1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

b. Define the domain set of X. Is the function concave for all over the domain?

a.) $\frac{dy}{dx} = 3x^2 - 10x + 7$

solve function x ; $(3x-7)(x-1) = 0$

$\therefore x = \frac{7}{3}, 1 \quad > 0$ It is an increasing function.

b.) From a.)

$\frac{d^2y}{d^2x} = 6x - 10$

$x = \frac{7}{3}$; $6\left(\frac{7}{3}\right) - 10 = 4 > 0$

↳ convex at $x = \frac{7}{3}$ minimum point

$x = 1$; $6(1) - 10 = 4 < 0$

↳ concave at $x = 1$ maximum point

$\therefore$ The answer is No, there are both convex and concave.

only labor can change

2. Suppose that a firm's short-run production function is given by

L is the only variable

$$Q(L) = 6L^2 - L^3$$

where $Q(L)$ is the output level, and L is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).

b. What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?

c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (MRP) function.

a) $AP_L = \dfrac{\overset{\text{output}}{Q(L)}}{\underset{\text{unit of labor}}{L}} = \dfrac{6L^2 - L^3}{L} = \dfrac{6L^2}{L} - \dfrac{L^3}{L} = 6L - L^2$

$MP_L = \dfrac{dQ}{dL} = Q'(L) = 12L - 3L^2$

b) ★ จะหา max / min ของอะไร ให้เราสร้าง equation ของสิ่งนั้นออกมาก่อน ★

The average output per labor $= AP_L = 6L - L^2$

find $\dfrac{dAP_L}{dL}$, set result = 0 and solve for L

from $AP_L = 6L - L^2$

$\dfrac{dAP_L}{dL} = 6 - 2L$

$0 = 6 - 2L$

$2L = 6 \qquad \therefore L = 3$

It's a critical value.

Check the maximize of the average output per labor from second order derivative test.

from $\frac{dAP_L}{dL} = 6 - 2L$

$$\frac{d^2AP_L}{dL^2} = -2$$

$\left.\frac{d^2AP_L}{dL^2}\right|_{L=3} = -2 < 0$ because there has NO L for us to plug in

so $L = 3$ give the maximize of the average output per labor

Suppose we have $L = L_0$ is the critical value, so we can check by plug $L = L_0$ in $\frac{d^2AP_L}{dL^2}$

if $\frac{d^2AP_L}{dL^2} > 0$, it shows that $L = L_0$ causes the minimize

if $\frac{d^2AP_L}{dL^2} < 0$, it shows that $L = L_0$ causes the maximize.

Then $L = L_0$ neither max nor min.

c) because $AP_L = \frac{Q(L)}{L}$

so the slope of $AP_L = \frac{d}{dL}\left[\frac{Q(L)}{L}\right]$

<use Quotient rule> $\frac{\text{ล่าง (ดิฟบน)} - \text{บน(ดิฟล่าง)}}{\text{ล่าง}^2}$

$$= \frac{L \cdot \left[\frac{d}{dL} Q(L)\right] - Q(L) \cdot 1}{L^2}$$

$$= \frac{L \cdot MP_L - Q(L)}{L^2}$$

$$= \frac{\cancel{L} \cdot MP_L}{\cancel{L}^2} - \frac{Q(L)}{L^2}$$

$$= \frac{MP_L}{L} - \frac{Q(L)}{L \cdot L}$$

AP_L

$$= \frac{MP_L}{L} - \frac{AP_L}{L}$$

$$\frac{d(AP_L)}{dL} = \frac{MP_L - AP_L}{L}$$

$\frac{d(AP_L)}{dL} = 0$ when $MP_L - AP_L = 0$

$$\boxed{MP_L = AP_L}$$

$\frac{d(AP_L)}{dL} =$ undefined when $\cancel{L = 0}$ L must higher than zero only!

when AP_L is maximize, so we can conclude that MP_L curve must cross the AP_L curve.

d) from demand function $Q = 100 - 2P$

$$2P = 100 - Q$$

$$P = \frac{100 - Q}{2}$$

revenue product function = price · quantity

$$= P \cdot Q$$

$$R(Q) = \left[\frac{100 - Q}{2}\right] \cdot Q$$

$$R(Q) = 50Q - \frac{1}{2}Q^2$$

$$MR = \frac{dR(Q)}{dQ} = 50 - \frac{2}{2}Q$$

$$MR = 50 - Q$$

from production function $Q = 6L^2 - L^3$

$$MR = 50 - (6L^2 - L^3)$$

$$MR = 50 - 6L^2 + L^3$$

$$MRP = MR \times MP_L$$

$$MRP = \left[50 - 6L^2 + L^3\right] \cdot \left[12L - 3L^2\right]$$

3

a) revenue maximizing

$$TR = P \cdot Q$$
$$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right] Q$$
$$TR = 40Q + 105 - \frac{3}{2}Q^3$$

