Solution: Quiz 5

1. Let $A = \{1, 2, 3\}$ and let $\mathcal{P}(A)$ be the set of all subsets of the set A and let X be a subset of $\mathcal{P}(A)$, which defined as

$$X = \{x \in \mathcal{P}(A) |\ x \cap \{1, 2\} = \emptyset\}.$$

Define a relation r as

$$r = \{(x, y) \in X \times \mathcal{P}(A) \mid \ x \subseteq y\}.$$

(a) Determine the sets $\mathcal{P}(A)$ and X.

(b) Draw the arrow diagram of r.

(c) Is r a function? If so, is it onto and/or one-to-one? Justify your answer briefly.

Solution:

(a) Determine the sets $\mathcal{P}(A)$ and X.

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

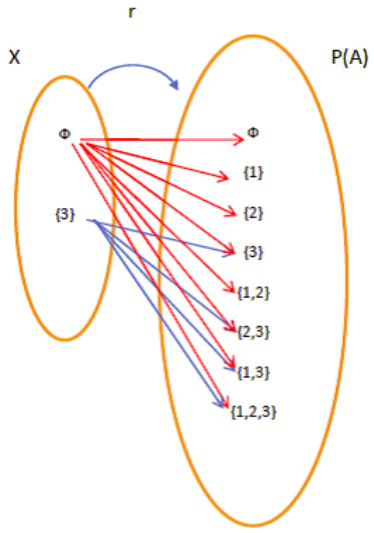
For $X = \{x \in \mathcal{P}(A) |\ x \cap \{1, 2\} = \emptyset\}$, the element of x cannot contain 1 and/or 2. That is,

$$X = \{\emptyset, \{3\}\}$$

(b) Draw the arrow diagram of r.

- Since $\emptyset \in X$ is a subset of every set, there is a map from $\emptyset$ to every element in $P(A)$ by the relation r.
- For $x = \{3\}$, $x \subseteq \{3\}$, $\{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

That is, the diagram of the relation r is given as shown below.



(c) Is r a function? If so, is it onto and/or one-to-one? Justify your answer briefly.

No. r is not a function because there is an element in the domain that gets mapped more than once (in this case both elements $\emptyset$, $\{3\}$ in the domain get mapped multiple times).