

Instructions

- (1) Please read the instruction carefully.
- (2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at least a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
- (3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

- (1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
- (2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
- (3) Default decimal point is 4.
- (4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
- (5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
- (6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

- (1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
- (2) Login to BE Moodle, head into the course, then the assignment topic.
- (3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

Question 1. (12 points) Economic model of Crime.

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with *avgsen*. Based on Model (1.1), test whether the average sentence served from prior convictions has an impact on the number of arrests in the current year (1986). Show your work. (Use $\alpha = 0.05$)

1.b) What is the overall significance of the regression from Model (1.1) and Model (1.2)? What test do you use? (Use $\alpha = 0.01$)

1.c) If we are interested in testing whether “ethnic background and legal income” has an impact on the number of arrests in the current year (1986), what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use $\alpha = 0.05$)

Estimate the model (1.1) reports in the Table 1.1

$$narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + u_i \quad (1.1)$$

Table 1.1

Source	SS	df	MS	Number of obs	=	2,725
Model	85.9532425	5	17.1906485	F(5, 2719)	=	24.29
Residual	1924.39391	2,719	.707757967	Prob > F	=	0.0000
				R-squared	=	0.0428
				Adj R-squared	=	0.0410
Total	2010.34716	2,724	.738012906	Root MSE	=	.84128

narr86	Coefficient	Std. err.	t	P> t	[95% conf. interval]
pcnv	-.1512246	.040855			
avgsen	-.0070487	.0124122			
tottime	.0120953	.0095768			
ptime86	-.0392585	.0089166			
qemp86	-.1030909	.0103972			
_cons	.7060607	.0331524			

Omitted for the purpose of this exam

Estimate the model (1.2) reports in the Table 1.2

$$narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + \beta_4 inc86_i + \beta_5 black_i + \beta_6 hispan_i + u_i \quad (1.2)$$

where

$narr86_i$	= the number of arrests in the current year (1986)
$pcnv_i$	= the proportion of prior arrests that led to a conviction
$avgsen_i$	= the average sentence served from prior convictions (in months)
$tottime_i$	= months spent in prison since age 18 prior to 1986
$ptime86_i$	= months spent in prison in 1986
$qemp86_i$	= the number of quarters that the man was legally employed in 1986
$inc86_i$	= legal income, 1986, (hundred dollars)
$black_i$	= 1 if black ethnic background
$hispan_i$	= 1 if Hispanic ethnic background

Table 1.2

Source	SS	df	MS	Number of obs	=	2,725
Model	145.390104	8	18.173763	F(8, 2716)	=	26.47
Residual	1864.95705	2,716	.686655763	Prob > F	=	0.0000
				R-squared	=	0.0723
				Adj R-squared	=	0.0696
Total	2010.34716	2,724	.738012906	Root MSE	=	.82865

narr86	Coefficient	Std. err.	t	P> t	[95% conf. interval]
pcnv	-.1332344	.0403502			
avgsen	-.0113177	.0122401			
tottime	.0120224	.0094352			
ptime86	-.0408417	.008812			
qemp86	-.0505398	.0144397			
inc86	-.0014887	.0003406			
black	.3265035	.0454156			
hispan	.1939144	.0397113			
_cons	.5686855	.0360461			

Omitted for the purpose of this exam

1. a) $\text{narrr8}_i = 0.7061 - 0.1512\text{pcnv}_i - 0.0070\text{avgsen}_i + 0.0121\text{tottime}_i - 0.393\text{ptime8}_i - 0.1031\text{gemp86}$

Kornchanok Vuttipakdee (BB)

6304640516

→ test the average sentence served

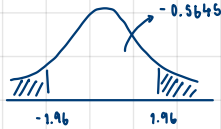
• Let $H_0: \beta_3 = 0$

$H_1: \beta_3 \neq 0$

• $t_{\text{cal}} = \frac{\hat{\beta}_3 - \beta_3}{\text{se}\hat{\beta}_3} = \frac{-0.0070 - 0}{0.0124} = -0.5645$

