Question Book for Time Value of Money

4–1 Future values for various compounding frequencies Delia Martin has $10,000 that she can deposit in any of three savings accounts for a 3-year period. Bank A compounds interest on an annual basis, bank B compounds interest twice each year, and bank C compounds interest each quarter. All three banks have a stated annual interest rate of 4%.

a. What amount would Ms. Martin have at the end of the third year, leaving all interest paid on deposit, in each bank?

b. What *effective annual rate* (*EAR*) would she earn in each of the banks?

c. On the basis of your findings in parts **a** and **b**, which bank should Ms. Martin deal with? Why?

d. If a fourth bank (bank D), also with a 4% stated interest rate, compounds interest continuously, how much would Ms. Martin have at the end of the third year? Does this alternative change your recommendation in part **c**? Explain why or why not.

4–2 Future values of annuities Ramesh Abdul wishes to choose the better of two equally costly cash flow streams: annuity X and annuity Y. X is an *annuity due* with a cash inflow of $9,000 for each of 6 years. Y is an *ordinary annuity* with a cash inflow of $10,000 for each of 6 years. Assume that Ramesh can earn 15% on his investments.

a. On a purely subjective basis, which annuity do you think is more attractive? Why?

b. Find the future value at the end of year 6, FVA_6, for both annuity X and annuity Y.

c. Use your finding in part **b** to indicate which annuity is more attractive. Why? Compare your finding to your subjective response in part **a**.

4–3 Present values of single amounts and streams You have a choice of accepting either of two 5-year cash flow streams or single amounts. One cash flow stream is an ordinary annuity, and the other is a mixed stream. You may accept alternative A or B—either as a cash flow stream or as a single amount. Given the cash flow stream and single amounts associated with each (see the accompanying table), and assuming a 9% opportunity cost, which alternative (A or B) and in which form (cash flow stream or single amount) would you prefer?

	Cash flow stream	
End of year	Alternative A	Alternative B
1	$700	$1,100
2	700	900
3	700	700
4	700	500
5	700	300
	Single amount	
At time zero	$2,825	$2,800

4–4 Deposits needed to accumulate a future sum Judi Janson wishes to accumulate $8,000 by the end of 5 years by making equal annual end-of-year deposits over the next 5 years. If Judi can earn 7% on her investments, how much must she deposit at the *end of each year* to meet this goal?

4–1 Using a time line The financial manager at Starbuck Industries is considering an investment that requires an initial outlay of $25,000 and is expected to result in cash inflows of $3,000 at the end of year 1, $6,000 at the end of years 2 and 3, $10,000 at the end of year 4, $8,000 at the end of year 5, and $7,000 at the end of year 6.

a. Draw and label a time line depicting the cash flows associated with Starbuck Industries' proposed investment.

b. Use arrows to demonstrate, on the time line in part **a**, how compounding to find future value can be used to measure all cash flows at the end of year 6.

c. Use arrows to demonstrate, on the time line in part **b**, how discounting to find present value can be used to measure all cash flows at time zero.

d. Which of the approaches—future value or present value—do financial managers rely on most often for decision making? Why?

4–2 Future value calculation *Without referring to tables or to the preprogrammed function on your financial calculator,* use the basic formula for future value along with the given interest rate, *i,* and the number of periods, *n,* to calculate the future value interest factor in each of the cases shown in the following table. Compare the calculated value to the value in Appendix Table A–1.

Case	Interest rate, *i*	Number of periods, *n*
A	12%	2
B	6	3
C	9	2
D	3	4

4–3 Future value tables Use the future value interest factors in Appendix Table A–1 in each of the cases shown in the following table to estimate, to the nearest year, how long it would take an initial deposit, assuming no withdrawals,

a. To double.

b. To quadruple.

Case	Interest rate
A	7%
B	40
C	20
D	10

4–4 Future values For each of the cases shown in the following table, calculate the future value of the single cash flow deposited today that will be available at the end of the deposit period if the interest is compounded annually at the rate specified over the given period.

