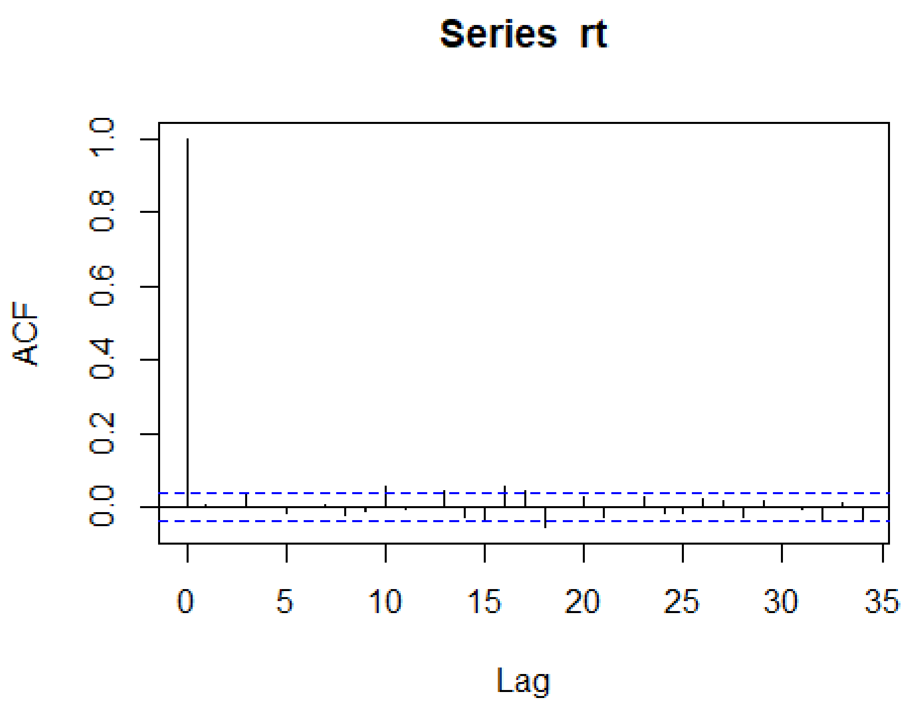
TANAPAT INPRINGANANTA 6104640914

Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 13, 2017.You may download the data using quantmod. Let r_t be the log returns, which can be obtained via

rt <- diff(log(as.numeric(CAT[,6])))

(a) Are there any serial correlations in the log return series r_t ? Why?

```
#1.a
a = as.numeric(CAT[,6])
rt = diff(log(a))
acf(rt)
```



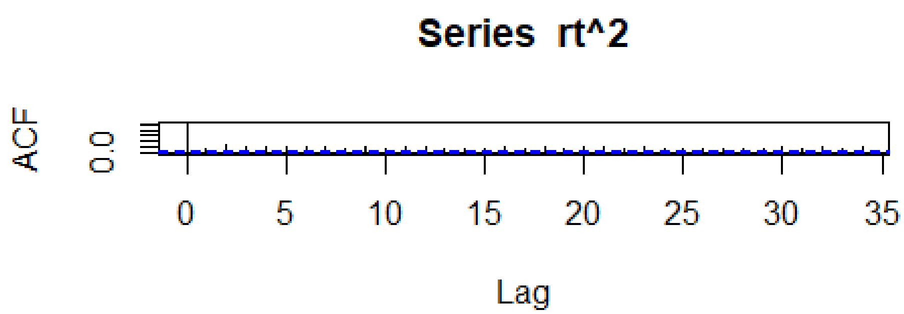

```
pacf(rt)

Box.test(rt,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt
## X-squared = 16.291, df = 10, p-value = 0.09159
```

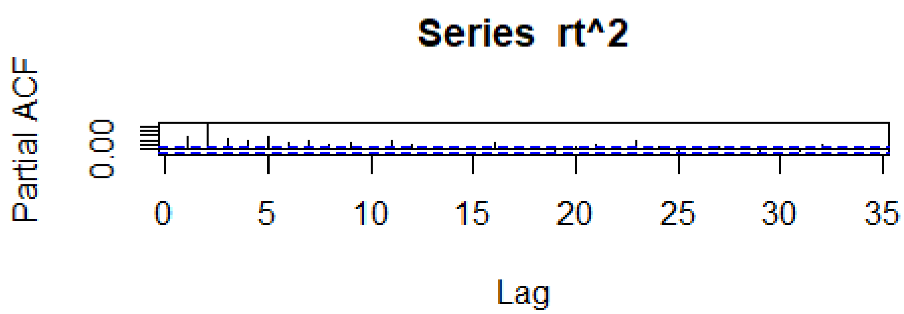
```
#According to the Ljung-Box Test, with null hypothesis p0=p1=p2=...=pt=0
#The calculated p-value is greater than 0.05, we can reject null hypothesis
at 95% confidence interval
#The log return series of rt contain serial correlations problem.
```

(b) Are there any ARCH effects in the log return series r_t (the linear depedence of squared returns)? Why?

```
Box.test(rt^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt^2
## X-squared = 917.21, df = 10, p-value < 2.2e-16
```

```
#The calculated p-value by performing Ljung-box Test is lower than 0.05
#Therefore, we cannot reject H0: There is no ARCH effect in the series at 95%
confidence interval
```

