^{u_i}$; we can linearize the function by taking log on both sides.

We will find that the slope and elasticity has a very interesting properties, by differentiation.

- Slope

- Elasticity

(4) *Functional forms*

1. Log-linear model

Original dataset

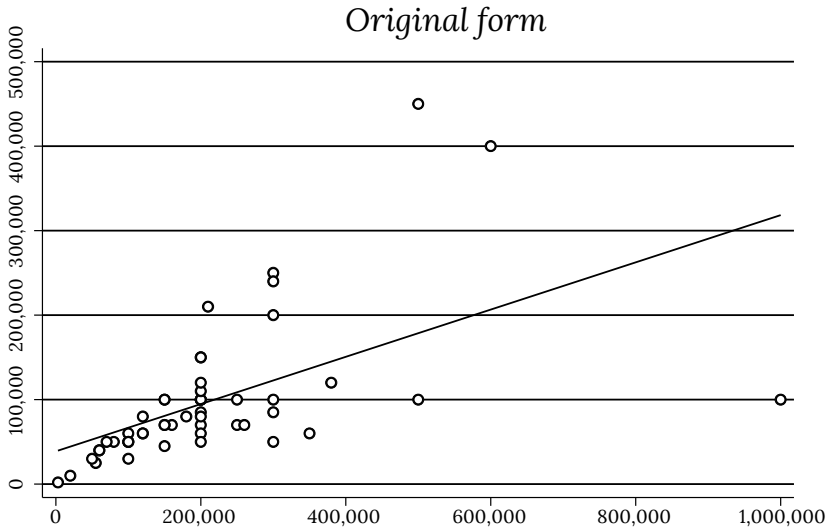
Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

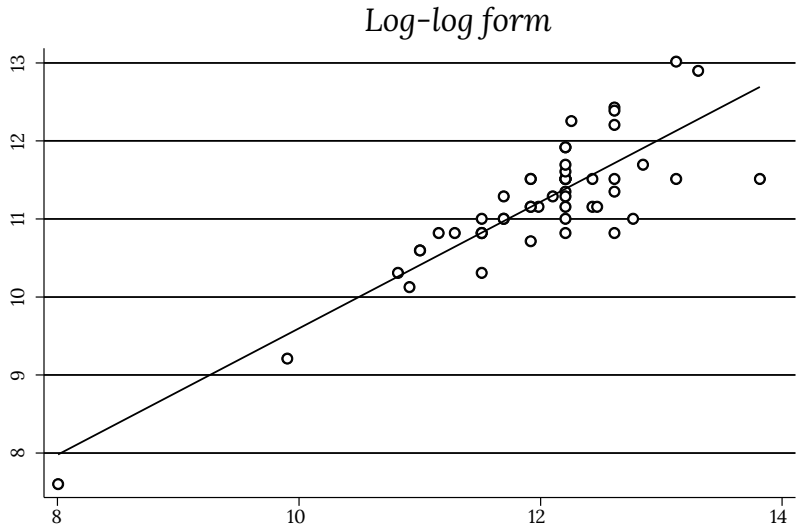
Transformed dataset

Observation	X_i	Y_i
1	12.20607	11.35041
2	11.51293	11.0021
3	12.10071	11.28978
4	11.0021	10.59663
5	11.51293	10.30895
6	12.12236	11.51293
7	.	.
8	.	.
.	.	.

(4) *Functional forms*

1. Log-linear model





(4) *Functional forms*

1. Log-linear model

Original form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Log-log form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =    137.35
       Model |  26.2503087         1  26.2503087   Prob > F        =    0.0000
    Residual |  9.55577384        50  .191115477   R-squared       =    0.7331
-------------+----------------------------------   Adj R-squared   =    0.7278
       Total |  35.8060825        51  .702080049   Root MSE        =    .43717

------------------------------------------------------------------------------
      ln_exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      ln_inc |   .8111731    .069214    11.72   0.000     .6721526    .9501936
       _cons |   1.485608   .8318879     1.79   0.080    -.1852878    3.156504
------------------------------------------------------------------------------
```

(4) *Functional forms*

1. Log-linear model

To interpret this model, we should write the result down as an equation

- $\ln \widehat{consmp}_i = 1.4856 + 0.8112 \ln inc_i$

- An increase by **one percent** of income is associated with expenditure increase of $\widehat{\boldsymbol{\beta}}_{\boldsymbol{2}}$ **percent**.

(4) *Functional forms*

2. Log-lin model

The two following models can be called semi-log model. The first one here is log-lin model which takes a form of

- $\ln Y = \beta_1 + \beta_2 X_i$

We will find the slope and elasticity again.

- Slope

- Elasticity

(4) *Functional forms*

2. Log-lin model

Original dataset

Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

Transformed dataset

Observation	X_i	Y_i
1	200,000	11.35041
2	100,000	11.0021
3	180,000	11.28978
4	60,000	10.59663
5	100,000	10.30895
6	500,000	11.51293
7	.	.
8	.	.
.	.	.

(4) *Functional forms*

2. Log-lin model

Original form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Log-lin form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.94
       Model |  10.9184499         1  10.9184499   Prob > F        =    0.0000
    Residual |  24.8876325        50  .497752651   R-squared       =    0.3049
-------------+----------------------------------   Adj R-squared   =    0.2910
       Total |  35.8060825        51  .702080049   Root MSE        =    .70552

------------------------------------------------------------------------------
      ln_exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   2.83e-06   6.04e-07     4.68   0.000     1.62e-06    4.04e-06
       _cons |   10.60821   .1613665    65.74   0.000      10.2841    10.93233
------------------------------------------------------------------------------
```

(4) *Functional forms*

2. Log-lin model

To interpret $\hat{\beta}_1$, now we can assume that income is zero from this equation

- $\ln \widehat{consmp}_i = 10.60821 + 0.00000283 inc_i$

(4) *Functional forms*

2. Log-lin model

Now consider interpreting $\hat{\beta}_2$ from the same result.

