EE320
Semester 1/2020
Group H.W. #3

Related chapter:

Chapter 7(Derivatives of More-Than-One Independent Variable Function) and 8(Optimization without Constraint: More-Than-One Independent Variable Cases)

Instruction:

Please form a group with maximum of 4 persons and follow the following instructions.

In the "EE320 PRACTICE PROBLEM SET, semester 1/2020",

(A.) Please work on "**PRACTICE PROBLEM SET 5** (Multivariate calculus**:** basis derivative and applications)"

Question 5 "Change in equilibrium price and quantity"
Question 7 "MRS"
Note: "Correction of lecture about MRS_{xy}" In class, I mistakenly stated that $MRS_{xy} = \frac{dy}{dx} = -\frac{MU_x}{MU_y}$, when in fact, MRS_{xy} is defined as: $MRS_{xy} = -\frac{dy}{dx} = \frac{MU_x}{MU_y}$
Question 8 "MRTS"

You can choose additional question per one additional member (additional from 4 people) from question: 3, 4, 6, 9, 11, 12, 14, 15

(B.) Please work on "**PRACTICE PROBLEM SET 6** (Multivariate calculus**:** Unconstrained optimization)"

Before trying the following questions, please read about Cournot model in the supplementary section below.

Question 3 "Oligopolists and merger"
Question 5 "Multiproduct, firm collusion, and antitrust"
Question 10 "Cournot equilibrium"
Question 11 "Profit function"

You can choose additional question per one additional member (additional from 4 people) from question: 1, 4, 6, 8, 9

You can write your answers on iPad and export to pdf, type in word document, use google docs, or write on paper and scan into pdf.

Please upload Microsoft Word® (DOC and DOCX), Google Docs via Google Drive™, or Portable Document Format (PDF) to the **turnitin.com** by **November 26, 2020.**

To hand in your work, please do the followings:

(1.) Go to turnitin.com and create a new student profile or choose ‘enroll in a class’ if you already have a Turnitin account, with the class ID and enrollment key below:

Class ID: 26193043
Class enrollment key: 32002
You will have class “EE320_SEM1/2020_SEC046402” on your Turnitin page.

(2.) Upload your answer to “GROUP H.W.#3”. Select “Single file upload”.

Supplementary section on “Cournot model of Oligopoly: Quantity setters”

Cournot model explains strategic interaction between the oligopolists in the market.

Imagine a market for one kind of good which can be produced by two firms: firm 1, producing Q_1, and firm 2, producing Q_2.

Suppose inverse market demand is $P(Q) = \alpha - Q$, where $Q = Q_1 + Q_2$.

Also suppose that each firm total cost is. $C_i(Q_i) = cQ_i$, where $i = 1,2$, $c < \alpha$.

Firm 1’s profit function is:

$$\pi_1(Q_1, Q_2) = P(Q)Q_1 - cQ_1$$
$$\pi_1(Q_1, Q_2) = P(Q_1 + Q_2)Q_1 - cQ_1$$
$$\pi_1(Q_1, Q_2) = (\alpha - Q_1 - Q_2)Q_1 - cQ_1$$

Notice that firm 1’s profit depends on firm 2’s production quantity through inverse market demand function which depends on total quantity sold in the market.

Firm 2’s profit function is:

$$\pi_2(Q_1, Q_2) = P(Q)Q_2 - cQ_2$$
$$\pi_2(Q_1, Q_2) = P(Q_1 + Q_2)Q_2 - cQ_2$$
$$\pi_2(Q_1, Q_2) = (\alpha - Q_1 - Q_2)Q_2 - cQ_2$$

Notice that firm 2’s profit depends on firm 1’s production quantity through inverse market demand function on total quantity sold in the market.

FOC for firm 1, given Q_2 , is:

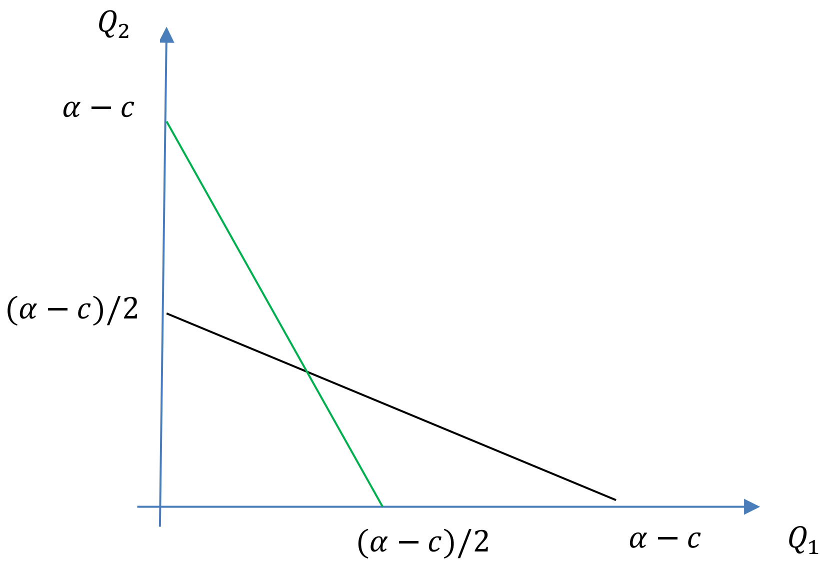
$$\frac{\partial \pi_1(Q_1, Q_2)}{\partial Q_1} = (\alpha - 2Q_1 - Q_2) - c = 0$$

That is, $Q_1^* = \frac{\alpha - c - Q_2}{2}$. This is called firm 1's best response function (the green line)

FOC for firm 2, given Q_1 , is:

$$\frac{\partial \pi_2(Q_1, Q_2)}{\partial Q_2} = (\alpha - Q_1 - 2Q_2) - c = 0$$

That is, $Q_2^* = \frac{\alpha - c - Q_1}{2}$. This is called firm 2's best response function (the black line).



The Nash equilibrium tells the exact market share of each firm in the market and happens when each firm's action is a best response to the other firm's action (the crossing point of the two best response functions). This can be found by:

Input Q_2^* in firm 1's best response function:

$$Q_1^* = \frac{\alpha - c - \left(\frac{\alpha - c - Q_1^*}{2}\right)}{2}$$

Hence, $Q_1^* = \frac{\alpha - c}{3}$, which gives $Q_2^* = \frac{\alpha - c}{3}$.