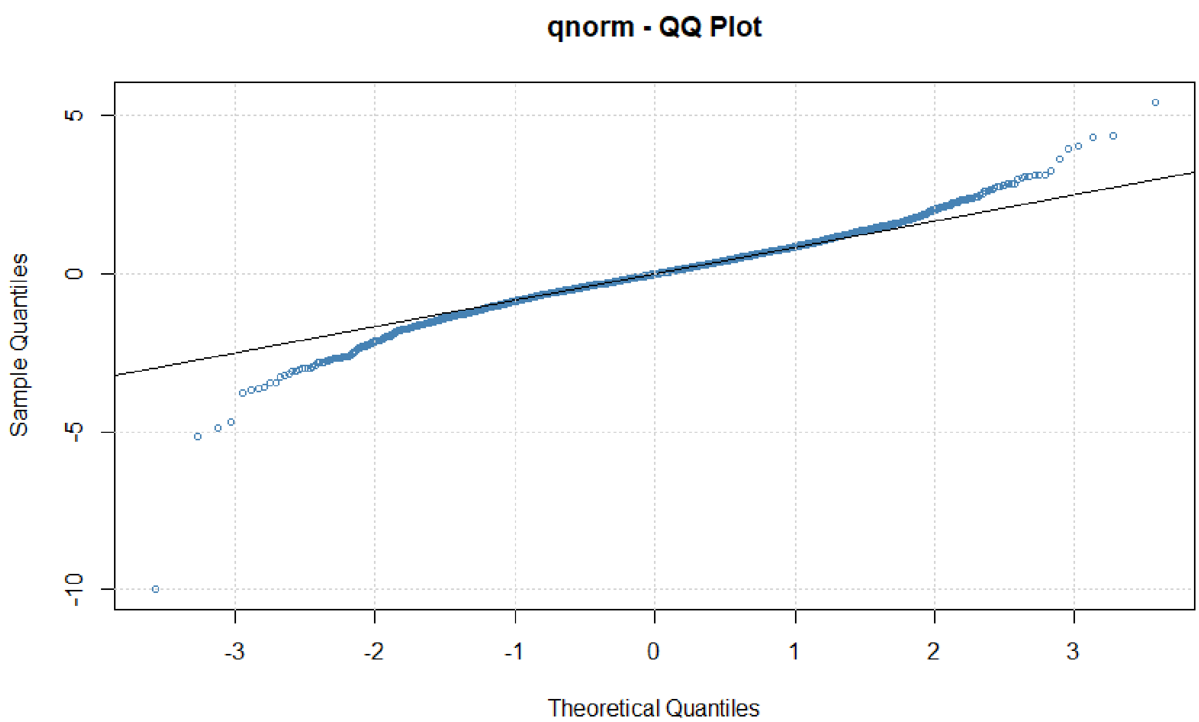




(c) Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

```
#1.c
m2 <- garchFit(~arma(1,0)+garch(1,1), data=rt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001e071dc0>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1
## 4.8298e-04  1.6866e-02  4.4779e-06  4.9720e-02  9.3866e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.830e-04   3.075e-04    1.571 0.116297
## ar1    1.687e-02   2.006e-02    0.841 0.400353
## omega  4.478e-06   1.278e-06    3.503 0.000461 ***
## alpha1 4.972e-02   8.191e-03    6.070 1.28e-09 ***
## beta1  9.387e-01   1.031e-02   91.048  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7378.56    normalized:  2.599916
```

```
##
## Description:
##  Sat May 01 18:39:56 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                  Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3298.441  0
##  Shapiro-Wilk Test  R    W      0.9663735 0
##  Ljung-Box Test     R    Q(10)  12.37554  0.2607088
##  Ljung-Box Test     R    Q(15)  14.79514  0.4662719
##  Ljung-Box Test     R    Q(20)  19.20107  0.5087928
##  Ljung-Box Test     R^2  Q(10)  0.980939  0.9998424
##  Ljung-Box Test     R^2  Q(15)  3.682825  0.9986048
##  Ljung-Box Test     R^2  Q(20)  6.9285    0.996913
##  LM Arch Test       R    TR^2   2.723165  0.9972029
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.196308 -5.185823 -5.196314 -5.192526
```



Mean equation: r_t = 4.8298e-04 + 1.6866e-02r_{t-1}

std. error: 3.075e-04

Variance equation: σ_t^2 = 4.478e-06 + 4.972e-02a_{t-1}^2 + 9.3866e-01σ_{t-1}^2

Std.error : (2.006e-02) (1.278e-06) (1.031e-02)

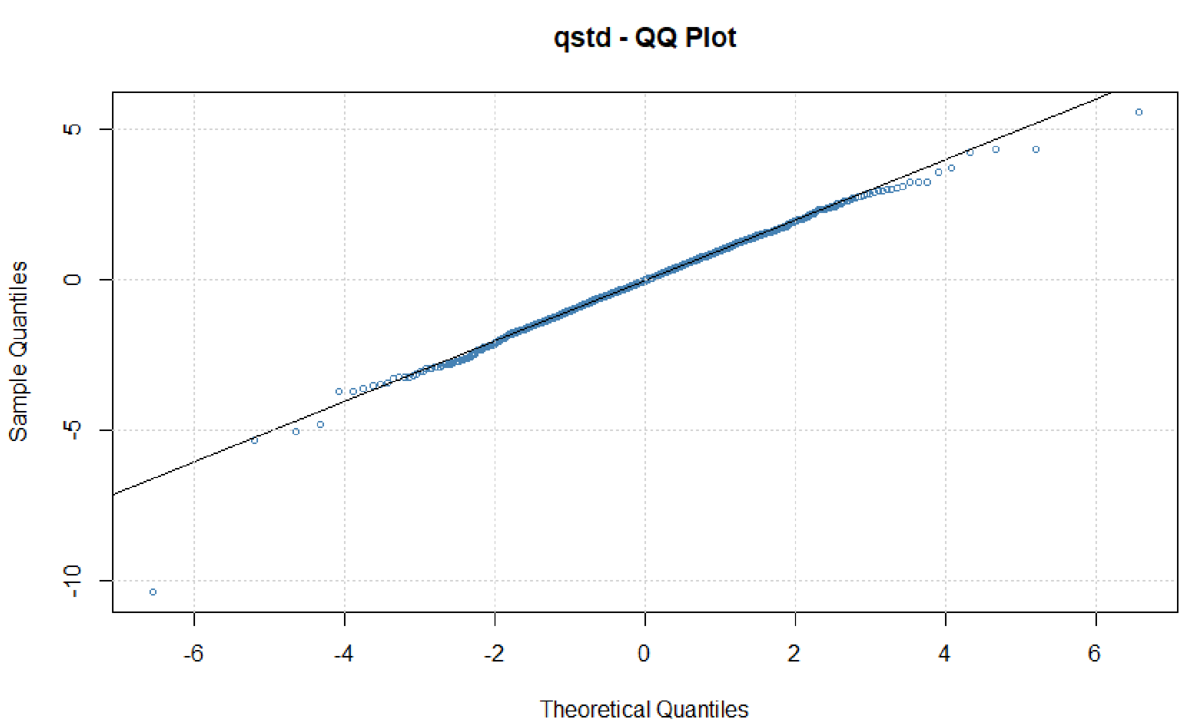
The model is not adequate, as the normality assumption is rejected from the fat-tail distribution occurred in QQplot.

(d) Build a GARCH(1,1) model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

```
#1.d
m3 <- garchFit(~garch(1,1), data=rt,trace=F, cond.dist="std")

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001d827de8>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9780e-04  4.2035e-06  7.2376e-02  9.2033e-01  5.0958e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.978e-04   2.702e-04    2.212  0.02695 *
## omega  4.203e-06   1.571e-06    2.675  0.00747 **
## alpha1 7.238e-02   1.374e-02    5.267 1.39e-07 ***
## beta1  9.203e-01   1.472e-02   62.503  < 2e-16 ***
## shape  5.096e+00   4.825e-01   10.561  < 2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7507.106    normalized:  2.64521
##
## Description:
##  Sat May 01 18:39:56 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4056.502  0
##  Shapiro-Wilk Test  R    W      0.9639091 0
##  Ljung-Box Test     R    Q(10)  14.77968  0.140303
##  Ljung-Box Test     R    Q(15)  16.74279  0.3344718
##  Ljung-Box Test     R    Q(20)  20.39783  0.433304
##  Ljung-Box Test     R^2  Q(10)  2.953085  0.9825066
##  Ljung-Box Test     R^2  Q(15)  5.482428  0.9871938
##  Ljung-Box Test     R^2  Q(20)  9.458146  0.9769677
##  LM Arch Test       R    TR^2   4.273688  0.977976
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.286897 -5.276412 -5.286903 -5.283115
```



The existence of fat-tail distribution is eliminated, the model is adequate.