Case	Single cash flow	Interest rate	Deposit period (years)
A	$ 200	5%	20
B	4,500	8	7
C	10,000	9	10
D	25,000	10	12
E	37,000	11	5
F	40,000	12	9

4–5 Future value You have $1,500 to invest today at 7% interest compounded annually.

a. Find how much you will have accumulated in the account at the end of (1) 3 years, (2) 6 years, and (3) 9 years.

b. Use your findings in part **a** to calculate the amount of interest earned in (1) the first 3 years (years 1 to 3), (2) the second 3 years (years 4 to 6), and (3) the third 3 years (years 7 to 9).

c. Compare and contrast your findings in part **b.** Explain why the amount of interest earned increases in each succeeding 3-year period.

4–6 Inflation and future value As part of your financial planning, you wish to purchase a new car exactly 5 years from today. The car you wish to purchase costs $14,000 today, and your research indicates that its price will increase by 2% to 4% per year over the next 5 years.

a. Estimate the price of the car at the end of 5 years if inflation is (1) 2% per year, and (2) 4% per year.

b. How much more expensive will the car be if the rate of inflation is 4% rather than 2%?

4–7 Future value and time You can deposit $10,000 into an account paying 9% annual interest either today or exactly 10 years from today. How much better off will you be at the end of 40 years if you decide to make the initial deposit today rather than 10 years from today?

4–8 Future value calculation Misty need to have $15,000 at the end of 5 years in order to fulfill her goal of purchasing a small sailboat. She is willing to invest the funds as a single amount today but wonders what sort of investment return she will need to earn. Use your calculator or the time value tables to figure out the approximate annually compounded rate of return needed in each of these cases:

a. Misty can invest $10,200 today.

b. Misty can invest $8,150 today.

c. Misty can invest $7,150 today.

4–9 Single-payment loan repayment A person borrows $200 to be repaid in 8 years with 14% annually compounded interest. The loan may be repaid at the end of any earlier year with no prepayment penalty.

a. What amount will be due if the loan is repaid at the end of year 1?

b. What is the repayment at the end of year 4?

c. What amount is due at the end of the eighth year?

4–10 Present value calculation *Without referring to tables or to the preprogrammed function on your financial calculator,* use the basic formula for present value, along with the given opportunity cost, *i,* and the number of periods, *n,* to calculate the present value interest factor in each of the cases shown in the accompanying table. Compare the calculated value to the table value.

Case	Opportunity cost, *i*	Number of periods, *n*
A	2%	4
B	10	2
C	5	3
D	13	2

4–11 Present values For each of the cases shown in the following table, calculate the present value of the cash flow, discounting at the rate given and assuming that the cash flow is received at the end of the period noted.

Case	Single cash flow	Discount rate	End of period (years)
A	$ 7,000	12%	4
B	28,000	8	20
C	10,000	14	12
D	150,000	11	6
E	45,000	20	8

4–12 Present value concept Answer each of the following questions.

a. What single investment made today, earning 12% annual interest, will be worth $6,000 at the end of 6 years?

b. What is the present value of $6,000 to be received at the end of 6 years if the discount rate is 12%?

c. What is the most you would pay today for a promise to repay you $6,000 at the end of 6 years if your opportunity cost is 12%?

d. Compare, contrast, and discuss your findings in parts **a** through **c.**

4–13 Present value Jim Nance has been offered a future payment of $500 three years from today. If his opportunity cost is 7% compounded annually, what value should he place on this opportunity today? What is the most he should pay to purchase this payment today?

4–14 Present value An Iowa state savings bond can be converted to $100 at maturity 6 years from purchase. If the state bonds are to be competitive with U.S. Savings Bonds, which pay 8% annual interest (compounded annually), at what price must the state sell its bonds? Assume no cash payments on savings bonds prior to redemption.

