Assignment #1

Instructions:
- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ใบ้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}\left(\hat{\beta}_1\right)$ and $\operatorname{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$Y_i = \beta_1 + \beta_2 X_i + U_i$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{7524}{5564} = 1.3523$$

$$\hat{\beta}_1 = 21.03 - 1.3523(12.20)$$

$$= 21.03 - 16.4981$$

$$= 4.5319$$

$$Y_i = 4.5319 + 1.3523 X_i + U_i$$

The ordinary least square model is a method to estimate the co-efficient that we want by use SRF (sample regression function) re-arrange the function to make $\hat{U}_i$ become Endogernous varible. Then partial differentiate to find the combination of $\hat{\beta}_1$ & $\hat{\beta}_2$ that will minimize the distance between the Line and the data point ($\hat{U}_i^2$).

b) Find r^2 and explain its meaning.

we can find $\hat{r}^2$ by $\frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$

$$= \frac{\sum (\hat{Y}_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2} = 1 - \frac{\sum \hat{U}_i^2}{\sum (Y_i - \bar{Y})^2}$$

$$= 1 - \frac{873.14}{882.92}$$

$$r^2 = 0.01108$$

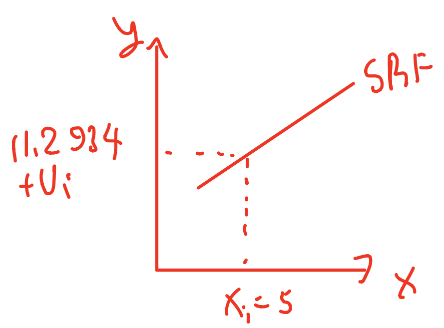
c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$$Y_i = 4.5319 + 1.3523 X_i$$

$$Y_i = 4.5319 + 1.3523(5)$$

$$Y_i = 11.2934$$

This mean that at $x_i = 5$ the value of $\hat{Y}_i$ that will fit on SRF will be $11.2934 + U_i$



d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}(\hat{\beta}_1)$ and $\operatorname{var}(\hat{\beta}_2)$

$$\operatorname{var}(u_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k}$$

$$= \frac{873.14}{30-2}$$

$$= 31.1835$$

$$\sum x_i^2 = \sum (X_i - \bar{X})^2 = 1098.8$$

$$\operatorname{var}(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2 = \frac{(5564)(31.1835)}{(30)(1098.8)} = 52.634$$

$$\operatorname{var}(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2} = \frac{\sigma^2}{\sum x_i^2} = \frac{31.1835}{1098.8} = 0.2840$$

e) $H_0: \beta_2 = 0$ null hypothesis

$\beta_2 \neq 0$ alternative hypothesis.

$\alpha = 0.05$ d.f. = 28

$$t = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$$

$$= \frac{-0.1585}{0.1085}$$

$$= 0.9407$$

lower bound : $t_{\frac{\alpha}{2}} = -2.048$

upper bound : $t_{\frac{\alpha}{2}} = 2.048$

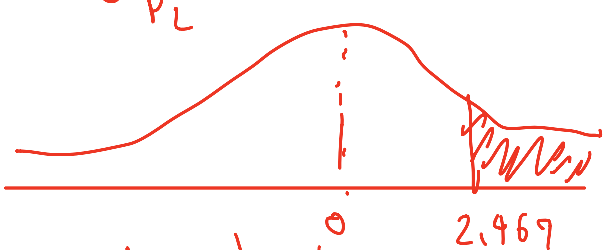
∴ H_0 is not rejected because t_{cal} is in the area of acceptance.

(f) $H_0 : P_2 \leq$ null hypothesis

$H_1 : \beta_2 > 0$ altenative hypothesis

$\alpha = 0.01$ d.f. $= 28$

$$t(a) = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{0.1585 - 0}{0.1085} = 0.9407$$



upper bound : $t_{0.01} = 2.467$

$\therefore$ H_0 is not rejected because t_{cal} is in the area of Acceptance

(2a) yes, the regression function have negative slope as x increased, y will increase

(2b) $E(y \mid X_0 = 5) = 7{,}830 - 502.4(5)$

$$\hat{y}_0 = 7{,}830 - 2512$$

$$= 5324$$

$$var(\hat{y}_0) = 6^2 \left[\frac{\frac{1}{n} + (X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]$$

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$$6^2(\hat{y}_0) = (212{,}877) \left[\frac{1/11 + (5 - 7.45)^2}{78.93} \right]$$

$$= 35{,}582.5356$$

$$6(\hat{y}_0) = 188.032$$

given $\alpha = 0.05$

$n - k = 11 - 2 = 9$

$$P_r[5{,}324 - 2.202(188.0333) \le y_0 < 5{,}324 + 2.202(188.0333)]$$

$$= P_r[4{,}897.3115 \le y_0 \le 5{,}750.6885$$

$$= 0.95 \text{ or } 95\%$$

(c) SRF; $\hat{y}_i = 7830 - 50.24(10X)$

$\quad$ se $\quad (52) \quad (41.18)$

(d) $X = 10 \quad y = 2{,}812$

$$\frac{dy}{dx} \cdot \frac{X}{y} = 502.4 \times \frac{10}{2{,}812} = 1.7866$$