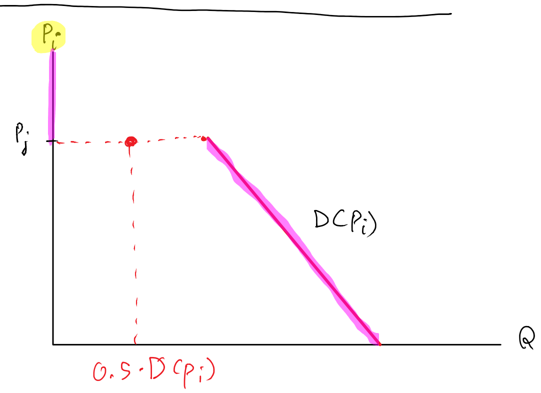
BERTRAND COMPETITION (1883)

- 2 FIRMS
- FIRMS COMPETE OVER PRICES JUST ONCE AND THEY MAKE THEIR PRICE DECISION AT THE SAME TIME. (THAT IS B/F OBSERVING THE PRICE OF ITS RIVAL)
- THE PRODUCT IS HOMOGENEOUS (=PERFECT SUBSTITUTES) ⇒ BUYERS PURCHASE FROM THE FIRM THAT OFFERS THE LOWEST PRICE.
- IDENTICAL MARGINAL COST FOR THE TWO FIRM ($MC = c$)
- THE TWO FIRMS SATISFY ALL THE DEMAND (i.e., THERE IS NO CAPACITY CONSTRAINTS)

THE DEMAND FUNCTION FOR FIRM i



RECALL THAT DEMAND FOR FIRM i DEPENDS ON BOTH P_i AND P_j

$$D_i(P_i, P_j) = \begin{cases} 0 & \text{IF } P_i > P_j \\ 0.5 \cdot D(P_i) & \text{IF } P_i = P_j \\ D(P_i) & \text{IF } P_i < P_j \end{cases}$$

THE NASH EQUILIBRIUM TO THIS GAME IS A SET OF PRICES, P_i AND P_j THAT SATISFIES THE 2 FOLLOWING CONDITIONS:

$P_i^B, P_j^B \xrightarrow{\text{MUST SATISFY}}$

$$\boxed{\begin{array}{ll} \Pi_i(P_i^B, P_j^B) \geq \Pi_i(P_i, P_j^B) & \text{FOR ANY } P_i \\ \Pi_j(P_i^B, P_j^B) \geq \Pi_j(P_i^B, P_j) & \text{FOR ANY } P_j \end{array}}$$

OPTIMAL PRICE STRATEGY j CHOOSES

OTHER PRICE STRATEGIE j MIGHT CHOOSE

EQUILIBRIUM? SOLUTION FOR THIS GAME IS $\underbrace{P_i = P_j = c}$!!!

BERTRAND PARADOX:

① TWO FIRMS ARE ENOUGH TO ELIMINATE MARKET POWER.
② COMPETITION BET. THE TWO FIRM LEADS TO COMPLETE DISSIPATION OF PROFITS.

$\Pi_i = \Pi_j = 0$

LET'S SHOW THAT $P_i = P_j = c$ IS A STABLE AND UNIQUE EQUILIBRIUM IN THE BERTRAND MODEL.

$i = 1, j = 2$

PROOF: FOR $P_1 > P_2 > c$. THIS IS NOT AN EQUILIBRIUM. AT THIS PRICE $\Pi_1 = 0$. SO FIRM 1 HAS AN INCENTIVE TO CHARGE $P_2 - \varepsilon$ AND IT WILL CAPTURE ALL DEMAND.

$$\Pi_1 = D_1(P_2 - \varepsilon) \cdot (P_2 - \varepsilon - c) > 0 \quad \text{FOR A SMALL } \varepsilon.$$

IT WILL CAPTURE ALL DEMAND.

$$\Pi_1 = \underbrace{D_1(P_2-\varepsilon)}_{Q_1} \cdot (\underbrace{P_2-\varepsilon}_{P_1} - \underset{MC}{c}) > 0 \quad \text{FOR A SMALL } \varepsilon.$$

FOR $P_1 = P_2 > c$. THIS IS NOT AN EQUILIBRIUM, BOTH FIRMS WILL DEVIATE

EX: B/F — A/F

$P_1 = P_2$ — $P_1 - \varepsilon$

$\Pi_1 = \frac{1}{2} D(P_1) \cdot (P_1 - c)$ — $\Pi_1 = D(P_1-\varepsilon) \cdot (P_1 - \varepsilon - c)$

