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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$\bar{X} = \dfrac{63+72+78+81+87+75+75+90}{8} = 77.625$

$\bar{Y} = \dfrac{2.8+3.4+3+3.5+3.6+3.0+2.7+3.7}{8} = 3.2125$

Table 1.a

Student	Y_i	X_i	$y_i - \bar{y}$	$x_i - \bar{x}$	$\hat{u}_i$
1	2.8	63	-0.4125	-14.625	0.0857
2	3.4	72	0.1875	-5.625	0.3791
3	3	78	-0.2125	0.375	-0.2253
4	3.5	81	0.2875	3.375	0.1725
5	3.6	87	0.3875	9.375	0.0681
6	3.0	75	-0.2125	-2.625	-0.1231
7	2.7	75	-0.5125	-2.625	-0.4231
8	3.7	90	0.4875	12.375	0.0659

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x - \bar{x})^2} = \frac{(63-77.625)(2.8-3.2125)+(72-77.625)(3.4-3.2125)+\dots+(90-77.625)(3.7-3.2125)}{(-14.625)^2+(-5.625)^2+\dots+(12.375)^2}$$

$$= \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 3.2125 - (0.0341)(77.625)$$

$$\hat{\beta}_0 = 0.5681$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ $\qquad$ $\hat{u}_i = y_i - \hat{y}_i$

$\hat{y}_i = 0.5681 + 0.0341 x_i$ $\qquad$ $\hat{u}_i = y_i - 0.5681 - 0.0341 x_i$

$$\sum_{i=0}^{10} \hat{u}_i = \left(2.8-0.5681-0.0341(63)\right) + \left(3.4-0.5681-0.0341(72)\right) + \dots + \left(3.7-0.5681-0.0341(90)\right)$$

$$= -0.0002 \approx 0$$

This is not zero because β_1 and β_0 were rounded

$SST = \sum_i (x_i - \bar{x})^2 = 551.875$ | $\sum x_i^2 = 48,717$ $SSR = \sum (\hat{y} - \bar{y})^2 = 1.02875$

$\sigma^2 = Var(u_i | X_i) = E[(u_i - E(u_i))^2] = E(u_i^2)$

1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$var(\hat{u}_i) = \sigma^2 = \frac{SSR}{n-2} = \frac{1.02875}{8-2} = 0.1715$$

$$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST} = \frac{(0.1715)(48717)}{(8)(551.875)} = 2.0398$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{\sigma^2}{551.875} = 0.00033$$

2. Data is listed in the table

X_i	Y_i	$X_i - \bar{X}$	$y_i - \bar{y}$	$\hat{u}_i$
10	0	-10	-9.1	-0.1455
12	2	-8	-7.1	0.0636
14	5	-6	-4.1	1.2727
16	6	-4	-3.1	0.4818
18	7	-2	-2.1	-0.3091
22	10	2	0.9	-0.8909
24	10	4	0.9	-2.6818
26	15	6	5.9	0.5273
28	16	8	6.9	-0.2636
30	20	10	10.9	1.9455

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\bar{X} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x - \bar{x})^2} = \frac{(-10)(-9.1) + (-8)(-7.1) + \ldots + (10)(10.9)}{(-10)^2 + (-8)^2 + \ldots + (10)^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - 0.8955(20)$$

$$\hat{\beta}_0 = -8.81$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \beta_0 + \beta_1 X_i$

$= -8.81 + 0.8955 X_i$

$u_i = y_i - \hat{y}_i$

$= y_i + 8.81 - 0.8955 X_i$

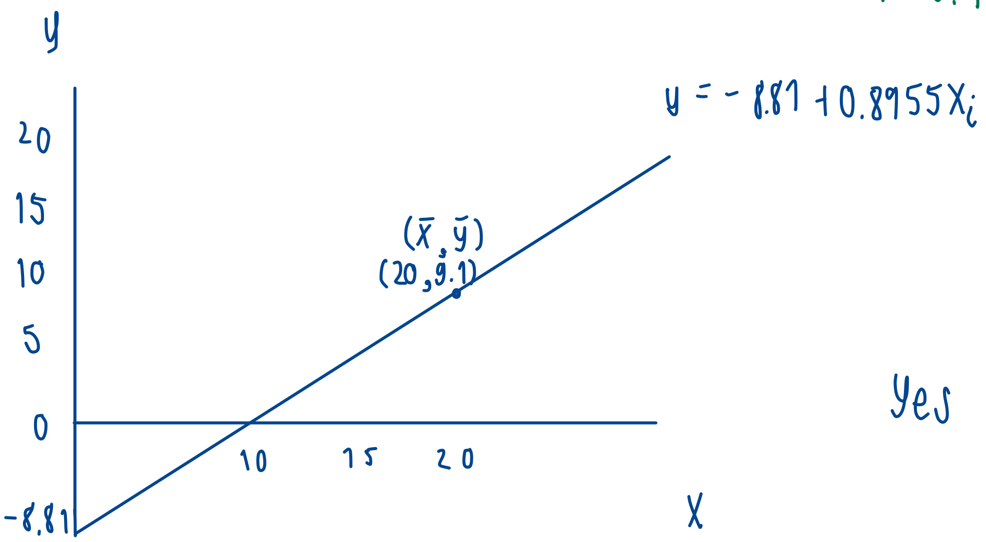
$\Sigma \hat{u}_i = (0 + 8.81 - 0.8955(10)) + (2 + 8.81 - 0.8955(12)) + \ldots + (20 + 8.81 - 0.8955(30))$

$= 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$y = -8.81 + 0.8955(20)$

$y = 9.1$



yes

2.4 If X_i =16, what is the predicted Y?

$E(y) = -8.81 + 0.8955(16)$

$= 5.518$

2.5 Find $var(\hat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$SSR = \Sigma(y - \bar{y})^2 = 366.9$

$SST = \Sigma(x - \bar{x})^2 = 440$

$var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{366.9}{10-2} = 45.8625$

$Var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST} = \frac{(45.8625)(4440)}{10(440)} = 46.2794$

$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{46.8625}{440} = 0.104233$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$y_i = \beta_1 X_i + u_i$$

$$\text{argmin } \Sigma_i (y_i - \tilde{\beta}_1 x_i)^2$$

F.O.C. w.r.t. $\tilde{\beta}_1$

$$\frac{\partial \, \Sigma_i (y_i - \tilde{\beta}_1 x_i)^2}{\partial \, \tilde{\beta}_1} = 0$$

$$\Sigma_i 2 (y_i - \tilde{\beta}_1 x_i)(-x_i) = 0$$

$$\Sigma_i x_i (y_i - \tilde{\beta}_1 x_i) = 0(-2)$$

$$\Sigma_i x_i y_i - \Sigma_i \tilde{\beta}_1 x_i = 0$$

$$\Sigma_i x_i y_i = \Sigma_i \tilde{\beta}_i x_i^2$$

$$\hat{\beta}_1 = \frac{\Sigma_i x_i y_i}{\Sigma_i x_i^2}$$

Substitute $\quad y_i = \beta_1 X_i + u_i$

$$\hat{\beta}_1 = \frac{\Sigma_i x_i (\beta_1 x_i + u_i)}{\Sigma_i x_i^2}$$

$$\hat{\beta}_1 = \frac{\Sigma_i \beta_1 x_i^2}{\Sigma_i x_i^2} + \frac{\Sigma_i u_i x_i}{\Sigma_i x_i^2}$$

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{\Sigma u_i x_i}{\Sigma_i x_i^2}\right) \quad \text{by SLR4} \quad E(u_i | x_i) = 0$$

$$E(\hat{\beta}_1) = \beta_1$$