Question 1 Total product function

Suppose that $TP(L) = 120L^2 - L^3$, where TP is the total output. Answer the following questions

a. Find the expression for AP and MP.

$AP(L) = 120L - L^2 \ and \ MP(L) = 240L - 3L^2$

b. Find the value of L that maximizes AP, MP and TP, respectively.

Maximize AP ⇒ $FOC: \ AP'(L) = 120 - 2L = 0 \ ==> \ L = 60$

$SOC: \ AP''(60) = -2 < 0)$

Maximize MP ⇒ $FOC: MP'(L) = 240 - 6L = 0 \ ==> \ L = 40$

$SOC: \ (MP''(40) = -6 < 0)$

Maximize TP ⇒ $FOC: \ TP'(L) = MP(L) = 240L - 3L^2 = 0 ==> \ L = 0, 80.$

$SOC: \ (TP''(L) = 240 - 6L \ ==> \ TP''(80) < 0.$

c. Define the domain set of L that justifies the production function.

One should be noted that (i) L must be greater than or equal to zero. (negative L doesn't make sense. Second, firm will not produce in the region that L is greater than 80 units. When L exceeds 80 units, TP decreases.

Question 2

Let the total cost function be:

$$TC(Q) = 2Q^2 - 8Q + 10.$$

a) Determine whether *TC(Q)* is a convex or concave function.

Ans. TC'(Q) = 4Q-8. ⇒ TC"(Q)=4>0. Thus, TC(Q) is a convex function.

b) Find the quantity *Q** that minimizes the total cost.

Ans. TC'(Q) = 4Q-8=0. ⇒Q*=2.

c) Verify that *TC(Q*)* is the lowest cost by using the second derivative test.

Ans. TC"(2)=4>0. Thus, TC(2) =2 is the minimum.

Question 3

Let the demand function be given by $Q = 60 - 2P$ where P is the unit price and Q is the amount of quantity. Assume that the production function of a firm is given by $Q = 5\sqrt{L}$ where L is the number of workers hired. Consider the following questions.

a. Suppose that wage is $50 per each worker hired. This is fixed, regardless to how many workers hired by a firm. Write down the equation that summarizes the total cost of hiring labor by "L" workers.

$Cost = W * L = 50L$

b. Normally, our cost function is written in terms of Q. Based on “a”, can you find a way to represent the total cost equation in the terms of Q, rather than L?
Note that $Q = 5\sqrt{L} \Rightarrow Q^2 = 25L \Rightarrow L = \frac{1}{25}Q^2$.
$Cost = 50L = 50\left(\frac{1}{25}Q^2\right) = \frac{1}{2}Q^2$

c. Use the cost function in “b”, and solve for the level of profit-maximizing output/price. Verify your answer
First, rewrite demand into p-equal form. That is, $P = \frac{60-Q}{2}$
$\pi(Q) = \frac{60-Q}{2}Q - \frac{1}{2}Q^2$
FOC: $\pi'(Q) = 30 - Q - Q = 0$
$Q = 15$.
SOC: $\pi''(Q) = -2 < 0 \Rightarrow$ This warrants that $Q = 15$ is actually the profit-maximizing level of output.

d. At the level of profit-maximizing output, how many workers should the firm hire?
$Q = 15 \implies L = \frac{1}{25}(15)^2 = 9\ units.$

<u>Question 4</u> *Deriving the market supply*

Suppose the cost function of a representative firm can be given by: $C(Q) = 3Q^2 + 5Q + 75$ where Q is the level of output produced. Answer the following question

a. Derive the expression for MC, AVC, and ATC.
$MC(Q) = 6Q + 5$
$AVC(Q) = \frac{TVC}{Q} = 3Q + 5$
$ATC(Q) = 3Q + 5 + \frac{75}{Q}$

b. Find the level of Q that results in the lowest total cost.
That is, when MC = 0.
$MC(Q) = 6Q + 5 = 0 \implies Q = 0.$
Second order condition is also satisfied because $C''(Q) = MC'(Q) = 6 > 0$

c. Find the supply equation of the representative firm. Make sure you specify the range of price that justifies your equation as the one representing the supply equation.
$P = MC(Q)$
$P = 6Q + 5$
When Q = 0 , P = 5. That is, firm needs to earn at least $5 if it would stay active in the market.