4–15 Present value and discount rates You just won a lottery that promises to pay you $1,000,000 exactly 10 years from today. Because the $1,000,000 payment is guaranteed by the state in which you live, opportunities exist to sell the claim today for an immediate single cash payment.

a. What is the least you will sell your claim for if you can earn the following rates of return on similar-risk investments during the 10-year period?
(1) 6%
(2) 9%
(3) 12%

b. Rework part **a** under the assumption that the $1,000,000 payment will be received in 15 rather than 10 years.

c. On the basis of your findings in parts **a** and **b**, discuss the effect of both the size of the rate of return and the time until receipt of payment on the present value of a future sum.

4–16 Present value comparisons of single amounts In exchange for a $20,000 payment today, a well-known company will allow you to choose *one* of the alternatives shown in the following table. Your opportunity cost is 11%.

Alternative	Single amount
A	$28,500 at end of 3 years
B	$54,000 at end of 9 years
C	$160,000 at end of 20 years

a. Find the value today of each alternative.

b. Are all the alternatives acceptable, i.e., worth $20,000 today?

c. Which alternative, if any, will you take?

4–17 Cash flow investment decision Tom Alexander has an opportunity to purchase any of the investments shown in the following table. The purchase price, the amount of the single cash inflow, and its year of receipt are given for each investment. Which purchase recommendations would you make, assuming that Tom can earn 10% on his investments?

Investment	Price	Single cash inflow	Year of receipt
A	$18,000	$30,000	5
B	600	3,000	20
C	3,500	10,000	10
D	1,000	15,000	40

4–18 Future value of an annuity For each case in the accompanying table, answer the questions that follow.

Case	Amount of annuity	Interest rate	Deposit period (years)
A	$ 2,500	8%	10
B	500	12	6
C	30,000	20	5
D	11,500	9	8
E	6,000	14	30

a. Calculate the future value of the annuity assuming that it is
(1) an ordinary annuity.
(2) an annuity due.

b. Compare your findings in parts **a**(1) and **a**(2). All else being identical, which type of annuity—ordinary or annuity due—is preferable? Explain why.

4–19 Present value of an annuity Consider the following cases.

Case	Amount of annuity	Interest rate	Period (years)
A	$ 12,000	7%	3
B	55,000	12	15
C	700	20	9
D	140,000	5	7
E	22,500	10	5

a. Calculate the present value of the annuity assuming that it is
(1) an ordinary due.
(2) an annuity due.

b. Compare your findings in parts **a**(1) and **a**(2). All else being identical, which type of annuity—ordinary or annuity due—is preferable? Explain why.

4–20 Ordinary annuity versus annuity due Marian Kirk wishes to select the better of two 10-year annuities, C and D. Annuity C is an ordinary annuity of $2,500 per year for 10 years. Annuity D is an annuity due of $2,200 per year for 10 years.

a. Find the *future value* of both annuities at the end of year 10, assuming that Marian can earn (1) 10% annual interest and (2) 20% annual interest.

b. Use your findings in part **a** to indicate which annuity has the greater future value at the end of year 10 for both the (1) 10% and (2) 20% interest rates.

c. Find the *present value* of both annuities, assuming that Marian can earn (1) 10% annual interest and (2) 20% annual interest.

d. Use your findings in part **c** to indicate which annuity has the greater present value for both (1) 10% and (2) 20% interest rates.

e. Briefly compare, contrast, and explain any differences between your findings using the 10% and 20% interest rates in parts **b** and **d.**

4–21 Future value of a retirement annuity Hal Thomas, a 25-year-old college graduate, wishes to retire at age 65. To supplement other sources of retirement income, he can deposit $2,000 each year into a tax-deferred individual retirement arrangement (IRA). The IRA will be invested to earn an annual return of 10%, which is assumed to be attainable over the next 40 years.

a. If Hal makes annual end-of-year $2,000 deposits into the IRA, how much will he have accumulated by the end of his 65th year?

b. If Hal decides to wait until age 35 to begin making annual end-of-year $2,000 deposits into the IRA, how much will he have accumulated by the end of his 65th year?

c. Using your findings in parts **a** and **b**, discuss the impact of delaying making deposits into the IRA for 10 years (age 25 to age 35) on the amount accumulated by the end of Hal's 65th year.

d. Rework parts **a**, **b**, and **c**, assuming that Hal makes all deposits at the beginning, rather than the end, of each year. Discuss the effect of beginning-of-year deposits on the future value accumulated by the end of Hal's 65th year.

