

(i) Find Cournot Equilibrium when there are three firms in the Market.

$$P = a - bQ ; Q = q_1 + q_2 + q_3$$

$$C_1 = C_2 = C_3 = C$$

What is the equilibrium price? : P^*

What are firms' profit? $\pi_1 = \pi_2 = \pi_3 = ?$

$$P = a - bq_1 - bq_2 - bq_3$$

C is constant ; $C_1 = C_2 = C_3 = C$

$$\pi_1 = TR_1 - TC_1$$

$$= [a - bq_1 - bq_2 - bq_3]q_1 - C_1$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$$

$$a - bq_2 - bq_3 = 2bq_1$$

$$q_1 = \frac{a}{2b} - 0.5q_2 - 0.5q_3$$

$$\pi_2 = TR_2 - TC_2$$

$$= [a - bq_1 - bq_2 - bq_3]q_2 - C_2$$

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 = 0$$

$$a - bq_1 - bq_3 = 2bq_2$$

$$q_2 = \frac{a}{2b} - 0.5q_1 - 0.5q_3$$

$$\pi_3 = TR_3 - TC_3$$

$$= [a - bq_1 - bq_2 - bq_3]q_3 - C_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$$

$$a - bq_1 - bq_2 = 2bq_3$$

$$q_3 = \frac{a}{2b} - 0.5q_1 - 0.5q_2$$

$$\text{Plug } q_3 \text{ in } q_1 ; q_1 = \frac{a}{2b} - 0.5q_2 - 0.5\left[\frac{a}{2b} - 0.5q_1 - 0.5q_2\right]$$

$$q_1 = \frac{a}{2b} + 0.25q_1 - 0.5q_2$$

$$0.75q_1 = \frac{a}{2b} - 0.5q_2$$

$$= 0.25 \frac{a}{b} - 0.5q_2$$

$$q_1 = \frac{1}{3}\left(\frac{a}{b} - q_2\right)$$

$$\text{Plug } q_3 \text{ in } q_2 ; q_2 = \frac{a}{2b} - 0.5q_1 - 0.5\left[\frac{a}{2b} - 0.5q_1 - 0.5q_2\right]$$

$$q_2 = 0.25 \frac{a}{b} - 0.25q_1 + 0.25q_2$$

$$0.75q_2 = 0.25\left(\frac{a}{b} - q_1\right)$$

$$q_2 = \frac{1}{3}\left(\frac{a}{b} - q_1\right)$$

$$\text{Plug } q_2 \text{ in } q_1 ; q_1 = \frac{1}{3}\left[\frac{a}{b} - \left(\frac{1}{3}\left(\frac{a}{b} - q_1\right)\right)\right]$$

$$q_1 = \frac{1}{3}\left(\frac{a}{b} - \frac{a}{3b} + \frac{q_1}{3}\right)$$

$$9q_1 = \frac{2a}{b} - \frac{a}{b} + q_1$$

$$8q_1 = \frac{a}{b}$$

$$q_1 = \frac{a}{8b} //$$

$$q_2 = \frac{a}{8b} - \frac{a}{16b}$$

$$12q_2 = \frac{2a}{b} - \frac{a}{b}$$

$$q_2 = \frac{a}{8b} //$$

$$q_3 = \frac{a}{2b} - 0.5 \frac{a}{8b} - 0.5 \frac{a}{8b}$$

$$q_3 = \frac{a}{4b} //$$

$$\therefore P = a - b\left[\frac{a}{8b} + \frac{a}{8b} + \frac{a}{4b}\right]$$

$$P = a - \frac{3}{4}a$$

$$P = \frac{a}{4} //$$

$$\pi_1 = Pq_1 - C_1$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{8b}\right) - C_1$$

$$= \frac{a^2}{32b} - C_1 //$$

$$\pi_2 = Pq_2 - C_2$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{8b}\right) - C_2$$

$$= \frac{a^2}{32b} - C_2 //$$

$$\pi_3 = Pq_3 - C_3$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_3$$

$$= \frac{a^2}{16b} - C_3 //$$

② If there are N firms ...

$$q_i^* = f(N), P = f(N), \pi_i = f(N)$$

$$P = a - bQ; Q = q_n$$

$$C_n = C$$

$$\pi_i = P(Q)q_i - C_i q_i = [a - b(q_n)]q_i - C_i q_i$$

Assume that $q_1 + q_2 + \dots + q_n = A$

$$P = a - b(q_1 + q_2 + \dots + q_n)$$

$$P = a - bq_1 - bq_2 - \dots - bq_n$$

$$\pi_1 = (a - bq_1 - bq_2 - \dots - bq_n)q_1 - C_1$$

⋮

$$\pi_n = (a - bq_1 - bq_2 - \dots - bq_n)q_n - C_n$$

$$\frac{\partial \pi_i}{\partial q_i} = a - bq_1 - \dots - bq_n = 0$$

$$\frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_n) = q_1$$

$$\frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_{n-1}) = q_n$$

$$q_1 - 0.5q_n = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_n)$$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A$$

$$q_2 = \frac{a}{b} - A$$

⋮

$$q_n = \frac{a}{b} - A$$

According to $A = q_1 + q_2 + \dots + q_n$; $A = n\left(\frac{a}{b} - A\right)$

$$A = n\frac{a}{b} - nA$$

$$(n+1)A = n\frac{a}{b}$$

$$A = \frac{na}{(n+1)b}$$

$$q_i = \frac{a}{(n+1)b} *$$

$$P = a - b(A)$$

$$= a - b\left[\frac{na}{(n+1)b}\right]$$

$$= a - \frac{n}{n+1}a$$

$$= \frac{na + a - na}{n+1}$$

$$P = \frac{a}{n+1} *$$

$$\pi_i = Pq_i - C_i$$

$$= \left(\frac{a}{n+1}\right)\left(\frac{a}{(n+1)b}\right) - C_i$$

$$\pi_i = \frac{a^2}{(n+1)^2 b} - C_i *$$

③ From Q₂, What happen if $N \rightarrow \infty$
 $N = 1$

if $N \rightarrow \infty$; $q_i = \frac{a}{(n+1)b} \rightarrow 0$
→ Each firm reach closer to zero unit
 $A = nq_i \rightarrow \infty$
→ Every firm combined with Q together will be nearly ∞ unit
 $P = \frac{a}{n+1} \rightarrow 0$
→ When supply increase, price will decrease until reach nearly zero
 $\pi_i = \frac{a^2}{(n+1)^2b} - c_i \rightarrow -c_i$
→ Each firm loses their profit and fixed cost is equal to C

if $N \rightarrow 1$; $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ Since $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$
→ Monopoly can sell less quantity
 $A = nq_i = Q$ Since $n=1$
→ Firm will become a monopoly
 $P = \frac{a}{n+1} = \frac{a}{2}$ Since $P_m = \frac{a}{2} > P = \frac{a}{n+1}$
→ Monopoly sets a higher price
 $\pi_i = \frac{a^2}{(n+1)^2b} - c_i = \frac{a^2}{4b} - c_i$ But $\pi_m > \pi_i = \frac{a^2}{(n+1)^2b} - c_i$
→ Monopoly can get higher profits