8 How the Hypothesis Testing is done in Practice

1. Check the values of $t-statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t-statistics$ are to test $H_0 : \beta_i = 0$

   $\Rightarrow$ If the d.f. > 30, then when $t > 1.96$, we can reject H_0 with 5% sig. level (z table)

   $\Rightarrow$ **When $t > 1.96$**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When $t < 1.96$** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $H_0 : \beta_i = 5$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

Sales

Othercompany performance

CEO characteristics

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	—	.112 (.050)	.100 (.049)
profmarg	—	−.0023 (.0022)	−.0022 (.0021)
ceoten	—	—	.0171 (.0055)
comten	—	—	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353

simple regression with 12

Multiple Regression Analysis : Further Issues

1 Data scaling on OLS statistics

When we change the unit of measurment of a variable, the value of estimators would chance accordingly. For example

$$\widehat{bweght} = \widehat{\beta}_0 + \widehat{\beta}_1 cigs + \widehat{\beta}_2 fa\min c,$$

where
$bwght$ = child birth weight, in grams.
$cigs$ = number of cigarettes smoked by the mother while pregnant, per day.
$fa\min c$ = annual family income, in thousands of dollars.

- what if we use bweght in kilograms?

1kg = 1000 g

$$\widehat{bweght_{kg}} = \frac{\widehat{bweght_g}}{1000} = \frac{\hat{\beta}_0}{1000} + \frac{\hat{\beta}_1 cigs}{1,000} + \frac{\hat{\beta}_2 faminc}{1,000} = \hat{\alpha}_0 + \hat{\alpha}_1 cigs + \hat{\alpha}_2 faminc$$

$$\rightarrow \hat{\alpha}_j = \frac{\hat{\beta}_j}{1000}$$

p. 35

- what if we use famic in USD (instead of 1000 USD)

$$bweght_g = \hat{\beta}_0 + \hat{\beta}_1 cigs + \frac{\hat{\beta}_2}{1,000} faminc_{USD}$$

the value of this variable is going to be 1000 times larger than faminc

$$= \hat{\beta}_0 + \hat{\beta}_1 cigs + \hat{\theta}_2 faminc_{USD}$$

$$\Rightarrow \hat{\theta}_2 = \frac{\hat{\beta}_2}{1,000}$$

in other words $\hat{\theta}_2$ = impact of 1 USD ↑ in income

$\hat{\beta}_2$ = impact of 1,000 USD ↑ in income

- what if we use bweght in kg & income in THB

$$bweght_{kg} = \frac{\hat{\beta}_0}{1000} + \frac{\hat{\beta}_1}{1,000} cigs + \frac{\frac{\hat{\beta}_2}{1,000}}{30,000} faminc_{THB}$$

This value is going to be 30,000 times more than faminc

$\frac{d\ln x}{dx} = \frac{1}{x} \rightarrow d\ln x = \frac{1}{x} dx$

2 More on functional forms

$\Delta y = y_1 - y_2$

- Logarithmic Functional Form

usually natural log

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + u$$

$\beta_1 = \frac{d\log(y)}{d\log(x_1)} = \frac{\frac{1}{y} dy}{\frac{1}{x_1} dx_1} = \frac{\frac{1}{y}\Delta y \times 100}{\frac{1}{x_1}\Delta x_1 \times 100} = \frac{\%\,\Delta y}{\%\,\Delta x}$

With the log y and log x format, the coefficient is going to be the elasticity? (x_1: elasticity of y)

$\beta_2 = \frac{d\log(y)}{dx_2} = \frac{\frac{1}{y} dy}{dx_2} = \frac{\frac{1}{y}\Delta y}{\Delta x_2}$

⇒ If we want the upper term to be $\%\Delta$, then

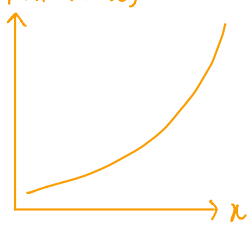
$100\beta_2 = \frac{100 \frac{1}{y}\Delta y}{\Delta x_2} = \frac{\%\Delta y}{\Delta x_2}$

$100\beta_2 = \%\Delta$ in y given that x_2 increases by 1 unit.

- Models with Quadratics (squares)

→ Capture increasing/decreasing marginal effect (slope of the relationship between x & y is not constant.