(e) Write down the fitted model.

Mean equation: $r_t = 5.9780e^{-04}$

Std.error: (2.702e-04)

Variance equation: σ_t^2 = 4.2035e-06 + 7.2376e-02a_{t-1}^2+9.2033e-01σ_{t-1}^2

Std.error: (1.571e-06) (1.374e-02) (1.472e-02)

(f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted GARCH(1,1) model with standardized Student-t innovations.

```
predict(m3,5)

##   meanForecast meanError         standardDeviation
## 1 0.0005977987 0.01515760        0.01515760
## 2 0.0005977987 0.01524072        0.01524072
## 3 0.0005977987 0.01532280        0.01532280
## 4 0.0005977987 0.01540384        0.01540384
## 5 0.0005977987 0.01548387        0.01548387
```

Question2.

Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file m-kovw-5116.txt. Obtain the log return series of KO stock.

(a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

```
#2.a
O=read.table("m-kovw-5116.txt", header=T)
KOrt <- (log(as.numeric(O[,3]+1)))
t.test(KOrt)

##
##  One Sample t-test
##
## data:  KOrt
## t = 4.9853, df = 779, p-value = 7.628e-07
```

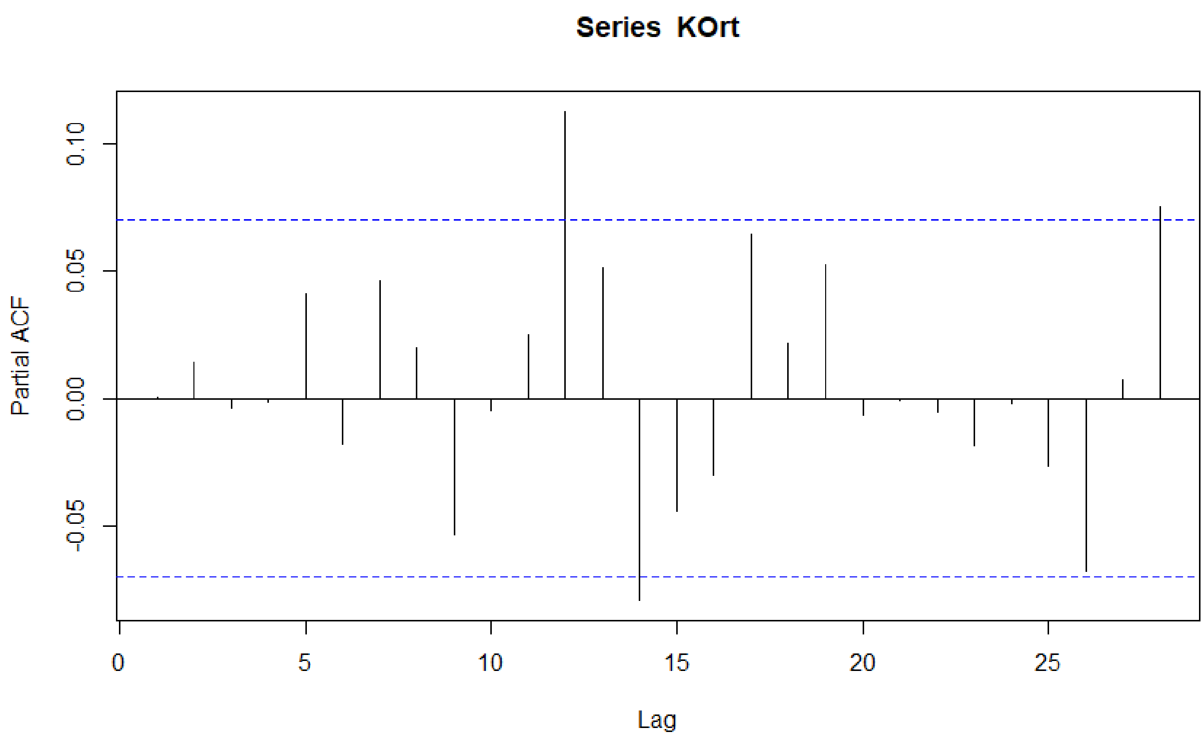
```
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
## sample estimates:
##  mean of x
## 0.01031992
```

Since the calculated p-value performed in t-test = 7.628e-07 is lower than 0.05, we can reject null hypothesis which $E(r_{t)} = 0$ at 95% confidence interval. Therefore, the expected return is differed from zero.

```
Box.test(KOrt,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  KOrt
## X-squared = 5.9201, df = 10, p-value = 0.8219
```

Since the calculated p-value = 0.8219 is greater than 0.05, we cannot reject null hypothesis at 95% confidence interval. Therefore, there is no problem of serial correlation.




```
Box.test(KOrt^2,lag=10,type='Ljung')
```

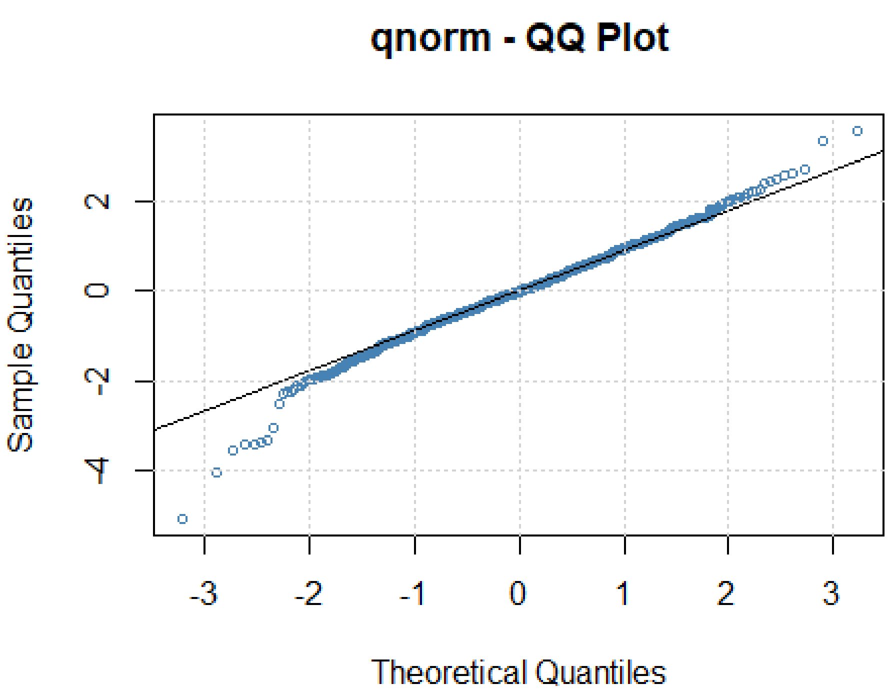
```
##
##  Box-Ljung test
##
## data:  KOrt^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16
```

The calculated p-value = 2.2e-16 is less than 0.05, we can reject null hypothesis at 95% confidence interval. There is a ARCH effect on the series.

