

2.6 Additional examples

Q. 10

$$p(x) = ? = D(x)$$

given $x = 120$, $p = \$1200$ where x = no of units rented

each \$50 increases $\uparrow$, $x \downarrow$ by 3

e.g. \$100 $\uparrow$, $x \downarrow$ by 6

} linear change
 $\therefore$ expect $p(x)$ to be
 a linear function

$$p(x) = mx + c$$

$$1200 = m(120) + c \quad \text{--- (1)}$$

$$1250 = m(120-3) + c \quad \text{--- (2)} \quad \text{from the statement that } p \uparrow \$50, x \downarrow \text{ by } 3$$

$$(2) - (1) \quad 50 = -3m$$

$$m = \frac{-50}{3} \quad \text{insert this in (1)}$$

$$1200 = \frac{-50}{3} \times 120 + c$$

$$c = 1200 + 2000 = 3200$$

$$\therefore p = \frac{-50}{3}x + 3200 \quad \text{--- (3)}$$

$$R = p \cdot x$$

But the question asks for what p that gives maximum R

$\therefore$ have to write R as a function of p not as a function of x

Note The question also hint this by saying "write a function describing $\overset{R}{\text{revenue}}$ in terms of monthly $\overset{p}{\text{rent}}$ "

Next change x in terms of p

$$\text{From (3)} \quad \frac{50}{3}x = 3200 - p$$

$$x = 192 - \frac{3}{50}p \quad \text{--- (4)}$$

From $R = p \cdot x$ and (4)

$$R = p \left(192 - \frac{3}{50} p \right)$$

$$= -\frac{3}{50} p^2 + 192p$$

$$= -\frac{3}{50} (p^2 - 3200p)$$

$$= -\frac{3}{50} (p^2 - 2(1600)p + 1600^2) + 1600^2 \times \frac{3}{50}$$

$$= \left(-\frac{3}{50} \right) (p - 1600)^2 + 153600$$

$\Rightarrow$ negative $\therefore$ opens downward $\Rightarrow$



This parabola will have a vertex at $(1600, 153600)$

$\therefore$ the price should be changed is at \$1600