COVID-19

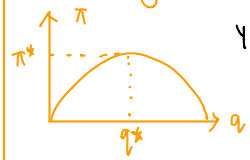


$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$

(+) (+)

decreasing returns



$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$

(+) (−)

$\pi = (P - MC)q$; Assume $MC = 10$, $D: P = 100 - q$

$\pi = (100 - q - 10)q$

FOC: $\pi' = 100 - 2q - 10 = 0$

$90 - 2q = 0$

β_1 "+"

β_2 "−"

Example : Effects of Pullution on Housing Prices

$$\log(price) = \beta_0 + \beta_1 \log(nox) + \beta_2 \log(dist) + \beta_3 rooms + \beta_4 room^2 + \beta_5 stratio + u$$

where
price = housing price
nox = level of pollution
dist = distance from downtown
rooms = number of rooms
stratio = average student per teacher ratio
The estimation result is given by

In the US or many other countries, students can apply to schools in the area without competition; taking any test.
So, the lower stratio, the better the school.

```
regress lprice lnox dist rooms rooms_sq stratio
```

Source	SS	df	MS
Model	51.4933152	5	10.298663
Residual	33.0889098	500	.06617782
Total	84.582225	505	.167489554

Number of obs	= 506
F(5, 500)	= 155.62
Prob > F	= 0.0000
R-squared	= 0.6088
Adj R-squared	= 0.6049
Root MSE	= .25725

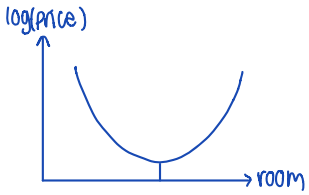
log(price) → lprice
log(nox) → lnox

lprice	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnox	-.9767545	.0995938	-9.81	0.000	-1.172429	-.7810806
dist	-.0321972	.0094013	-3.42	0.001	-.050668	-.0137264
rooms	-.5528032	.1612965	-3.43	0.001	-.8697056	-.2359007
rooms_sq	.0624697	.0124867	5.00	0.000	.0379368	.0870025
stratio	-.0486667	.0058131	-8.37	0.000	-.0600879	-.0372455
_cons	13.59154	.5650901	24.05	0.000	12.4813	14.70178

|t| > 1.96
→ All variable are significant
all < 0.05

Consider the effect of "room"

$$\frac{d\log(price)}{d\,room} = \beta_3 + 2\beta_4 room = -0.553 + 2(0.062)(rooms)$$



- at how many room dose 1 additional room has a positive impact on log (price)?

$0 = -0.553 + 2(0.062)(rooms)$

$room = 4.4$

answer ⇒ at ~~4.4~~ 5 ; round up room or more

What wold be the % change in price when the number of room increases from 5 to 6?

$$\frac{d\log(price)}{d\,room} = -0.553 + 2(0.062)rooms$$

$$\frac{100 \cdot \frac{1}{price}\Delta price}{\Delta room} = 100(-0.553 + 2(0.062)(5))$$

$= 100 \times 0.067 = 6.7\%$ increase.

What about the Δ in price when # room increase from 5 to 7.

For 6-7 → $100(-0.553 + 2(0.062)(6)) = 19.1\%$.

total % Δ in price = $6.7\% + 19.1\% = 25.8\%$.

3 Models with Interaction Terms

⇒ used when the impact of one variable depends on the value (level) of another variable.

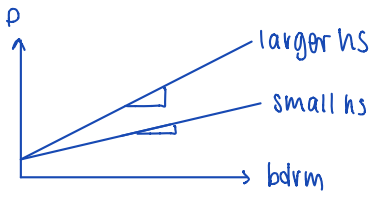
Consider

$$price = \beta_0 + \beta_1 sqrft + \beta_2 bdrms + \beta_3 sqrft \times bdrms + \beta_4 bthrms + u$$

(x_1, x_2, x_3 = $x_1 \times x_2$, x_4)

where

$price$ = housing price
$sqrft$ = house size (square feet)
$bdrms$ = number of bedrooms
$bthrms$ = number of bathrooms



$\frac{\Delta price}{\Delta bdrms} = \beta_2 + \beta_3 sqrft$

⇒ if $\beta_2 > 0$ then, an additional bedroom would increase price more for a large house!