• critical values when d.f. is $2725 - 6 = 2719$

$t_{0.05} = \pm 1.96$



∴ Cannot reject H_0 which means we can make sure that average sentence served parameter is not zero 95% of the time

b) Model 1.1

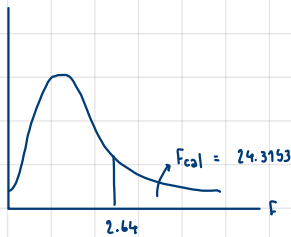
→ Use F-distribution to calculate

• Let $H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 =$

$H_1: \text{Otherwise}$

• $F_{\text{cal}} = \frac{R^2/(K-1)}{1-R^2/(n-k)} = \frac{0.0429/(6-1)}{(1-0.0429)/(2725-6)} = 24.3153$

• $F_{\text{upper}, d(6, 2719)} \text{ when } d = 0.01 = 2.64$



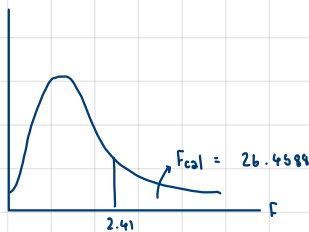
∴ $F_{\text{cal}} > F_{\text{upper}}(6, 2719)$

We can reject H_0 , in other word, we can make sure that $\beta_2, \beta_3, \beta_4, \beta_5$ and β_6 are not simultaneously equal to zero, 99 times out of 100

Model 1.2

• $F_{\text{cal}} = \frac{0.0723/9}{(1-0.0723)/2716} = 26.4588$

• $F_{\text{upper}, (9, 2716)} \text{ when } d = 0.01 = 2.41$



∴ $F_{\text{cal}} > F_{\text{upper}}(9, 2716)$

We can reject H_0 , in other word, we can make sure that $\beta_2, \beta_3, \beta_4, \beta_5$ and β_6 are not simultaneously equal to zero, 99 times out of 100

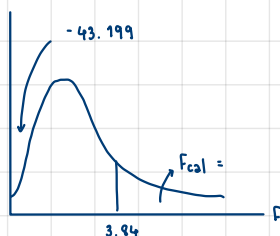
c) Use F test for marginal contribution of these two variables add in model 2

• Let H_0 : ethnic background and legal income has no marginal contribution to the model

H_1 : otherwise

• $F_{\text{cal}} = \frac{R_{\text{new}}^2 - R_{\text{old}}^2/m}{1 - R_{\text{new}}^2/(n - k_{\text{new}})} = \frac{(0.0723 - 0.0429)/3}{(1 - 0.0723)/(2725 - 9)} = 29.7888$

• $F_{\text{upper}, (3, 2716)} \text{ when } d = 0.05 = 2.6$



∴ $F_{\text{cal}} > F_{\text{upper}}$

We can reject H_0 , in other word, we can make sure that ethnic background and legal income have marginal contribution to the model 95% of the times

$$2. a) \cdot \beta_1: \begin{array}{l} H_0: \beta_1 = 0 \\ H_1: \beta_1 \neq 0 \end{array} \quad \beta_2: \begin{array}{l} H_0: \beta_2 = 0 \\ H_1: \beta_2 \neq 0 \end{array} \quad \beta_3: \begin{array}{l} H_0: \beta_3 = 0 \\ H_1: \beta_3 \neq 0 \end{array} \quad \beta_4: \begin{array}{l} H_0: \beta_4 = 0 \\ H_1: \beta_4 \neq 0 \end{array}$$