d. If there are 60 identical firms in the market, derive the market supply curve.
$Q = \frac{P-5}{6} \implies Q^s{}_M = 60 * \left(\frac{P-5}{6}\right) = 10(P-5)$ when P is greater than or equal to zero.
$P = 5 + \frac{Q}{10}$

Question 5

Given the following function

$$f(x) = 2x^3 + 8x^2 - 32x - 50$$

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a. Find the critical value(s) of x and the corresponding stationary value(s) of *f(x)*.

Ans. $f'(x) = 6x^2 + 16x - 32 = 0 \Rightarrow x^* = -4, 4/3$

b. Evaluate whether the stationary value(s) found in part a) are relative maxima or minima or inflection points by using *the first-derivative test*.

Ans. f(-4) is a maximum because f'(-5)=38 > 0 and f'(-3)=-26 < 0.

f(4/3) is a minimum because f'(1)=-10 < 0 and f'(2)=24 > 0.

Question 6

A competitive firm receives a price p for each unit of its output, and pays a price w for each unit of its only variable input. It also incurs set up costs of F. Its output from using x units of variable input is $f(x) = \sqrt{x}$. Determine the firm's revenue, cost, and profit functions.

Ans. For $x > 0$, $R = p\sqrt{x}$; $C = wx + F$; $\pi(x) = p\sqrt{x} - wx - F$.

$\pi'(x) = 0$ when $w = p/2\sqrt{x} \Rightarrow x = p^2/4w^2$. Check: $\pi''(x) < 0$

Question 7

The price a firm obtains for a commodity varies with demand Q according to the formula $P(Q) = 18 - 0.006Q$. Total cost is $C(Q) = 0.004Q^2 + 4Q + 4500$.

a. Find the firm's profit and the value Q which maximizes profit.

Ans. $\pi(Q) = Q.P(Q) - C(Q) = -0.01Q^2 + 14Q - 4500 \Rightarrow Q^* = 700$

b. Find a formula for the elasticity of $P(Q)$ w.r.t. Q, and find the particular value Q^* of Q at which the elasticity is equal to -1.

Ans. This is the price elasticity w.r.t. demand (Q=Qd):

$$E_d = \frac{Q}{P}\frac{dP}{dQ} = \frac{Q}{18-0.006Q}(-0.006) = -1 \Rightarrow Q^* 1500$$

c. Show that the marginal revenue is 0 at Q*.

Ans. MR = 18 -0.012Q ⇒ at Q*=1500, MR = 0.

Question 8 Monopoly and Subsidy program

A monopolist firm faces the market demand equation given by $P = 150 - 0.5Q$ and operates under a technology with cost function given by $TC = 100 + 3Q + 7Q^2$. Consider the following questions

a. By using the derivative method, find the level of profit-maximizing output and price. Verify that your answer is the correction solution that results in maximized profit.

$MR = 150 - Q$

$MC = 3 + 14Q$

FOC:

$MR = MC$

$150 - Q = 3 + 14Q$

$Q = \frac{147}{15}$

SOC:

$\pi'' = MR' - MC' < 0$

$MR' = -1$

$MC' = 14$

$\pi'' = -15 < 0$

So, $Q = \frac{147}{15}$ is the level of profit-maximizing output.

Continue with all the information given above, but now add another assumption to the questions. That is, we now assume that government subsidizes the monopolist for $3 for each unit of output.

b. Write the cost function of the monopolist when subsidization is taken into account.
$TC = 100 + 3Q + 7Q^2 - 3Q = 100 + 7Q^2$.

c. Find the level of profit-maximizing output and price under the subsidy program.
Redo the same as in (a), but now we use the new TC as obtained in (b).
FOC:
$MR = MC$
$150 - Q = 14Q$
$Q = 10.$

SOC. $\pi'' = -15 < 0$. Thus, $Q = 10$ is the level of profit-maximizing output.
Given this Q, price would be $145. (This can be obtained by plugging Q into the demand equation.)