4 More on the Goodness-of-Fit and Selection of Regressors

- Adding more regressors ALWAYS improve fit R^2 always↑
  - But we lose the "degree of freedom"
    (d.f. = free data point used to estimate the parameter)
    ⇝ 1 data point is sacrificed every time we estimate a parameter
  - using just R^2 would not punish "having too many regressor"
  - We use adjusted_R^2 or $\bar{R}^2$ when we want to punish adding too many regressors.

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{SSR/n}{SST/n}$$

$$adj\,R^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)}$$

↳ If we have k, d.f. = n-k-1 ↓, SSR/n-k-1 ↑ → adj_R^2 ↓

Using adjusted R-squared to choose between non-nested models (one model is not a subset of another).

Consider Model 1

$$\begin{aligned} \widehat{salary} &= \underset{(223.90)}{830.63} + \underset{(0.0089)}{0.0163}sales + \underset{(11.08)}{19.63}roe \\ n &= 209, \quad R^2 = 0.029, \quad \bar{R}^2 = 0.020 \end{aligned}$$

Consider Model 2

$$\begin{aligned} \widehat{\log(salary)} &= \underset{(0.29)}{4.36} + \underset{(0.033)}{0.2751}\log(sales) + \underset{(0.004)}{0.0179}roe \\ n &= 209, \quad R^2 = 0.282, \quad \bar{R}^2 = 0.275 \end{aligned}$$

27.5% of variation in Y is explained
so, this model is better?

Multiple Regression Analysis with Qualitative Information:

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories
- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	wage	educ	exper	female	married
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
. . .	. . .	. . .	. . .	. . .	. . .
525	11.56	16	5	0	1
526	3.50	14	5	1	0

3 Models with a single dummy independent variable

Consider

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + u. \quad (1)$$

where

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

In this case, the δ_0 notation is used to highlight the interpretation of the parameters multiplying dummy variables. In other cases, we can use any notation that is the most convenient.

1) $E(wage|female,educ) = E(\beta_0 + \delta_0 female + \beta_1 educ + u|female,educ)$

$= \beta_0 + \delta_0 female + \beta_1 educ + E(u|female,educ)$

$= \beta_0 + \delta_0 female + \beta_1 educ$ $\quad \Downarrow = 0$ MLR1-4 satisfied

2) Thus

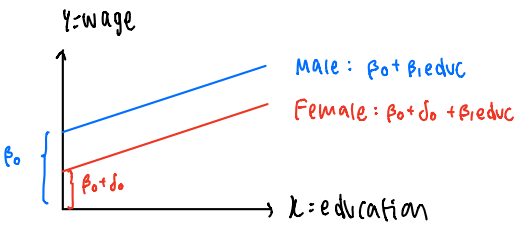
♀ : $E(wage|female=1,educ) = \beta_0 + \delta_0(1) + \beta_1 educ = \beta_0 + \delta_0 + \beta_1 educ$

♂ : $E(wage|female=0,educ) = \beta_0 + \delta_0(0) + \beta_1 educ = \beta_0 + \beta_1 educ$

$\delta_0 = E(wage|female=1,educ) - E(wage|female=0,educ)$

or $E(wage|Female,educ) - E(wage|male,educ)$

* given the same value of educ (same education level), δ_0 is the different in the expected wage of female & male.



→ By the way we model this regression function, "female" is going to give a constant impact on wage, regradless of the level of educ.

4 It is not possible to include all of the dummy alternatives in the same model (as long there in an intercept in the model)

- If we include all alternatives of a dummy variable in the same model, we will face the "perfect collinearity" problem.

For example:

$$wage = \beta_0 x_0 + \delta_0 female + \beta_1 educ$$

x_0: intercept × 1

$$x_0 = x_1 + x_3$$

$$1 = female + male$$

$$female = male + 1$$

id	female	male	x_0
	1	0	1
	1	0	1
	0	1	1
	0	1	1
	1	0	1
	0	1	1
	1	0	1

or

If there are "n" categories, we omit "1" category, to avoid multicollinearity

$$1 = winter + spring + summer + fall$$

$$winter = 1 - spring - summer - fall$$

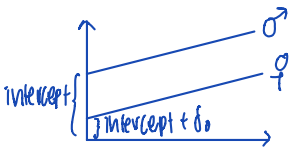
$$winter = \begin{cases} 1 & \text{if winter} \\ 0 & \text{otherwise} \end{cases}$$

$$spring = \begin{cases} 1 & \text{if spring} \\ 0 & \text{otherwise} \end{cases}$$