Kornchanok Vuttipakdee (BB)
6304640516

$$\cdot t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{9.1748 - 0}{0.0035} = 2,621.3714$$

$$t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{0.587 - 0}{0.0072} = 81.5279$$

$$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{se(\hat{\beta}_3)} = \frac{-0.0336 - 0}{0.005} = -6.72$$

$$t_{cal}(\beta_4) = \frac{\hat{\beta}_4 - \beta_4}{se(\hat{\beta}_4)} = \frac{0.0444 - 0}{0.0102} = 4.3529$$

$$\cdot \text{Critical values when d.f. is } n-k = 99979 - 4 = 99,974 \text{ and } \alpha = 0.05$$

$$t_{d/2} = 0.025 = \pm 1.96,$$

• for all parameters, $|t_{cal}| > |\text{critical values}|$

$\therefore$ we can reject H_0 of all parameters and say that all of them are significantly different from zero
95% of the times

b) A civil servant and state employee earns $100 \times (e^{\hat{\beta}_2} - 1) = 79.8585\%$ more than the others

c) Overall, wage drop by $100 \times (e^{\hat{\beta}_3} - 1) = 3.3042\%$ on average

d) For control group, their wage is

$$\ln \text{wage}_i = 9.1748 + 0.587(1) - 0.0336(1) + 0.0444(1)(1)$$

For the others, their wage is

$$\ln \text{wage}_i = 9.1748 + 0.587(0) - 0.0336(1) + 0.0444(0)(1)$$

The difference between the two is that interaction term is not included in the others' wage.

As a result, the control group is better-off by $100 \times (e^{\hat{\beta}_3 \hat{\beta}_4} - 1) = 1.0959\%$.

While, the treatment group is worse-off by $100 \times (e^{\hat{\beta}_3} - 1) = 3.3042\%$.

Economically, the other groups might work less due to private firms' plan or regulations during pandemic which might vary according to each firm's status. While, the same change is not applied in civil servants' work.

Question 3. (8 points) Multicollinearity.

As cheese ages, several chemical processes take place that determine the taste of the final product. The data given pertain to concentrations of various chemicals in a sample of 30 mature cheddar cheeses and subjective measure of taste for each sample.

Estimate the model (3.1) reports in the Table 3.1

$$\text{Taste} = \beta_0 + \beta_1 \text{acetic} + \beta_2 \text{h2s} + \beta_3 \text{lactic} + u \quad (3.1)$$

Where *Taste* = Measures of taste for each sample

acetic = The natural logarithm of concentration of acetic

h2s = The natural logarithm of concentration of hydrogen sulfide

lactic = Lactic

Table 3.1

Source	SS	df	MS	Number of obs	=	30
Model	5020.64468	3	1673.54823	F(3, 26)	=	16.47
Residual	2642.24237	26	101.624706	Prob > F	=	0.0000
				R-squared	=	0.6552
				Adj R-squared	=	0.6154
Total	7662.88705	29	264.237485	Root MSE	=	10.081

taste	Coefficient	Std. err.	t	P> t	[95% conf. interval]
acetic	1.538645	3.000501			
h2s	3.915242	1.153106			
lactic	18.80235	8.342614			
_cons	-34.13491	15.67628			

Omitted for the purpose of this exam

	acetic	h2s	lactic	Variable	VIF	1/VIF
acetic	1.0000			lactic	1.83	0.546648
h2s	0.2700	1.0000		h2s	1.72	0.582609
lactic	0.3607	0.6448	1.0000	acetic	1.15	0.867477
				Mean VIF	1.57	

3.a) Is there evidence of multicollinearity in the data? How do you know? Explain your answers in detail and state the critical value for hypothesis testing to receive full points.

3.b) What is the property of BLUE? If there is the multicollinearity problem, is the OLS estimators still retain the property of BLUE? If not, which properties are violated?

Question 4. (8 points) Heteroscedasticity.

