**EE320 Placement test**

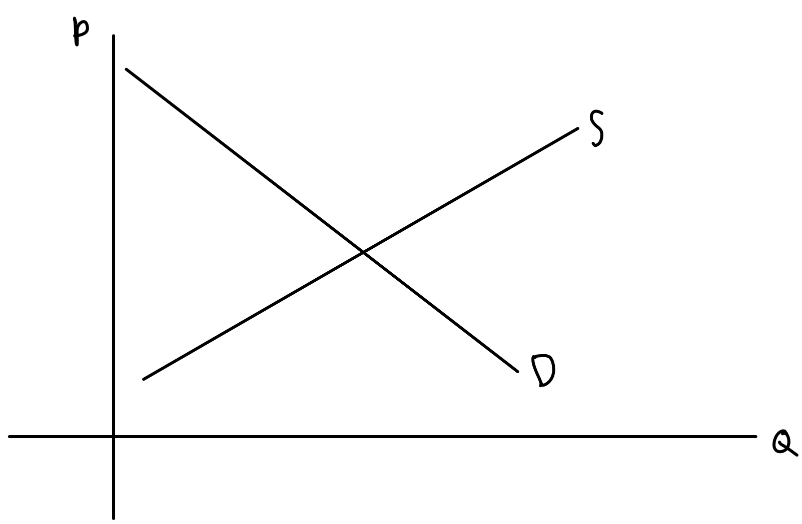
1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.

   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.

   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.

   1.3) If “a” increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.




$D = P = 10 - Q^2$

$S = P = Q - a$

$= Q - 14$

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q)' = \left(\ln(Q^2+1) + \frac{3Q}{Q-1}\right)$$

$$= \frac{d}{dQ}\left(\ln(Q^2+1)\right) + \frac{d}{dQ}\left(\frac{3Q}{Q-1}\right)$$

$$= \frac{1}{Q^2+1} \times 2Q + \frac{3(Q-1) - 3Q}{(Q-1)^2}$$

$$= \frac{2Q}{Q^2+1} - \frac{3}{(Q-1)^2}$$

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\frac{\partial \pi}{\partial Q} = -Q^2 - 2Q + 8$$

$$0 = -Q^2 - 2Q + 8$$

$$0 = (-Q-4)(Q-2)$$

$$Q = -4, 2$$

$$\frac{\partial \pi'}{\partial Q} = -2Q - 2 < 0 \quad (\text{max})$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B $\quad \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} =$ undefinde

4.2 A*B $\quad \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} (8\times1)+(9\times4) & (8\times2)+(9\times5) & (8\times3)+(9\times6) \\ (10\times1)+(11\times4) & (10\times2)+(11\times5) & (10\times3)+(11\times6) \end{bmatrix}$

$$= \begin{bmatrix} 42 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A) $\quad = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} = 88 - 90 = -2$

4.4 det(B) $\quad = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = (5 + 12) - (8 + 15) = 17 - 23 = -6$

4.5 det(C) $\quad = \left[\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array}\right] = (45 + 84 + 96) - (105 + 48 + 72) = 225 - 225 = 0$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = ax^{a-1} + \left(\frac{x+y}{x}\right) \cdot \left[(x+y) - (x)(1+y)\right]$$

$$\frac{\partial U}{\partial y} = x^a by^{b-1} + \left(\left[\frac{(x+y)}{x}\right]\right)\left[(x+y)(x) - (x)(x+1)\right\}$$