- At least one alternative has to be dropped. We treat the dropped alternative as the "BASE GROUP" or "BASELINE" or "BENCHMARK GROUP".

in this case, male

1 female
0 male




```
. regress lwage female male married educ exper
note: male omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  4,    521) =   75.27
       Model |  54.3265253     4  13.5816313            Prob > F      =  0.0000
    Residual |  94.0032262   521  .180428457           R-squared     =  0.3663
-------------+------------------------------           Adj R-squared =  0.3614
       Total |  148.329751   525  .28253286            Root MSE      =  .42477

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3251146   .0377061    -8.62   0.000    -.3991892    -.25104
        male |           0  (omitted)
     married |   .1380145   .0411197     3.36   0.001     .0572338    .2187953
        educ |   .0872644   .0071554    12.20   0.000     .0732075    .1013213
       exper |   .0076213   .0015314     4.98   0.000     .0046129    .0106297
       _cons |   .4690918   .1040575     4.51   0.000      .264668    .6735156
------------------------------------------------------------------------------
```

female workers are expected to have less wage compared to male worker

5 Using dummy variables for multiple categories

Case 1 We can use many dummy variables in the same model

Consider a model which includes 2 dummy variables– $female$ and $married$.

$\{1$ female, 0 otherwise

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 married + \beta_1 educ + \beta_2 \exp er + \beta_3 \exp er^2 + \beta_4 tenure + \beta_5 tenure^2 + u.$$

```
regress lwage female married educ exper expersq tenure tenursq
```

Source	SS	df	MS
Model	65.6482326	7	9.37831895
Residual	82.6815188	518	.159616832
Total	148.329751	525	.28253286

Number of obs	= 526
F(7, 518)	= 58.76
Prob > F	= 0.0000
R-squared	= 0.4426
Adj R-squared	= 0.4351
Root MSE	= .39952

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-.2901838	.0361121	-8.04	0.000	-.3611279	-.2192396
married	.0529219	.0407561	1.30	0.195	-.0271456	.1329894
educ	.0791547	.0068003	11.64	0.000	.0657952	.0925143
exper	.0269535	.0053258	5.06	0.000	.0164907	.0374163
expersq	-.0005399	.0001122	-4.81	0.000	-.0007603	-.0003196
tenure	.0312962	.0068482	4.57	0.000	.0178426	.0447499
tenursq	-.0005744	.0002347	-2.45	0.015	-.0010355	-.0001134
_cons	.4177837	.0988662	4.23	0.000	.2235557	.6120116

$\hat{\beta}$

Comments:

1) δ_0 measures the expected difference between female & male workers given the same marital status and other factors

$$\frac{d\log(wage)}{d female} = \frac{\frac{1}{wage}\Delta wage}{d female} = -0.29$$

$$\frac{100 \cdot \frac{1}{wage}\Delta wage}{d female} = 100 \cdot -0.29$$

$$\frac{\%\Delta wage}{d female} = 29.02\%$$

- female workers are expected to earn less than male workers by 29.02%, holding other factors constant

2) δ_0 measure the impact of being married (marriage premium)

But since $|t| < 1.96$ or $p > 0.05$, we do not reject H_0 of no impact

	♀	♂
ma	marrfem	marrmale
si	singfem	singmale

Consider a model which includes dummy variables for each gender/marital status combination– $marrmale$, $marrfem$ and $singfem$. (& singmale ← used as the basecase)

$$\log(wage) = \beta_0 + \delta_0 marrmale + \delta_1 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u. \quad (8.1)$$

```
regress lwage marrmale marrfem singfem educ exper expersq tenure tenursq
```

Source	SS	df	MS		
Model	68.3617623	8	8.54522029	Number of obs	= 526
Residual	79.9679891	517	.154676961	F(8, 517)	= 55.25
				Prob > F	= 0.0000
				R-squared	= 0.4609
				Adj R-squared	= 0.4525
Total	148.329751	525	.28253286	Root MSE	= .39329