- $\ln \widehat{consmp}_i = 10.60821 + 0.00000283 inc_i$

- ***Approximate measure***: an increase by **one unit of income** is associated with expenditure increase of $\mathbf{100 \cdot \widehat{\beta}_2}$ **percent**. (suitable for small value of $\hat{\beta}_2$)

- ***Exact measure***: an increase by **one unit of income** is associated with expenditure increase of $\left(\mathbf{100 \cdot e^{\widehat{\beta}_2}}\right) - \mathbf{100}$ **percent**.

(4) *Functional forms*

2. Log-lin model

Here is a comparison between approximate and exact change.

$\widehat{\beta}_2$	$100 \cdot \widehat{\beta}_2$ (approximate)	$(100 \cdot e^{\widehat{\beta}_2}) - 100$ (exact)	Diff (approximate-exact)
0.005	0.5	0.5012521	-0.0012521
0.01	1	1.0050167	-0.0050167
0.015	1.5	1.5113065	-0.0113065
0.02	2	2.020134	-0.020134
0.025	2.5	2.5315121	-0.0315121
0.03	3	3.0454534	-0.0454534
0.035	3.5	3.5619709	-0.0619709
0.04	4	4.0810774	-0.0810774
0.045	4.5	4.602786	-0.102786
0.05	5	5.1271096	-0.1271096
0.055	5.5	5.6540615	-0.1540615
0.06	6	6.1836547	-0.1836547
0.065	6.5	6.7159024	-0.2159024
0.07	7	7.2508181	-0.2508181
0.075	7.5	7.7884151	-0.2884151
0.08	8	8.3287068	-0.3287068
.	.	.	.
.	.	.	.

(4) *Functional forms*

3. Lin-log model

The lin-log model takes a form of

- $Y = \beta_1 + \beta_2 \ln X_i$

We will find the slope and elasticity again.

- Slope

- Elasticity

(4) *Functional forms*

3. Lin-log model

Original dataset

Observation	X_i	Y_i
1	200,000	85,000
2	100,000	60,000
3	180,000	80,000
4	60,000	40,000
5	100,000	30,000
6	500,000	100,000
7	.	.
8	.	.
.	.	.

Transformed dataset

Observation	X_i	Y_i
1	12.20607	85,000
2	11.51293	60,000
3	12.10071	80,000
4	11.0021	40,000
5	11.51293	30,000
6	12.12236	100,000
7	.	.
8	.	.
.	.	.

(4) *Functional forms*

3. Lin-log model

Original form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.22
       Model |  1.0675e+11         1  1.0675e+11   Prob > F        =    0.0000
    Residual |  2.5157e+11        50  5.0314e+09   R-squared       =    0.2979
-------------+----------------------------------   Adj R-squared   =    0.2839
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70932

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .2797096   .0607242     4.61   0.000     .1577414    .4016778
       _cons |   38687.86   16223.66     2.38   0.021      6101.68    71274.03
------------------------------------------------------------------------------
```

Lin-log form

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(1, 50)        =     21.42
       Model |  1.0746e+11         1  1.0746e+11   Prob > F        =    0.0000
    Residual |  2.5086e+11        50  5.0172e+09   R-squared       =    0.2999
-------------+----------------------------------   Adj R-squared   =    0.2859
       Total |  3.5832e+11        51  7.0259e+09   Root MSE        =     70832

------------------------------------------------------------------------------
         exp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      ln_inc |   51899.89   11214.45     4.63   0.000     29375.01    74424.76
       _cons |  -524013.9   134787.1    -3.89   0.000    -794741.8     -253286
------------------------------------------------------------------------------
```

(4) *Functional forms*

3. Lin-log model

Again, it does not make sense interpreting $\hat{\beta}_1$, so we are going to consider only $\hat{\beta}_2$ from this equation.

- $\widehat{consmp}_i = -524{,}013.9 + 51{,}899.89 \ln inc_i$

- ***Approximate measure***: an increase by **one percent of income** is associated with expenditure increase of $\frac{\widehat{\boldsymbol{\beta}}_{\mathbf{2}}}{\mathbf{100}}$ **unit**. (suitable for small value of change of independent variable)

- ***Exact measure***: an increase by **one percent of income** is associated with expenditure increase of $\widehat{\boldsymbol{\beta}}_{\mathbf{2}} \cdot \mathbf{ln}(\mathbf{1.01})$ **unit**.

(4) *Functional forms*

4. Reciprocal model and log-reciprocal

These two model, respectively, take a form of

- $Y = \beta_1 + \beta_2(\frac{1}{X_i})$
- $\ln Y = \beta_1 - \beta_2(\frac{1}{X_i})$

Further details can be found in the textbook.