Question 1.1

Unconditional expectation of a_t : $E(a_t)$

using law of iterated expectation

$E[a_t] = E[E[a_t | F_{t-1}]]$

$= E[E[\sigma_t \varepsilon_t | F_{t-1}]]$

$= E[\sigma_t E[\varepsilon_t | \cancel{F_{t-1}}]]$; assume : $\varepsilon_t \overset{iid}{\sim} (0,1)$

$E[a_t] = 0$

unconditional expectation of r_t : $E[r_t]$

$r_t = 0.002 + a_t$

$E(r_t) = E(0.002) + E(a_t)$

$= 0.002$

Question 1.2

unconditional variance of a_t $Var(a_t)$

$Var(a_t) = E[a_t^2] = E[(a_t - E(\cancel{a_t}))^2]$ 0

$= E[a_t^2]$

$= E[E[a_t^2 | F_{t-1}]]$

$= E[\sigma_t^2]$

$= E[0.01 + 0.1 a_{t-1}^2]$

$E[a_t^2] = 0.01 + 0.1 E[a_{t-1}^2]$

From properties of weak stationarity ; $E[a_t^2] = E[a_{t-1}^2]$

$E[a_t^2] = \frac{0.01}{1-0.1} = 0.0111$

conditional variance of a_t : $Var(a_t | F_{t-1})$

$Var(a_t | F_{t-1}) = E[(a_t - E(\cancel{a_t} | F_{t-1}))^2 | F_{t-1}]$ 0

$= E[a_t^2 | F_{t-1}]$

$= E[\sigma_t^2 \varepsilon_t^2 | F_{t-1}]$

$= \sigma_t^2 E[\varepsilon_t^2 \cancel{|} F_{t-1}]$ 1

$= \sigma_t^2$

Question 1.3

$h = 100$

$r_t = 0.002 + a_t$

1-step ahead prediction of r_t

$r_{101} = 0.002 + a_{101}$

$E[r_{101} | F_{100}] = E[0.002 | F_{100}] + E[a_{101} | F_{100}]$

$= 0.002 + 0$

$= 0.002$

$\sigma_t^2 = 0.01 + 0.1\, a_{t-1}^2$

1-step ahead volatility

$\sigma_{101}^2 = 0.01 + 0.1\, a_{100}^2$

$E[\sigma_{101}^2 | F_{100}] = 0.01 + 0.1\, E[a_{100}^2 | F_{100}]$

$\sigma_{100}^2(1) = 0.01 + 0.1(0.015)^2 = 0.0100225$

Question 4

∞-step ahead volatility

$\lim_{l \to \infty} \sigma_{100}^2(l) = 0.0111$

∞-step ahead prediction $r_{100}(\infty)$

$\lim_{l \to \infty} r_{100}(l) = 0.002$