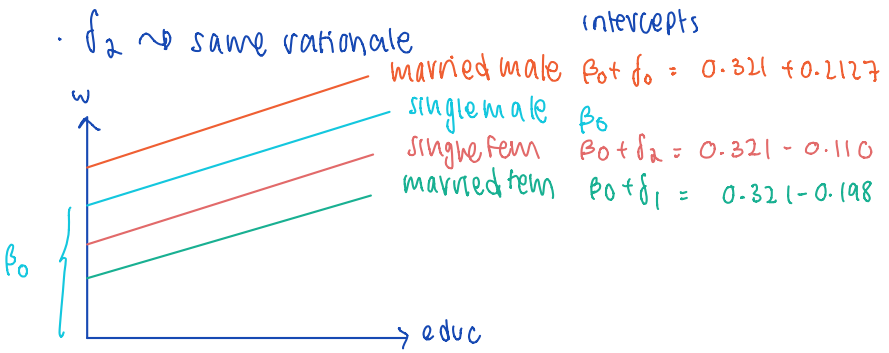
lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
marrmale	.2126757	.0553572	3.84	0.000	.103923	.3214284
marrfem	-.1982676	.0578355	-3.43	0.001	-.311889	-.0846462
singfem	-.1103502	.0557421	-1.98	0.048	-.219859	-.0008414
educ	.0789103	.0066945	11.79	0.000	.0657585	.092062
exper	.0268006	.0052428	5.11	0.000	.0165007	.0371005
expersq	-.0005352	.0001104	-4.85	0.000	-.0007522	-.0003183
tenure	.0290875	.006762	4.30	0.000	.0158031	.0423719
tenursq	-.0005331	.0002312	-2.31	0.022	-.0009874	-.0000789
_cons	.3213781	.100009	3.21	0.001	.1249041	.5178521

($\hat{\beta}$)

This regression is not the same as the previous one as it uses "single male" as the base group, the previous one use "male and single" as the base group

Comments:

- δ_0 measure the expect diff in wage of married male as compared to single male, holding other factor constant
- δ_1 measure the expected diff " female " female, "
- δ_2 ↝ same rationale



Case 2 We can use dummy variables to represent multiple categories of a variable
Consider the relationship between law school rankings and starting salaries

$$\begin{aligned}\log(salary) &= \beta_0 + \delta_0 top10 + \delta_1 r11\_25 + \delta_3 r26\_40 + \delta_4 r41\_60 + \beta_1 LSAT \\ &\quad + \beta_2 GPA + \beta_3 \log(libvol) + \beta_4 \log(cost) + u.\end{aligned}$$

where $top10$, $r11\_25$, $r26\_40$, $r41\_60$ would be equal to 1 when the variable $rank$ falls into the appropriate range.

** Rank below 60 would be the base case.

In many cases the "range of value" serves as a better explanatory variable than the "value" itself
↳ e.g. age may explain the model better if split into generations

```
. regress lsalary top10 r11_25 r26_40 r41_60 LSAT GPA llibvol lcost
```

Source	SS	df	MS		
Model	9.16538532	8	1.14567316	Number of obs	= 136
Residual	1.2109665	127	.009535169	F(8, 127)	= 120.15
				Prob > F	= 0.0000
Total	10.3763518	135	.076861865	R-squared	= 0.8833
				Adj R-squared	= 0.8759
				Root MSE	= .09765

lsalary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
top10	.5393428	.053542	10.07	0.000	.4333927	.6452928
r11_25	.4716199	.0390921	12.06	0.000	.3942637	.548976
r26_40	.2790977	.0346972	8.04	0.000	.2104383	.3477571
r41_60	.182382	.0283098	6.44	0.000	.126362	.238402
LSAT	.0060482	.0034919	1.73	0.086	-.0008616	.012958
GPA	.1305893	.0818678	1.60	0.113	-.0314122	.2925908
llibvol	.0725522	.0289213	2.51	0.013	.0153221	.1297824
lcost	.0249169	.0283224	0.88	0.381	-.031128	.0809619
_cons	8.363103	.4457314	18.76	0.000	7.481081	9.245125

Comments:

rank	top 10	11 - 25
1	1	0
2	1	0
3	1	0
...	1	0
10	1	1
11	0	1
...	0	1
25	0	0
...	0	0
40	0	0

1) δ_0 measure the diff in expected log(salary) of law_school graduate from a top 10 university compared to expected log(salary) of those who graduated from school ranked 61th and worse

2) δ_1 ~ same rational