4–22 Present value of a retirement annuity An insurance agent is trying to sell you an immediate-retirement annuity, which for a single amount paid today will provide you with $12,000 at the end of each year for the next 25 years. You currently earn 9% on low-risk investments comparable to the retirement annuity. Ignoring taxes, what is the most you would pay for this annuity?

4–23 Funding your retirement You plan to retire in exactly 20 years. Your goal is to create a fund that will allow you to receive $20,000 at the end of each year for the 30 years between retirement and death (a psychic told you would die after 30 years). You know that you will be able to earn 11% per year during the 30-year retirement period.

a. How large a fund will you need *when you retire* in 20 years to provide the 30-year, $20,000 retirement annuity?

b. How much will you need *today* as a single amount to provide the fund calculated in part **a** if you earn only 9% per year during the 20 years preceding retirement?

c. What effect would an increase in the rate you can earn both during and prior to retirement have on the values found in parts **a** and **b**? Explain.

4–24 Present value of an annuity versus a single amount Assume that you just won the state lottery. Your prize can be taken either in the form of $40,000 at the end of each of the next 25 years (i.e., $1,000,000 over 25 years) or as a single amount of $500,000 paid immediately.

a. If you expect to be able to earn 5% annually on your investments over the next 25 years, ignoring taxes and other considerations, which alternative should you take? Why?

b. Would your decision in part **a** change if you could earn 7% rather than 5% on your investments over the next 25 years? Why?

c. On a strictly economic basis, at approximately what earnings rate would you be indifferent between the two plans?

4–25 Perpetuities Consider the data in the following table.

Perpetuity	Annual amount	Discount rate
A	$ 20,000	8%
B	100,000	10
C	3,000	6
D	60,000	5

Determine, for each of the perpetuities:
a. The appropriate present value interest factor.
b. The present value.

4–26 Creating an endowment Upon completion of her introductory finance course, Marla Lee was so pleased with the amount of useful and interesting knowledge she gained that she convinced her parents, who were wealthy alums of the university she was attending, to create an endowment. The endowment is to allow three needy students to take the introductory finance course each year in perpetuity. The guaranteed annual cost of tuition and books for the course is $600 per student. The endowment will be created by making a single payment to the university. The university expects to earn exactly 6% per year on these funds.
a. How large an initial single payment must Marla's parents make to the university to fund the endowment?
b. What amount would be needed to fund the endowment if the university could earn 9% rather than 6% per year on the funds?

4–27 Future value of a mixed stream For each of the mixed streams of cash flows shown in the following table, determine the future value at the end of the final year if deposits are made into an account paying annual interest of 12%, assuming that no withdrawals are made during the period and that the deposits are made:
a. At the *end* of each year.
b. At the *beginning* of each year.

	Cash flow stream		
Year	A	B	C
1	$ 900	$30,000	$1,200
2	1,000	25,000	1,200
3	1,200	20,000	1,000
4		10,000	1,900
5		5,000	

4–28 Future value of a single amount versus a mixed stream Gina Vitale has just contracted to sell a small parcel of land that she inherited a few years ago. The buyer is willing to pay $24,000 at the closing of the transaction or will pay the amounts shown in the following table at the *beginning* of each of the next

5 years. Because Gina doesn't really need the money today, she plans to let it accumulate in an account that earns 7% annual interest. Given her desire to buy a house at the end of 5 years after closing on the sale of the lot, she decides to choose the payment alternative—$24,000 single amount or the mixed stream of payments in the following table—that provides the higher future value at the end of 5 years. Which alternative will she choose?

Mixed stream	
Beginning of year	Cash flow
1	$ 2,000
2	4,000
3	6,000
4	8,000
5	10,000

4–29 Present value—Mixed streams Find the present value of the streams of cash flows shown in the following table. Assume that the firm's opportunity cost is 12%.