The data on U.S. inflation rates (%) and unemployment rates (%), 1948-2006

Estimate the model (4.1) reports in the Table 4.1

$$Inf_t = \beta_1 + \beta_2 unem_t + u_t \quad (4.1)$$

where Inf_t = inflation rates (%)

$unem_t$ = unemployment rates (%)

Table 4.1

Source	SS	df	MS	Number of obs	=	59
Model	32.3284496	1	32.3284496	F(1, 57)	=	3.85
Residual	478.096987	57	8.38766644	Prob > F	=	0.0545
				R-squared	=	0.0633
				Adj R-squared	=	0.0469
Total	510.425437	58	8.80043856	Root MSE	=	2.8961

inf	Coefficient	Std. err.	t	P> t	[95% conf. interval]
unem	.5054734	.2574699			
_cons	1.010847	1.491583			

Omitted for the purpose of this exam

White's general test statistic: 1.0266 Chi-sq (2)

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity

$$\chi^2(1) = 1.12$$

Answer the following questions.

- 4.a)** Interpret the intercept and slope coefficients.
- 4.b)** According to the test statistics given after Table 4.1 below, is there any sufficient evidence to conclude that there is heteroscedasticity problem? Show your work on the hypothesis testing. (Use $\alpha = 0.05$)
- 4.c)** Given your test results in a), do the OLS estimators still retain the property of BLUE? If not, which properties are violated?

3. a) There is no evidence of multicollinearity in the data. According to rule of thumb, coefficient of correlation have to exceed 0.8 if there is multicollinearity. Furthermore, VIF do not exceed 10.

Kornchanok Vuttipakdee (BB)

6304640516

$$\begin{array}{llll} \beta_1: & H_0: \beta_1 = 0 & \beta_2: & H_0: \beta_2 = 0 \\ & H_1: \beta_1 \neq 0 & & H_1: \beta_2 \neq 0 \\ \beta_3: & H_0: \beta_3 = 0 & \beta_4: & H_0: \beta_4 = 0 \\ & H_1: \beta_3 \neq 0 & & H_1: \beta_4 \neq 0 \end{array}$$

$$t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{-34.1249 - 0}{15.6763} = -2.1775$$

$$t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{1.5396 - 0}{3.0005} = 0.5129$$

$$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{se(\hat{\beta}_3)} = \frac{3.9152 - 0}{1.1531} = 3.3954$$

$$t_{cal}(\beta_4) = \frac{\hat{\beta}_4 - \beta_4}{se(\hat{\beta}_4)} = \frac{19.8023 - 0}{8.3426} = 2.2539$$

- critical values when d.f. is $n-k = 30-4 = 26$ and $\alpha = 0.05$

$$t_{0.05(4,26)} = \pm 2.056$$

- t_{cal} for parameters β_1 , β_3 and β_4 exceed the critical value
 $\therefore$ we can reject null hypothesis and can say that they are significantly different from zero 95% of the time.
- t_{cal} for β_2 does not exceed critical value
 $\therefore$ we cannot reject null hypothesis and cannot say that β_2 is significantly different from zero 95% of the time.
- As almost all parameters are significantly different from zero and a high R^2 , there are no problem of multicollinearity in the model

- b) BLUE stands for best linear unbiased estimator which refers to estimator that has the lowest variance.

OLS estimators still retain the property of BLUE if there is multicollinearity problem. However, conclusion might be misleading.

4. a) $\hat{\beta}_1$ means that when unemployment rate = 0, inflation rate will be 1.01 on average.

$\hat{\beta}_2$ means that when unemployment rate increase by 1%, inflation rate will increase by 0.5% on average.

- b) We can use White's test

- Given $LM_{cal}(X^2) = 1.0266$
- Critical value of X^2 when $\alpha = 0.05$ is 3.8415
- Critical value > LM_{cal}

$\therefore$ we cannot reject null hypothesis at $\alpha = 0.05$. This means that heteroscedasticity is not in our model.

- c) According to 4.b), OLS estimator still retain property of BLUE and none of the properties is violated as there is no heterodasticity in our model.