maximizing $\frac{dTR}{dQ} = 40 - \frac{9}{2}Q^2 = 0$

$$Q^2 = 40 \times \frac{2}{9}$$
$$Q^2 = \frac{80}{9}$$
$$Q = \sqrt{\frac{80}{9}} = 2.98$$

plug Q in demand equation

$$P = 40 + \frac{105}{2.98} - \frac{3}{2}(2.98)^2$$
$$= 61.91$$

find elasticity

$\frac{dQ}{dP}$ demand

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$
$$\frac{dQ}{dP} = \frac{1}{\frac{dP}{dQ}}$$
$$\frac{dP}{dQ} = -105Q^{-2} - 3Q$$
$$\rightarrow \frac{dQ}{dP} = \frac{1}{-105Q^{-2} - 3Q}$$

elasticity demand

$$\frac{dQ}{dP} \cdot \frac{P}{Q}$$
$$= \frac{1}{-105Q^{-2} - 3Q} \cdot \frac{P}{Q} \quad 2.98$$
$$= \frac{1}{-105(2.98)^{-2} - 3(2.98)} \cdot \frac{61.91}{2.98} = -1$$

#

b) $\pi(Q) = TR(Q) - TC$

$$= PQ - TC$$
$$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - 6\left[6Q^3 - 81Q^2 - 175Q - 10\right]$$
$$= -75Q^3 - 972Q^2 - 2020Q + 90 \quad \#$$

c) $\pi = TR - TC$

$$= PQ - TC$$
$$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - \left[6Q^3 - 81Q^2 - 175Q - 10\right]$$
$$= 40Q + 105 - \frac{3}{2}Q^3 - 6Q^3 + 81Q^2 + 175Q + 10$$

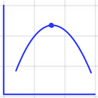
Max profit

$$\frac{d\pi}{dQ} = 40 + \frac{9}{2}Q^2 - 18Q^2 + 162Q + 175 = 0$$
$$= -13.5Q^2 + 162Q + 215 = 0$$
$$= 13.5Q^2 - 162Q - 215 = 0$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\frac{-162 \pm \sqrt{-162^2 - 4(13.5)(-215)}}{2(13.5)}$$
$$= -1.21, 13.21$$

Second order Condition

$$\frac{d^2\pi}{dQ^2} = 9Q - 36Q + 162$$
$$= -27(13.21) + 162 \quad -$$



D) lumpsum tax = $\pi = TR - TC - [Tax]$

$\frac{d\pi}{dQ} = \frac{d}{dQ}[TR - TC - Tax] = 0$ – with tax

$$\frac{dTR}{dQ} - \frac{dTC}{dQ} - \frac{dTax}{dQ} = 0$$

$$MR - MC - 0 = 0$$
$$MR = MC$$

$\pi = TR - TC$ – with out tax

$$\frac{d\pi}{dQ} = \frac{d}{dQ}[TR - TC] = 0$$
$$MR = MC$$

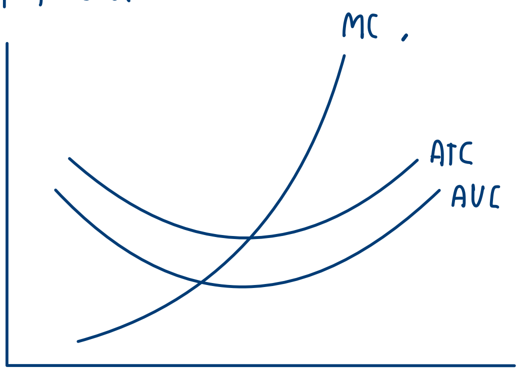
Produce at MR = MC [same : not effective]

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?

4a

Supply curve



$STC = 0.5Q^2 + 10Q + 5$

$MC = \frac{\partial TC}{\partial Q}$

$= \frac{dTC}{dQ}$

$MC = 1Q + 10$

Supply $P = MC$

$P = Q + 10$

$Q = -10 + P \rightarrow$ supply curve

Market supply

$Q_m = \sum_{i=1}^{n} Q_{mi}$

$n = 100$

$Q_m = 100(P-10)$

$Q_m^s = 100P - 1000$

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

4b

$Q_m^S = Q_m^D$

$100P - 1000 = 1100 - 50P$

$150P = 2100$

$P = 14$

substitute $P = 14$ in Q_m^s

$100(14) - 1000 = Q_m^s$

$Q_m = 400$

$\therefore$ market price is 14 units

market quantity 400 units by 100 firms.