A		B		C	
Year	Cash flow	Year	Cash flow	Year	Cash flow
1	−$2,000	1	$10,000	1–5	$10,000/yr
2	3,000	2–5	5,000/yr	6–10	8,000/yr
3	4,000	6	7,000		
4	6,000				
5	8,000				

4–30 Present value—Mixed streams Consider the mixed streams of cash flows shown in the following table.

	Cash flow stream	
Year	A	B
1	$ 50,000	$ 10,000
2	40,000	20,000
3	30,000	30,000
4	20,000	40,000
5	10,000	50,000
Totals	$150,000	$150,000

a. Find the present value of each stream using a 15% discount rate.
b. Compare the calculated present values and discuss them in light of the fact that the undiscounted cash flows total $150,000 in each case.

4–31 Present value of a mixed stream Harte Systems, Inc., a maker of electronic surveillance equipment, is considering selling to a well-known hardware chain the rights to market its home security system. The proposed deal calls for payments of $30,000 and $25,000 at the end of years 1 and 2 and for annual year-end payments of $15,000 in years 3 through 9. A final payment of $10,000 would be due at the end of year 10.

a. Lay out the cash flows involved in the offer on a time line.

b. If Harte applies a required rate of return of 12% to them, what is the present value of this series of payments?

c. A second company has offered Harte a one-time payment of $100,000 for the rights to market the home security system. Which offer should Harte accept?

4–32 Funding budget shortfalls As part of your personal budgeting process, you have determined that in each of the next 5 years you will have budget shortfalls. In other words, you will need the amounts shown in the following table at the end of the given year to balance your budget—that is, to make inflows equal outflows. You expect to be able to earn 8% on your investments during the next 5 years and wish to fund the budget shortfalls over the next 5 years with a single amount.

End of year	Budget shortfall
1	$ 5,000
2	4,000
3	6,000
4	10,000
5	3,000

a. How large must the single deposit today into an account paying 8% annual interest be to provide for full coverage of the anticipated budget shortfalls?

b. What effect would an increase in your earnings rate have on the amount calculated in part **a**? Explain.

4–33 Relationship between future value and present value—Mixed stream Using *only* the information in the accompanying table, answer the questions that follow.

Year (t)	Cash flow	Future value interest factor at 5% ($FVIF_{5\%,t}$)
1	$ 800	1.050
2	900	1.102
3	1,000	1.158
4	1,500	1.216
5	2,000	1.276

a. Determine the *present value* of the mixed stream of cash flows using a 5% discount rate.
b. How much would you be willing to pay for an opportunity to buy this stream, assuming that you can at best earn 5% on your investments?
c. What effect, if any, would a 7% rather than a 5% opportunity cost have on your analysis? (Explain verbally.)

4–34 Changing compounding frequency Using annual, semiannual, and quarterly compounding periods, for each of the following: (1) Calculate the future value if $5,000 is initially deposited, and (2) determine the *effective annual rate* (*EAR*).
a. At 12% annual interest for 5 years.
b. At 16% annual interest for 6 years.
c. At 20% annual interest for 10 years.

4–35 Compounding frequency, future value, and effective annual rates For each of the cases in the following table:
a. Calculate the future value at the end of the specified deposit period.
b. Determine the *effective annual rate, EAR*.
c. Compare the nominal annual rate, *i*, to the effective annual rate, EAR. What relationship exists between compounding frequency and the nominal and effective annual rates.

Case	Amount of initial deposit	Nominal annual rate, *i*	Compounding frequency, *m* (times/year)	Deposit period (years)
A	$ 2,500	6%	2	5
B	50,000	12	6	3
C	1,000	5	1	10
D	20,000	16	4	6

4–36 Continuous compounding For each of the cases in the following table, find the future value at the end of the deposit period, assuming that interest is compounded continuously at the given nominal annual rate.