(b) Build a AR(1)-GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

```
#2.b
m4<-garchFit(~arma(1,0)+garch(1,1), data=KOrt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m4)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = KOrt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001f22b1a8>
##  [data = KOrt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
```

```
##
## Description:
##  Sat May 01 18:39:56 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396

par(mfrow=c(1,1))
plot(m4, which=13)
```



Mean equation: $r_t = 0.0112544 - 0.02633742 r_{t-1}$

Std.error: (1.897e-03) (3.881e-02)

Variance: $\sigma_t^2 = 0.00018112 + 0.09535029a_{t-1}^2 + 0.84861593\sigma_{t-1}^2$

Std.error: (5.852e-05) (1.915e-02) (2.766e-02)

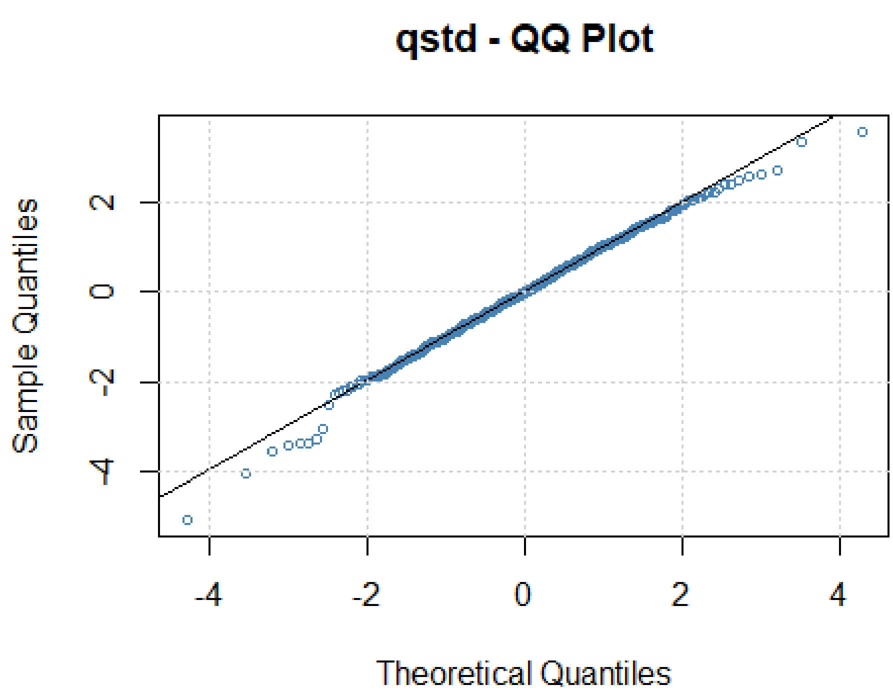
According to standard residuals test, all values are significant. However, the normality assumption in QQ-plot indicates a rejection. Therefore, the model is not adequate

(c) Fit a AR(1)-GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

```
m5<-garchFit(~arma(1,0)+garch(1,1), data=KOrt,trace=F, cond.dist="std")

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m5)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = KOrt, cond.dist = "s
td",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000002050f9d8>
##  [data = KOrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1         beta1         sha
pe
##  0.01124020  -0.01887601   0.00017395   0.09642927   0.85044151   7.478777
80
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1    -1.888e-02   3.691e-02   -0.511  0.60904
## omega   1.739e-04   6.596e-05    2.637  0.00836 **
```

```
## alpha1  9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1   8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape   7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
##  Sat May 01 18:39:56 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816987
##  LM Arch Test       R    TR^2   10.48089  0.5738501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```

```
plot(m5, which=13)
```



Mean Equation: $r_t = 0.01124020 - 0.01887601r_{t-1}$

Std.error: (1.810e-03) (3.691e-02)

Variance: $\sigma_t^2 = 0.00017395 + 0.09642927a_{t-1}^2 + 0.85044151\sigma_{t-1}^2$

Std.error: (6.896e-05) (2.338e-02) (3.267e-02)

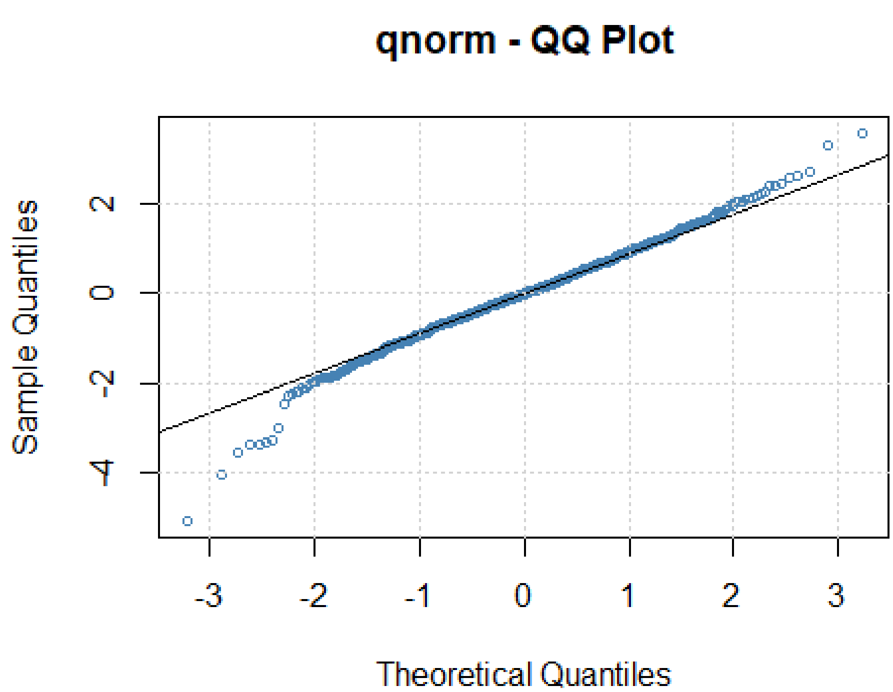
The normality assumption in QQ-plot seems not to have problem of fat-tails, the model is adequate

(d) Build a GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

```
#2.d
m6<-garchFit(~garch(1,1),data=KOrt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m6)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = KOrt, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001e59d9b0>
##  [data = KOrt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
```

```
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Sat May 01 18:39:57 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```

```
plot(m6,which=13)
```



Mean Equation: $r_t = 0.01098417$

Std.error: (1.846e-03)

Variance: $\sigma_t^2 = 0.00018497 + 0.09479925a_{t-1}^2 + 0.84780406\sigma_{t-1}^2$