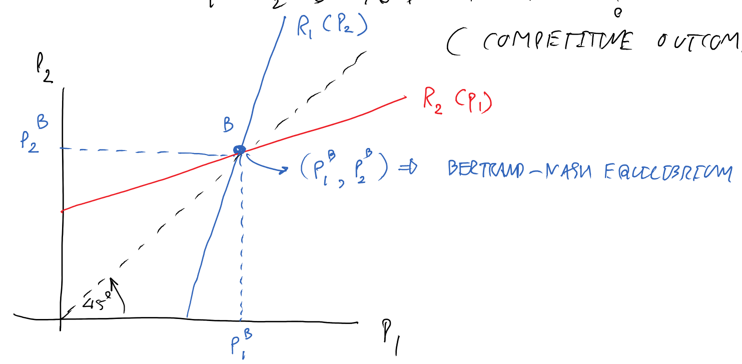
CAPTURES ALL PROFITS!

FOR $P_1 > P_2 = c$. THIS IS NOT AN EQUILIBRIUM. AT THESE PRICE, FIRM 2 GETS ALL DEMAND BUT $\Pi_2 = 0$. SO, FIRM 2 CAN INCREASE ITS PROFIT BY SETTING $P_2 = P_1 - \varepsilon$ WHERE ε IS VERY SMALL. THEN Π_2 WOULD BE EQUAL TO $\Pi_2 = D(P_1-\varepsilon)(P_1-\varepsilon-c) > 0$ FOR A SMALL ε.

FOR $P_1 = P_2 = c$. THIS IS A UNIQUE EQUILIBRIUM, BOTH HAVE NO INCENTIVE TO UNILATERALLY DEVIATE FROM $P_1 = c$, $P_2 = c$.

→ IMPLIES $\Pi_1 = \Pi_2 = 0$ AND NO DWL!

(COMPETITIVE OUTCOME)



NOW, LET'S RELAX SOME ASSUMPTIONS...

THE BERTRAND PARADOX CAN BE SOLVED IF WE CHANGE EACH ONE OF THE MAIN ASSUMPTIONS OF THE MODEL.

① PRODUCT DIFFERENTIATION: IF PRODUCTS ARE NOT HOMOGENEOUS (i.e., DIFFERENT BRANDS, DIFFERENT LOCATION. WE CAN FORESEE THAT A PRICE REDUCTION DOES NOT IMPLY THAT THE RIVAL GETS NO DEMAND, OR A PRICE INCREASE DOES NOT IMPLY THAT THE FIRM WILL LOSE ITS CUSTOMERS. (ISSUE OF CONSUMER ROYALTY)

THEREFORE $P = \underset{MC}{c}$ IS NO LONGER THE EQUILIBRIUM.

CONCLUSION: THE BERTRAND MODEL IS AN EXTREME CASE. ONCE WE INTRODUCE A MORE REALISTIC ASSUMPTION, THE COMPETITION BY PRICE SOFTENS AND THE EQUILIBRIUM PRICE CAN BE HIGHER THAN MARGINAL COST.

(2) TIME DIMENSION (REPEATED GAMES). IF THE FIRMS MEET IN THE MARKET REPEATEDLY, BOTH MAY REALIZE THAT PRICE WAR HURTS BOTH, $\pi_1 = \pi_2 = 0$. WE CAN SEE THAT IN REPEATEDLY PLAYED GAME, COOPERATION MAY EMERGE UNDER SOME CONDITIONS

(3) EDGEWORTH SOLUTION: INTRODUCING CAPACITY CONSTRAINT

UNDER CAPACITY CONSTRAINT, $(p_1^B, p_2^B) = (c, c)$ CANNOT BE AN EQUILIBRIUM ANYMORE (WHY?)

PROOF BY CONTRADICTION: SUPPOSE IT IS AN EQUILIBRIUM. THEN $\pi_1 = 0$ AND $\pi_2 = 0$, IF FIRM 1 RAISES THE PRICE, THEN FIRM 2 GETS ALL DEMAND. BUT, UNFORTUNATELY, FIRM 2 CANNOT SATISFY IT DUE TO CAPACITY CONSTRAINT!