

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$a. \sum_{i=1}^5 (a + bx_i) = 5a + b \sum_{i=1}^5 x_i$$

$$= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$$

$$b. \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$c. \sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$$

$$d. \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 [2x+2+3] = \sum_{x=1}^2 2x+5 = 2(1) + 2(2)+5 = 11$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$f(x) = p(x=x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$$

$$8b = 1$$

$$b = \frac{1}{8}$$

- b. Find the answer for $P(X \leq 2)$

$$P(X \leq 2) = 1 - P(X > 2)$$

$$= 1 - P(X=3) - P(X=4)$$

$$= 1 - (0.5)(\frac{1}{8}) - (0.25)(\frac{1}{8})$$

$$= 0.90625$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$P(-2 \leq X \leq 3) = 1 - P(X < -2)$$

$$= 1 - (0.25)(\frac{1}{8})$$

$$= 0.96875$$

- d. Find the answer for $P(X \geq 1)$

$$P(X \geq 1) = 1 - P(X < 1)$$

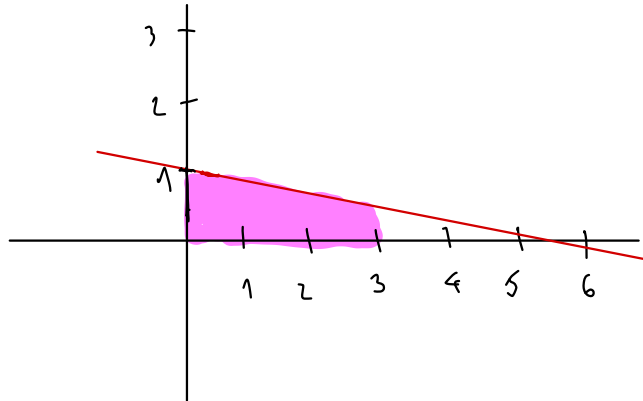
$$= 1 - P(X=-2) - P(X=-1) - P(X=0)$$

$$= 0.53125$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_1^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9} \right] = -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9} = \frac{16}{18} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$P(X \geq 2) = \int_2^3 f(x) dx$$

$$= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_2^3$$

$$= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9} \right] = -\frac{9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9} = \frac{7}{18}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} xf(x) dx \\ &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{x^3}{27} + \frac{6x^2}{18} \right]_0^3 \end{aligned}$$

$$= \left[-\frac{(3)^3}{27} + \frac{6(3)^2}{18} \right]$$

$$= -\frac{27}{27} + \frac{6 \cdot 9}{18}$$

$$= 2$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

X/Y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

- b. Find the marginal probability distribution function (PDF) of X

$$\frac{1}{6}$$

- c. Find the marginal probability distribution function (PDF) of Y

$$\frac{1}{2}$$

- d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

X	1	2	3	4	5	6
$P(X=x Y=1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- e. Find the expected value of X given Y is equal to 1

$$\frac{1}{6} \left[\left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(3 \cdot \frac{1}{6}\right) + \left(4 \cdot \frac{1}{6}\right) + \left(5 \cdot \frac{1}{6}\right) + \left(6 \cdot \frac{1}{6}\right) \right]$$

- f. Find the variance of X given Y is equal to 1

$$\left[\left(1 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(2 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(3 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(4 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(5 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(6 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] = \frac{10}{3}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{3} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{3} E(X_1 + X_2 + X_3) \\ &= \frac{1}{3} [E(X_1) + E(X_2) + E(X_3)] \\ &= \frac{1}{3} [\mu_X + \mu_X + \mu_X] \\ &= \frac{1}{3} 3\mu_X = \mu_X \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{3} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{9} \text{Var}(X_1 + X_2 + X_3) \\ &= \frac{1}{9} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) \\ &\quad + 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_1, X_3) \\ &\quad + 2\text{Cov}(X_2, X_3)] \\ &= \frac{1}{9} \cdot 9\sigma^2 = \sigma^2 \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{4} E(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{4} [E(X_1) + E(X_2) + E(X_3) + E(X_4)] \\ &= \frac{1}{4} [\mu_X + \mu_X + \mu_X + \mu_X] \\ &= \frac{1}{4} 4\mu_X = \mu_X \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{16} \text{Var}(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{16} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \text{Var}(X_4)] \\ &= \frac{1}{16} [4\sigma^2] \\ &= \frac{1}{4} \sigma^2 \end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\tilde{X} = \frac{1}{4}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$E(\tilde{X}) = E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4} E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4} [E(0.5X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)]$$

$$= \frac{1}{4} \cdot 4\mu_X = \mu_X$$

$\tilde{X}$ is an unbiased estimator of μ_X

$$\text{Var}(\tilde{X}) = \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4^2} \text{Var}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4^2} [\text{Var}(0.5X_1) + \text{Var}(X_2) + \text{Var}(0.5X_3) + \text{Var}(2X_4)]$$

$$= \frac{1}{4^2} [0.25\sigma_X^2 + \sigma_X^2 + 0.25\sigma_X^2 + 4\sigma_X^2]$$

$$= \frac{5.5\sigma_X^2}{16}$$

$$= 0.34\sigma_X^2$$

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$\bar{X}$ is more efficient estimator of μ_X than $\tilde{X}$ because it has smaller variance.