Std.error: (5.899e-05) (1.912e-02) (2.787-e02)

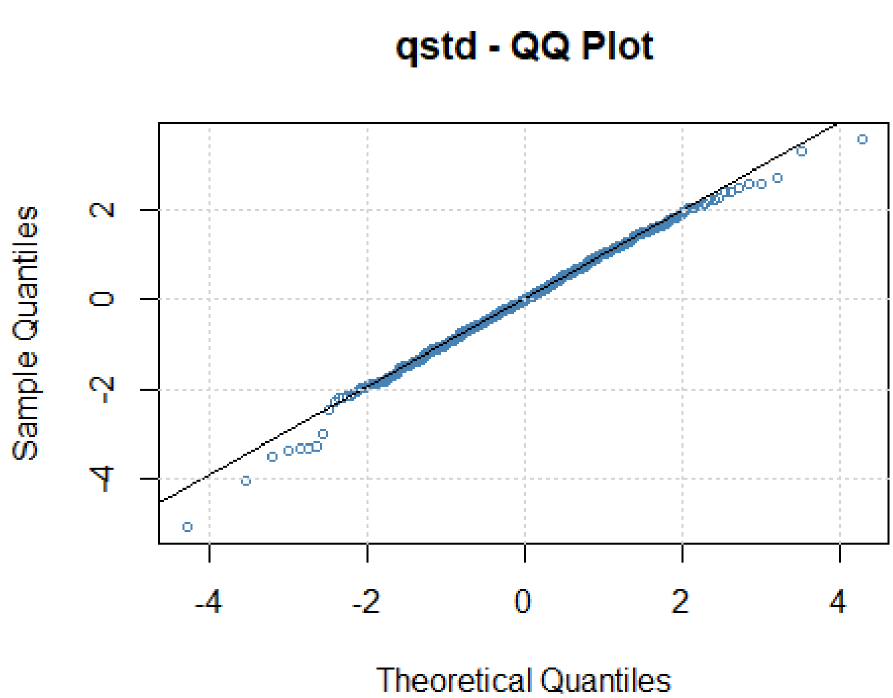
QQ-plot seems to reject normality assumption(fat-tails), the model is not adequate

(e) Fit a GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

```
#2.e
m7<-garchFit(~garch(1,1),data=KOrt,trace=F,cond.dist="std")

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m7)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = KOrt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001e21a5b0>
##  [data = KOrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
```

```
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Sat May 01 18:39:57 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

```
plot(m7,which=13)
```



Mean Equation: $r_t = 0.01105016$

Std.error: (1.757e-03)

Variance: $\sigma_t^2 = 0.00017528 + 0.09632874a_{t-1}^2 + 0.85006800\sigma_{t-1}^2$

Std.error: (6.627e-05) (2.337e-02) (3.277e-02)

The model is adequate according to QQ-plot.

(f) Compare the model (b)-(e) which model you select.

M4 BIC: -2.959016

M5 BIC: -2.986885

M6 BIC: -2.966858

M7 BIC: -2.994954

Model 7 is preferred since the model is adequate and yield the lowest BIC value

Question3.

Consider the daily returns of the stock *S&P*500 from January 2, 2005 to March 31, 2021. Let r_t be the percentage log returns.

(a) Is the expected value of r_t zero? Why? Are there any serial correlations in r_t? Why?

GOOGLE: GOOG

```
#3.a
getSymbols("GOOG",from="2005-01-02",to="2021-03-31")

## [1] "GOOG"

Grt=diff(log(as.numeric(GOOG[,6])))
```

```
t.test(Grt)

##
##  One Sample t-test
##
```

```
## data:  Grt
## t = 2.5191, df = 4086, p-value = 0.0118
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.0001634894 0.0013111404
## sample estimates:
##    mean of x
## 0.0007373149
```

Since the calculated p-value = 0.0118 is lower than 0.05, we can reject null hypothesis at 95% confidence interval. The expected value is differed from 0.

```
Box.test(Grt,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  Grt
## X-squared = 18.443, df = 10, p-value = 0.04793
```

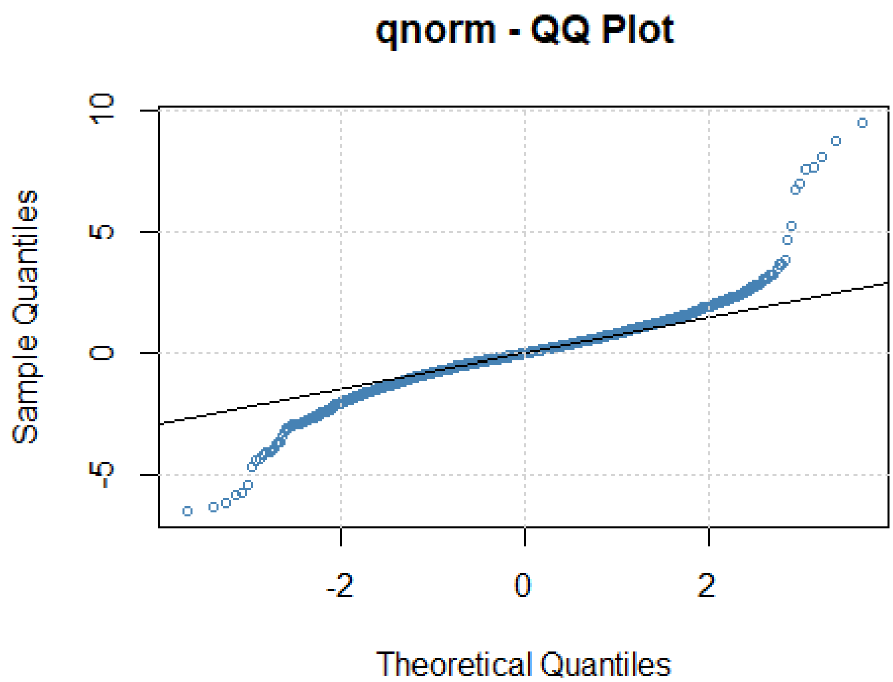
Since the calculated p-value in Ljung-Box test in lower than 0.05, we can reject null hypothesis at 95% confidence interval. The series has serial correlation problem

(b) Fit a Gaussian ARMA-GARCH model to the r_t series. Obtain the normal QQ- plot of the standardized residuals, and write down the fitted model. Is the model adequate? Why?