Case	Amount of initial deposit	Nominal annual rate, *i*	Deposit period (years), *n*
A	$1,000	9%	2
B	600	10	10
C	4,000	8	7
D	2,500	12	4

4–37 Compounding frequency and future value You plan to invest $2,000 in an individual retirement arrangement (IRA) today at a *nominal annual rate* of 8%, which is expected to apply to all future years.

a. How much will you have in the account at the end of 10 years if interest is compounded (1) annually? (2) semiannually? (3) daily (assume a 360-day year)? (4) continuously?

b. What is the *effective annual rate, EAR,* for each compounding period in part **a**?

c. How much greater will your IRA account balance be at the end of 10 years if interest is compounded continuously rather than annually?

d. How does the compounding frequency affect the future value and effective annual rate for a given deposit? Explain in terms of your findings in parts **a** through **c.**

4–38 Comparing compounding periods René Levin wishes to determine the future value at the end of 2 years of a $15,000 deposit made today into an account paying a nominal annual rate of 12%.

a. Find the future value of René's deposit, assuming that interest is compounded (1) annually, (2) quarterly, (3) monthly, and (4) continuously.

b. Compare your findings in part **a,** and use them to demonstrate the relationship between compounding frequency and future value.

c. What is the maximum future value obtainable given the $15,000 deposit, the 2-year time period, and the 12% nominal annual rate? Use your findings in part **a** to explain.

4–39 Annuities and compounding Janet Boyle intends to deposit $300 per year in a credit union for the next 10 years, and the credit union pays an annual interest rate of 8%.

a. Determine the future value that Janet will have at the end of 10 years, given that end-of-period deposits are made and no interest is withdrawn, if

(1) $300 is deposited annually and the credit union pays interest annually.

(2) $150 is deposited semiannually and the credit union pays interest semiannually.

(3) $75 is deposited quarterly and the credit union pays interest quarterly.

b. Use your finding in part **a** to discuss the effect of more frequent deposits and compounding of interest on the future value of an annuity.

4–40 Deposits to accumulate future sums For each of the cases shown in the following table, determine the amount of the equal annual end-of-year deposits necessary to accumulate the given sum at the end of the specified period, assuming the stated annual interest rate.

Case	Sum to be accumulated	Accumulation period (years)	Interest rate
A	$ 5,000	3	12%
B	100,000	20	7
C	30,000	8	10
D	15,000	12	8

4–41 Creating a retirement fund To supplement your planned retirement in exactly 42 years, you estimate that you need to accumulate $220,000 by the end of

42 years from today. You plan to make equal annual end-of-year deposits into an account paying 8% annual interest.

a. How large must the annual deposits be to create the $220,000 fund by the end of 42 years?

b. If you can afford to deposit only $600 per year into the account, how much will you have accumulated by the end of the 42nd year?

4–42 Accumulating a growing future sum A retirement home at Deer Trail Estates now costs $85,000. Inflation is expected to cause this price to increase at 6% per year over the 20 years before C. L. Donovan retires. How large an equal annual end-of-year deposit must be made each year into an account paying an annual interest rate of 10% for Donovan to have the cash to purchase a home at retirement?

4–43 Deposits to create a perpetuity You have decided to endow your favorite university with a scholarship. It is expected to cost $6,000 per year to attend the university into perpetuity. You expect to give the university the endowment in 10 years and will accumulate it by making annual (end-of-year) deposits into an account. The rate of interest is expected to be 10% for all future time periods.

a. How large must the endowment be?

b. How much must you deposit at the end of each of the next 10 years to accumulate the required amount?

4–44 Inflation, future value, and annual deposits While vacationing in Florida, John Kelley saw the vacation home of his dreams. It was listed with a sale price of $200,000. The only catch is that John is 40 years old and plans to continue working until he is 65. Still, he believes that prices generally increase at the overall rate of inflation. John believes that he can earn 9% annually after taxes on his investments. He is willing to invest a fixed amount at the end of each of the next 25 years to fund the cash purchase of such a house (one that can be purchased today for $200,000) when he retires.

a. Inflation is expected to average 5% a year for the next 25 years. What will John's dream house cost when he retires?

b. How much must John invest at the end of each of the next 25 years in order to have the cash purchase price of the house when he retires?

c. If John invests at the beginning instead of at the end of each of the next 25 years, how much must he invest each year?