MA217 Calculus for Social Science II

QUIZ II Wednesday 19th November 2014: 15.30-16.10 **(40 mins)**

There are 4 questions. The full mark is 40. **Show all your calculation steps and simplify your answers where possible.**

1. **(5 marks)** Students must give reasons. Random numerical answers will not gain marks.

(a) What is the largest possible number of pivots a 6x4 matrix can have? Why? If this linear system is a homogenous linear system ($\underline{\mathbf{b}} = \underline{0}$), how many solutions does this system have? Why?

(b) How many solutions does a consistent linear system of 3 equations and 4 unknowns have? Why?

(c) Suppose the coefficient matrix corresponding to a linear system is 4x6. How many pivot columns can the augmented matrix have if the linear system is inconsistent? Why?

2. **(15 marks)** For a linear system

$$\begin{aligned} 2x_2 + 3x_3 &= 1 + 4x_4 \\ \alpha\, x_1 + \beta\, x_2 &= 4 \\ 2x_1 + 2x_2 + 2x_4 &= 4 + 5x_3 \\ 2x_1 + 9x_4 &= 7 + 6x_3 \end{aligned}$$

where α and β are constant.

(a) Determine α and β that makes the linear system has a unique solution. Choose a value for α and β and determine x_4.

(b) If $\beta = 3$, identify α that makes the linear system consistent. Determine the solution. If the system has infinite number of solutions, give your answer in vector form. Show that your answer is correct.

3. **(15 marks)**

Let $\underline{\mathbf{A}} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$, determine the inverse of matrix $\underline{\mathbf{A}}$ using

(a) row operations method
(b) adjoint an co-factor method Note: keep the answer to 1 decimal place.

4. **(5 marks)**

Determine the determinant of the matrix $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 3 & 1 & 3 \\ 1 & 1 & 4 & 1 & 1 & 4 \\ 1 & 3 & 1 & 7 & 1 & 3 \\ 1 & 1 & 1 & 1 & 6 & 1 \\ 1 & 3 & 4 & 3 & 1 & 12 \end{bmatrix}$.

Numerical Answers

1.

(a) Max r = 4. If r = 4, the linear system unique solution. If r = 1-3, the linear system has infinite solutions.

(b) Cannot have unique solution. If r = 3, the linear system is consistent, hence it has infinite solutions. If r = 1 or 2, depending on $\underline{b}$, the system could have no solution if 0 = non-zero (inconsistent) or infinite solution if the system is consistent.

(c) 1 or 2 or 3 pivot columns, need at least 1 row to be 0 = non-zero for inconstancy

2.

(a) $\beta \neq \frac{3}{2}\alpha$, if choose $\alpha = 0, \beta = 1, x_4 = 4$

(b) If you choose to have unique solution, $\alpha \neq 2$ then you choose α and calculate unique solution e.g. if $\alpha = 0$, $x_1 = -5/2, x_2 = 19/6, x_3 = 0, x_4 = 4/3$

If you choose to have infinite solutions, $\alpha = 2$, $\underline{x} = \begin{bmatrix} 19/2 \\ -5/2 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -9 \\ 17/4 \\ -3/2 \\ 1 \end{bmatrix} C_4; C_4 \in \Re$

3. $\underline{\mathbf{A}}^{-1} = \begin{bmatrix} -0.7 & 0.2 & 0.3 \\ -1.3 & -0.2 & 0.7 \\ 0.8 & 0.2 & -0.2 \end{bmatrix}$

4. 720