```
m9 <- garchFit(~arma(1,0)+garch(1,1), data=Grt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m9)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = Grt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x0000000013c3cab0>
##  [data = Grt]
##
## Conditional Distribution:
##  norm
##
```

```
## Coefficient(s):
##         mu          ar1       omega      alpha1       beta1
## 9.4852e-04  1.4759e-03  1.3137e-05  8.1641e-02  8.8265e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     9.485e-04   2.474e-04    3.834 0.000126 ***
## ar1    1.476e-03   1.781e-02    0.083 0.933960
## omega  1.314e-05   2.120e-06    6.196 5.80e-10 ***
## alpha1 8.164e-02   1.113e-02    7.335 2.22e-13 ***
## beta1  8.827e-01   1.458e-02   60.527  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  10823.23    normalized:  2.648209
##
## Description:
##  Sat May 01 18:39:58 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  16490.15  0
##  Shapiro-Wilk Test  R    W      0.9216928 0
##  Ljung-Box Test     R    Q(10)  7.883341  0.6402309
##  Ljung-Box Test     R    Q(15)  10.34868  0.7972529
##  Ljung-Box Test     R    Q(20)  14.56513  0.8007269
##  Ljung-Box Test     R^2  Q(10)  3.490384  0.9674239
##  Ljung-Box Test     R^2  Q(15)  4.841025  0.993389
##  Ljung-Box Test     R^2  Q(20)  6.100821  0.9987543
##  LM Arch Test       R    TR^2   3.915296  0.9849168
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.293972 -5.286245 -5.293975 -5.291236
```

```
plot(m9,which=13)
```



Mean Equation: r_t = 9.4852e-04 + 1.4759e-03 r_{t-1}

Std.error: (2.474e-04) (1.781e-02)

Variance Equation: $\sigma_t^2 = 1.314e-05 + 8.167e-02a_{t-1}^2 + 8.827e-01\sigma_{t-1}^2$

Std.error: (2.120e-06) (1.113e-02) (1.485e-02)

According to QQ-plot the model is not adequate, since the QQ-plot has fat-tails problem.

(c) Build an ARMA-GARCH model with Student-t innovations for the log return series. Perform model checking and write down the fitted model.

```
m10 <- garchFit(~arma(1,0)+garch(1,1),data=Grt,trace=F,cond.dist='std')

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m10)

##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = Grt, cond.dist = "st
d",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001cfd0248>
##  [data = Grt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1       shape
## 1.0403e-03  1.5223e-02  5.2203e-06  6.3349e-02  9.2477e-01  3.8486e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.040e-03   2.008e-04    5.181 2.21e-07 ***
## ar1    1.522e-02   1.467e-02    1.038  0.29932
## omega  5.220e-06   1.608e-06    3.247  0.00117 **
## alpha1 6.335e-02   1.162e-02    5.453 4.96e-08 ***
## beta1  9.248e-01   1.356e-02   68.184  < 2e-16 ***
## shape  3.849e+00   2.404e-01   16.010  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  11231.79    normalized:  2.748174
##
## Description:
##  Sat May 01 18:39:58 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  20801.57  0
##  Shapiro-Wilk Test  R    W      0.9161251 0
##  Ljung-Box Test     R    Q(10)  7.477688  0.6797086
##  Ljung-Box Test     R    Q(15)  10.25969  0.8030944
##  Ljung-Box Test     R    Q(20)  13.87331  0.8368608
##  Ljung-Box Test     R^2  Q(10)  2.788602  0.9859705
##  Ljung-Box Test     R^2  Q(15)  4.718126  0.9942559
##  Ljung-Box Test     R^2  Q(20)  6.55367   0.9979126
##  LM Arch Test       R    TR^2   3.200098  0.9939588
##
## Information Criterion Statistics:
```

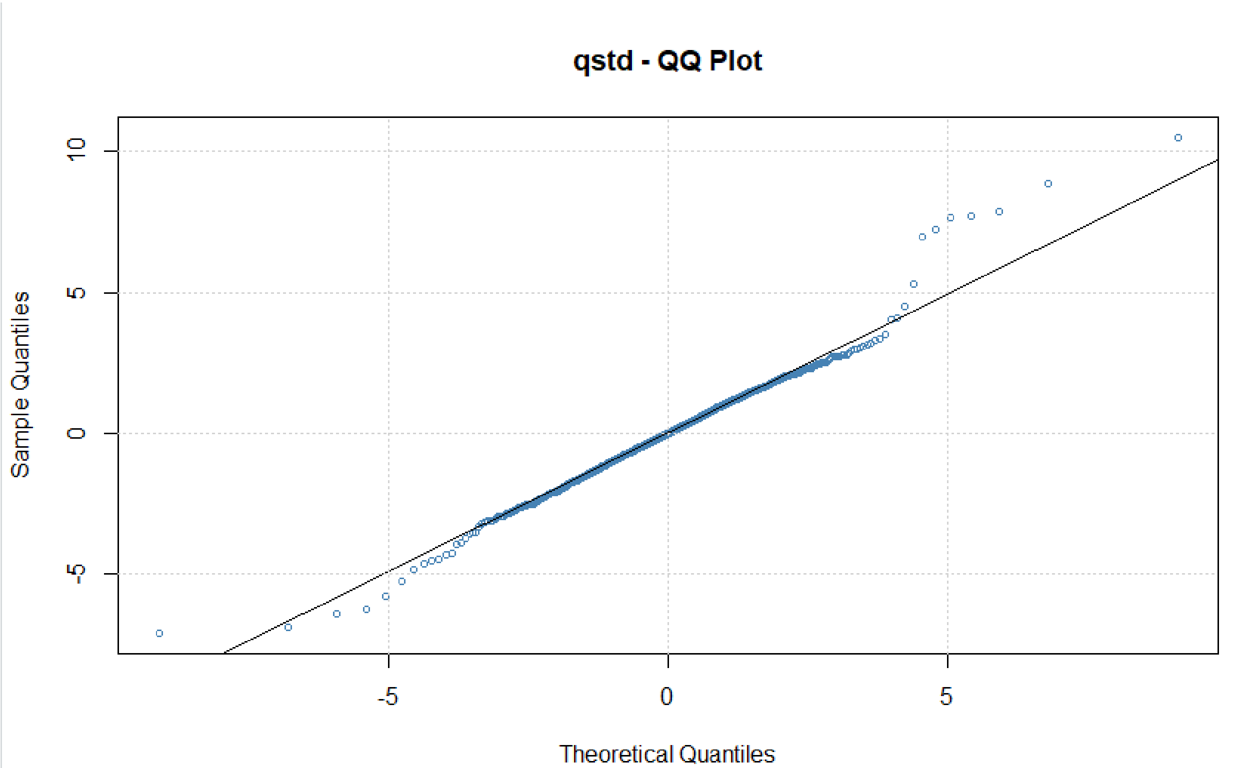
```
##        AIC        BIC        SIC       HQIC
## -5.493412 -5.484140 -5.493416 -5.490129
```

Mean Equation: r_t = 1.0403e-03 + 1.5223e-02 r_{t-1}

Std.error: (2.008e-04) (1.467e-02)

Variance Equation: $\sigma_t^2 = 5.22e - 06 + 6.335e - 02a_{t-1}^2 + 9.248e - 01\sigma_{t-1}^2$

Std.error: (1.608e-06) (1.162e-02) (1.356e-02)



According to QQ-plot the model is adequate, since the QQ-plot shows normality assumption

(d) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with Student-t innovations.