(Q of 1 firm $= \frac{400}{100} = 4$ units)

short-run profit

$\pi = TR - TC$

$= P \cdot Q - (0.5(4)^2 + 10(4) + 5)$

$= 14(4) - (53)$

$\pi = 3$

$\therefore$ short-run profit = 3 units

4c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

imposed a €3 tax on producers

$STC = 0.5Q^2 + 10Q + 5 + 3Q$
$= 0.5Q^2 + 13Q + 5$

$MC = \frac{dSTC}{dQ} = 1Q + 13$

$MC = P = Q + 13$

$Q = -13 + P$ → firm's supply

market supply

$Q_S = 100(-13 + P)$

$Q_S = -1300 + 100P$

new equilibrium

$Q_D = Q_S^{new}$

$1100 - 50P = -1300 + 100P$

$2400 = 150P$

$P = 16$

substitute $P=16$ in new market supply equation

$Q_S^{new} = -1300 + 100(16)$

$Q = 300$

∴ new price equilibrium €16

new quantity equilibrium = 300 units.

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?

4d consumer burden

$= p^{\sim} - p^{0}$

$= 16 - 14$

$= €2$ per unit $\rightarrow \frac{2}{3} \times 100 = 66.67\%$

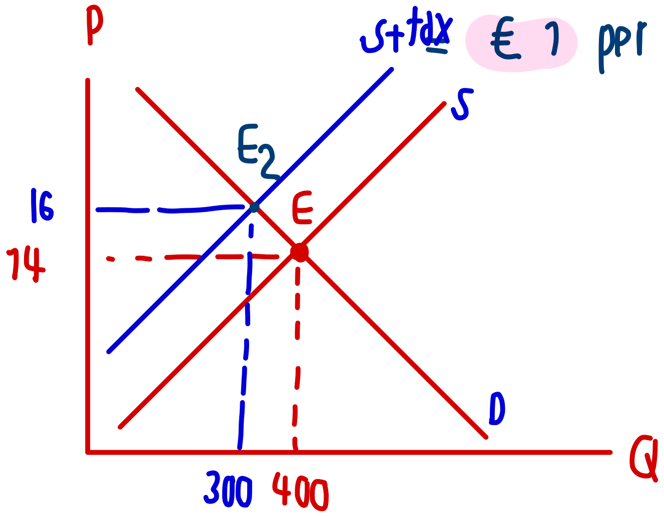
tax

producer burden = tax − consumer pay

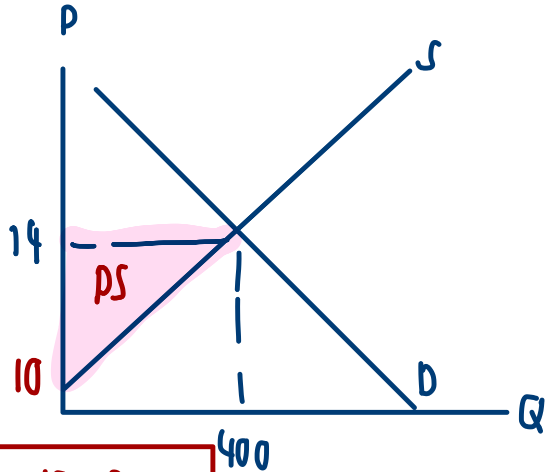
$= 3 - 2$

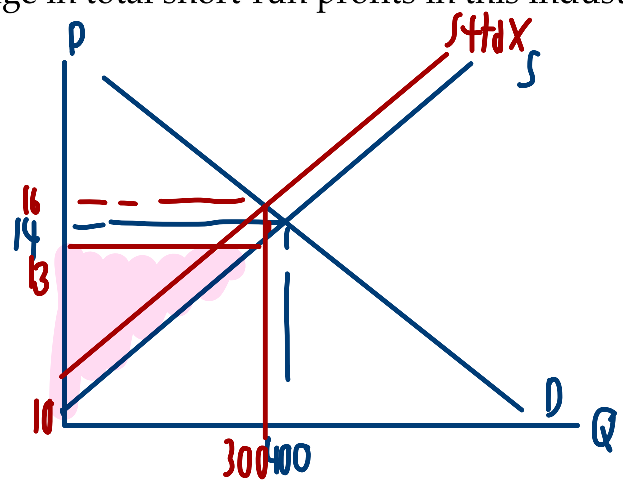
$= €1$ per unit $\rightarrow \frac{1}{3} \times 100 = 33.34\%$

tax



4e e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.





$\boxed{Q_s = -10 + P \quad Q = 0,\ P = 10}$

Producer surplus $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 4 \times 400$

$= 800$ units

new PS $= \frac{1}{2} \times 3 \times 300$

$= 450$ units

change in PS

$= 450 - 800$

$= -350$ units

change in total short-run profit

$$q = \frac{300}{100 \text{ firm}} = 3 \text{ units}$$

new $\pi = PQ - (TC + \text{tax})$

$= 16Q - (0.5Q^2 + 13Q + 5)$

$= 16(3) - (0.5(3)^2 + 13(3) + 5)$

$= -0.5$

before tax profit $= €3$

change in short-run profit $= -0.5 - 3$

$= -3.5$ #

total profit $= -3.5(100)$

$= -350$

$\therefore$ Total profit decreases €350. #