

Homework#1

Solution for question 8

Suppose that the real estimators for X_1 , X_2 , and X_3 are μ

$$8.1 \ E(\tilde{X}) = E\left(\frac{1}{3}X_1 + \frac{2}{3}X_2\right) = \frac{1}{3}E(X_1) + \frac{2}{3}E(X_2) = \frac{1}{3}\mu + \frac{2}{3}\mu = \mu$$

Hence, \tilde{X} is an unbiased estimator.

$$8.2 \ E(\bar{X}) = E\left(\frac{X_1 + X_2 + X_3}{3}\right) = \frac{1}{3}(E(X_1) + E(X_2) + E(X_3)) = \frac{\mu + \mu + \mu}{3} = \mu$$

Hence, \bar{X} is an unbiased estimator.

$$8.3 \ Var(\tilde{X}) = Var\left(\frac{1}{3}X_1 + \frac{2}{3}X_2\right) = \left(\frac{1}{3}\right)^2 Var(X_1) + \left(\frac{2}{3}\right)^2 Var(X_2) = \frac{5}{9}\sigma^2$$

$$Var(\bar{X}) = Var\left(\frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3\right) = \left(\frac{1}{3}\right)^2 Var(X_1) + \left(\frac{1}{3}\right)^2 Var(X_2) + \left(\frac{1}{3}\right)^2 Var(X_3) = \frac{1}{3}\sigma^2$$

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + [Bias(\hat{\theta})]^2$$

$$MSE(\tilde{X}) = \frac{5}{9}\sigma^2 \text{ and } MSE(\bar{X}) = \frac{1}{3}\sigma^2, \text{ since } Bias(\tilde{X}) = Bias(\bar{X}) = 0$$

8.4 Both \bar{X} and \tilde{X} are unbiased estimator where $Var(\tilde{X}) > Var(\bar{X})$ (and $MSE(\tilde{X}) > MSE(\bar{X})$)

Hence, \bar{X} is a better estimator than \tilde{X} .