```
#3.d
predict(m10)

##    meanForecast  meanError standardDeviation
## 1   0.001037308 0.01613715        0.01613715
## 2   0.001056135 0.01620475        0.01620289
## 3   0.001056421 0.01626946        0.01626759
```

```
## 4   0.001056426 0.01633314        0.01633126
## 5   0.001056426 0.01639582        0.01639394
```

Consider the monthly returns log returns of the CRSP decile 9 portfolio from January 1951 to December 2010. The simple returns are in the file m-deciles.txt under the name CAP9RET.

(a) Is the expected value of the CRSP decile 9 portfolio log return zero? Why? Is there any serial correlation in the log returns? Why? If necessary, find an ARMA model to remove the serial correlations.

```
A=read.table("m-deciles.txt", header=T)
Drt = log(A[,10]+1)
t.test(Drt)
```

```
##
##  One Sample t-test
##
## data:  Drt
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054
```

Since the calculated p-value = 2.873e-07 is lower than 0.05, we can reject null hypothesis at 95% confidence interval. The expected value is significant from 0.

```
Box.test(Drt,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  Drt
## X-squared = 24.257, df = 10, p-value = 0.006946
```

We can reject null hypothesis at 95% confidence interval since the calculated p-value is less than 0.05. The series has serial correlation problem.

```
m11 = auto.arima(Drt)
summary(m11)
```

```
## Series: Drt
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
```

```
##          ma1      ma2    mean
##       0.1660  -0.0558  0.0096
## s.e.  0.0373   0.0370  0.0020
##
## sigma^2 estimated as 0.002392:  log likelihood=1152.66
## AIC=-2297.32   AICc=-2297.26   BIC=-2279
##
## Training set error measures:
##                        ME     RMSE        MAE       MPE     MAPE      MASE
## Training set 1.149455e-06 0.048807 0.03675026 -137.9858 502.9481 0.7525704
##                      ACF1
## Training set 0.00044559
```

(b) Is there any ARCH effect in the log returns? Why?

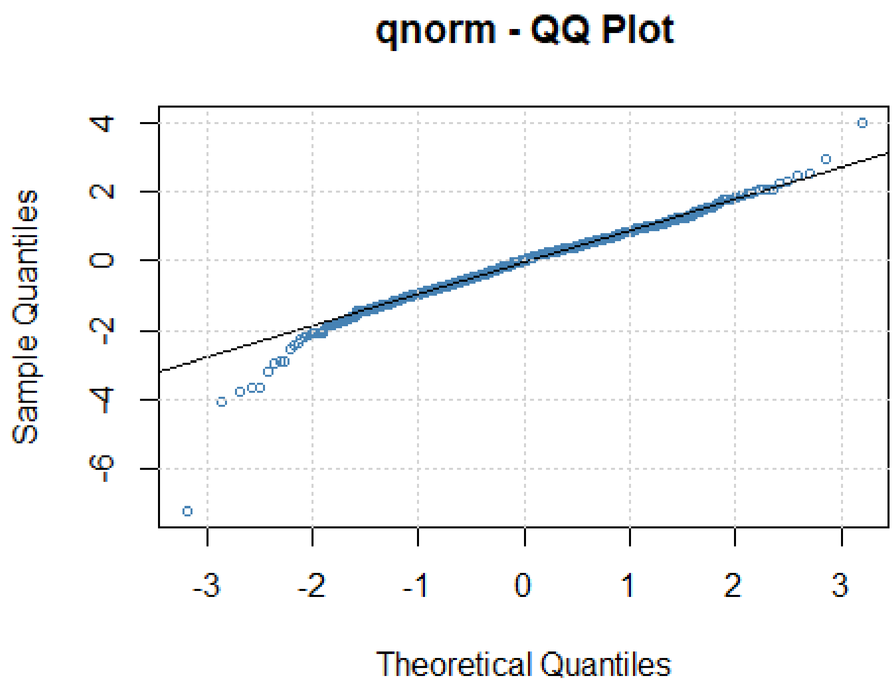
```
#4.b
Box.test(Drt^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  Drt^2
## X-squared = 19.824, df = 10, p-value = 0.03096
```

The calculated p-value is less than 0.05, we can reject null hypothesis in favor of alternative hypothesis at 95% confidence interval. There is ARCH effect in the log returns.

(c) Build a AR(1)-ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

```
#4.c
m12 <- garchFit(~arma(1,0)+garch(1,0),data=Drt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m12)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = Drt, trace = F)
##
## Mean and Variance Equation:
```

```
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x000000001d4cb210>
##  [data = Drt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      mu       ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1     0.1470670   0.0424301    3.466 0.000528 ***
## omega   0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Sat May 01 18:39:58 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480

plot(m12,which=13)
```



Mean Equation: r_t= 0.01053 + 0.14707r_{t-1}

Std.error: (0.0019275) (0.0424301)

Variance Equation: σ_t^2= 0.002 + 0.1815182a_{t-1}^2

Std.error: (0.0001482) (0.0651142)

The model is not adequate as QQ-plot shows fat-tails distribution.

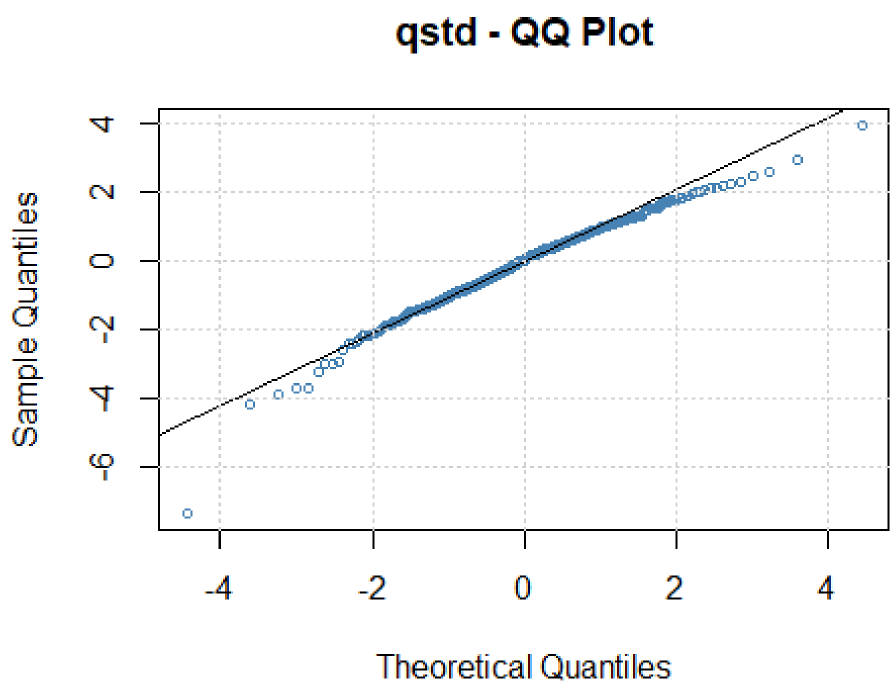
(d) Fit a AR(1)-ARCH(1) model with Standardized Student-t innovations to the log return series.

Perform model checking and write down the fitted model.

```
#4.d
m13 <- garchFit(~arma(1,0)+garch(1,0),data=Drt,trace=F,cond.dist="std")

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m13)

##
## Title:
##  GARCH Modelling
##
```

```
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = Drt, cond.dist = "st
d",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x000000001f5dd598>
##  [data = Drt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115905    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Sat May 01 18:39:59 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
##
## Information Criterion Statistics:
```

```
##        AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491

plot(m13,which=13)
```



Mean Equation: r_t= 0.0116189 + 0.1076982 r_{t-1}

Std.error: (0.0017710) (0.0403169)

Variance Equation: σ_t^2= 0.0019203 + 0.1900830a_{t-1}^2

Std.error: (0.0001818) (0.0713692)

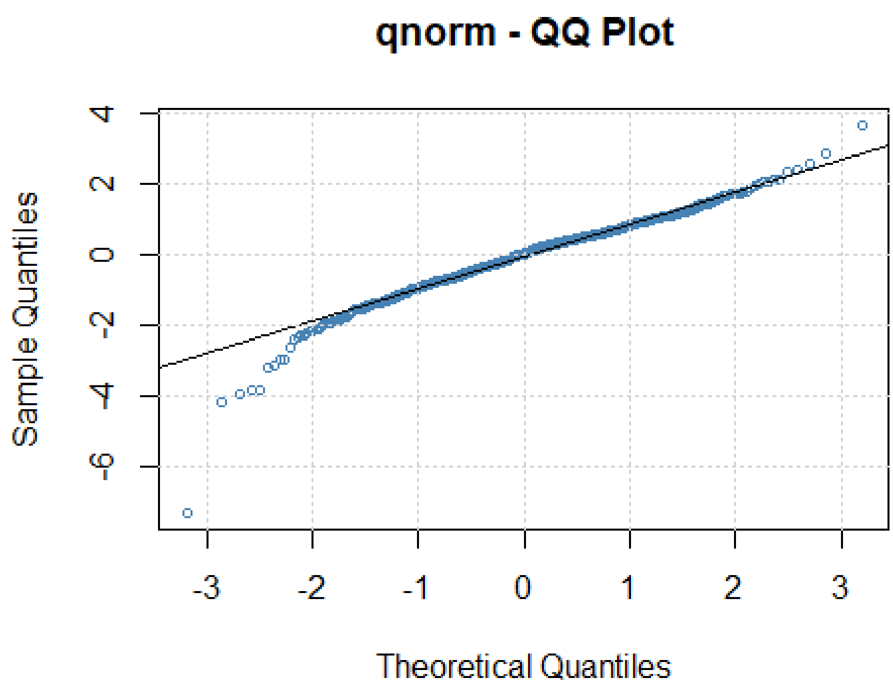
The model is not adequate as QQ-plot shows fat-tails distribution.

(e) Build a ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

```
#4.e
m14 <- garchFit(~garch(1,0),data=Drt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m14)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = Drt, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x0000000020699018>
##  [data = Drt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Sat May 01 18:39:59 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
```

```
##        AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102

plot(m14,which=13)
```



Mean Equation: r_t= 0.012346

Std.error: (0.001923)

Variance Equation: σ_t^2= 0.002016 + 0.000145a_{t-1}^2

Std.error: (0.000145) (0.062209)

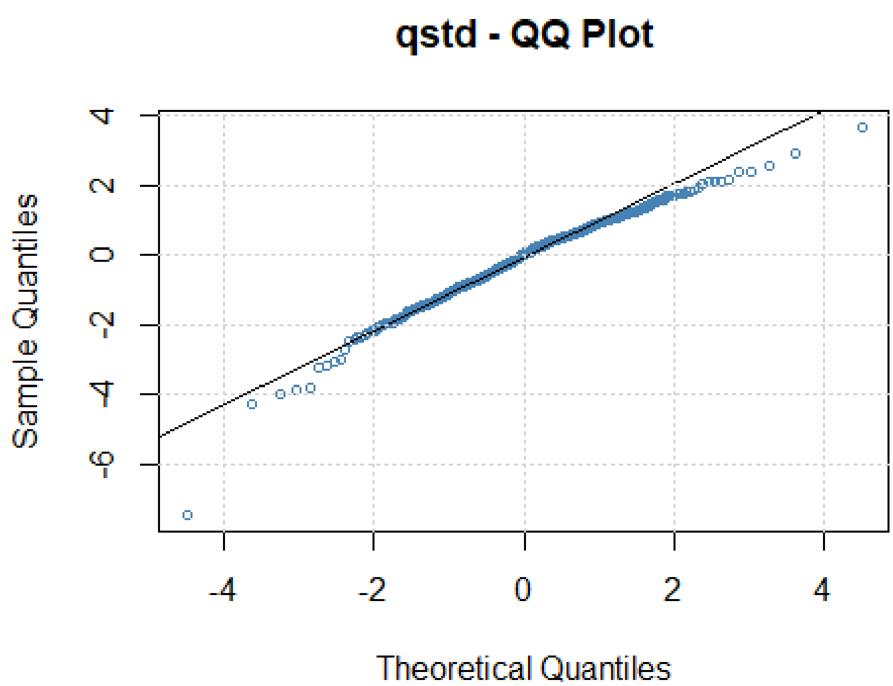
The model seems to be adequate.

(f) Fit a ARCH(1) model with Standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

```
#4.f
m15 <- garchFit(~garch(1,0),data=Drt,trace=F,cond.dist='std')

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m15)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = Drt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x00000000212009f0>
##  [data = Drt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Sat May 01 18:39:59 2021 by user: ryu_r
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705
##  Ljung-Box Test     R    Q(15)  22.99926  0.08415548
##  Ljung-Box Test     R    Q(20)  27.3947   0.1245202
##  Ljung-Box Test     R^2  Q(10)  6.667871  0.7563837
##  Ljung-Box Test     R^2  Q(15)  10.0157   0.8187514
##  Ljung-Box Test     R^2  Q(20)  11.63085  0.928196
##  LM Arch Test       R    TR^2   6.819315  0.8693193
```

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.275969 -3.250529 -3.276031 -3.266148

plot(m15,which=13)
```



Mean Equation: r_t= 0.013356

Std.error: (0.001685)

Variance Equation: σ_t^2= 0.001928 + 0.204163 a_{t-1}^2

Std.error: (0.000185) (0.072375)

The model is not adequate as the fat-tail problem occurred in the QQ-plot

(g) Compare the model (c)-(f) which model you select.

M14 is preferred since the model is addequated