EE 320 Introductory Mathematical Economics

Semester 1/2015

Practice Problem for Topic 4 - Answers

Question 1 Basic operations on matrix

1.1

Find $A^{-1}B$ when $A = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Answer:

$$A^{-1} = -\frac{1}{32}\begin{bmatrix} -50 & 23 & 11 \\ 54 & -21 & -17 \\ 12 & -10 & -2 \end{bmatrix}$$

$$A^{-1}B = -\frac{1}{32}\begin{bmatrix} -50 & 22 & -4 \\ 54 & -34 & 24 \\ 24 & -4 & -8 \end{bmatrix}$$

1.2

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$ Find $(ABC)^T$.

$$(ABC)^T = \begin{bmatrix} 6 & 12 \\ 3 & 18 \end{bmatrix}$$

Answer:

1.3

If A $= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ Find $A^{-1}B$.

Answer

$$A^{-1} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -0.4 & 0.6 \\ 0.8 & -0.2 \end{bmatrix}$$

1.4

Let A $= \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ Find determinant of A

Answer : det(A) = -32

1.5

If A $= \begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 1 \end{bmatrix}$ B $= \begin{bmatrix} 2 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$ Find $[A^T B]^{-1}$.

Answer: $\left[[A^T B]^{-1} = \frac{1}{13} \begin{bmatrix} 0 & -13 \\ 1 & 12 \end{bmatrix} \right]$

Question 2

Consider a simple macroeconomic model.

$C = a + bY_d; \quad 0 < b < 1$

$I = I_a + iY; \quad 0<i<1$

$G = G_0$

$T = T_0 + tY; \quad 0 < t < 1$

$R = R_0$

$Y_d = Y - T + R$

where R is the government transfer and G is the government purchase. All the remainings are defined as usual.

a) Determine *all* the endogenous and exogenous variables in the model.
   Y, C, I, T, Y_d
b) State the condition that characterizes the equilibrium of this model.
   Y = C + I + G
c) Simplify the model into a 3-variable system of equations that only includes on Y, C and I.
   Y = C + I + G_0
   $C = a + b(Y - T + R) = a + b(Y - T_0 - tY + R_0) = a - bT_0 + bR_0 + (1-t)bY$
   $I = I_a + iY$

d) Rewrite the system of equations in 2.3 in the form of matrix.

$$\begin{bmatrix} 1 & -1 & -1 \\ -(1-t)b & 1 & 0 \\ -i & 0 & 1 \end{bmatrix} \begin{bmatrix} Y \\ C \\ I \end{bmatrix} = \begin{bmatrix} G_0 \\ a - bT_0 + bR_0 \\ I_a \end{bmatrix}$$

$$A \qquad x \quad = \quad d$$

e) Solve for the solution of Y, C and I. Use the Cramer's rule method.

$\det(\mathrm{A}) = 1 - i - (1-t) * b$

$$Y = \frac{\begin{vmatrix} G_0 & -1 & -1 \\ a - bT_0 + bR_0 & 1 & 0 \\ I_a & 0 & 1 \end{vmatrix}}{\det(A)} = \frac{G_0 + I_a + (a - bT_0 + bR_0)}{det(A)}$$

$$C = \frac{\begin{vmatrix} 1 & G_0 & -1 \\ -(1-t) * b & a - bT_0 + bR_0 & 0 \\ -i & I_a & 1 \end{vmatrix}}{\det(A)} = \frac{G_0 + I_a + (a - bT_0 + bR_0)}{det(A)}$$

I = $\frac{\begin{vmatrix} 1 & -1 & G_0 \\ -(1-t)*b & 1 & a-bT_0+bR_0 \\ -i & 0 & I_a \end{vmatrix}}{\det(A)}$

$= \frac{I_a(1-b(1-t)+i*(a-bT_0+bR_0)+G_o*i}{det(A)}$

f) Compare the multipliers of G and R. Which one has a bigger impact? Why?

Multiplier of G is $\frac{1}{1-i-b(1-t)}$

Multiplier of R is $\frac{b}{1-i-b(1-t)}$

Multiplier of G > Multiplier of R. An increase in R doesn't cause an increase in output in the first round as the increase in G does. So, the effect of an increase in R is discounted by "b" time of the effect of an increase in G.

Question 3

Assume that we have three markets in the economy. Each market can be characterized by demand and supply equations as given below.

Demand for goods A : $q_A^d = 3 - P_A + P_B$ Supply for goods A : $q_A^s = P_A - 2$
Demand for goods B : $q_B^d = 3 - 2P_B + P_C$ Supply for goods B : $q_B^s = P_B - 1$
Demand for goods C : $q_C^d = 6 + 2P_A - P_C$ Supply for goods C : $q_C^s = 2P_C - 2$

a. State the equilibrium conditions for this multi-market economy, and write the model in the form of matrix.

$q_A{}^d = q_A{}^S, \quad q_B{}^d = q_B{}^S, \quad q_c{}^d = q_c{}^S$

Ax = d

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ P_C \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -8 \end{bmatrix}$$

det(A) = -16

b. Solve for the equilibrium solution by using the *Cramer's rule*.

$P_A = \frac{-90}{-16} = 5$

$P_B = \frac{-90}{-16} = 5$

$P_C = \frac{-96}{-16} = 6$

$[q_A = 3, q_B = 4, q_C = 10 \text{and} P_A = 5, P_B = 5, P_C = 6]$

Question 4

Consider a simple global economy model. There are but two countries, namely A and B. Model equations for each country are given below.

$$\begin{aligned} \text{Country A} : C_A &= 150 + 0.4Y_A \\ M_A &= 0.6Y_A \\ I_A &= 250 \end{aligned}$$

$$\begin{aligned} \text{Country B} : C_B &= 100 + 0.5Y_B \\ M_B &= 0.4Y_B \\ I_B &= 200 \end{aligned}$$

where Y is national income, C is consumption, M is import, and I is investment.

4.1) Let "X" be a new variable representing export of a country. That is, when I write X_A, this is referred to the value of export of country A. Based on the information given above, can we find the export function of both country A and country B?

$X_A = M_B = 0.4Y_B$

$X_B = M_A = 0.6Y_A$

4.2) State all the endogenous and exogenous variables.

$Y_A, Y_B, C_A, C_B, X_A, X_B, M_A, M_B$

4.3) State the equilibrium conditions for the global economy model.

Equilibrium in country A: $Y_A = C_A + I_A + X_A - M_A$

Equilibrium in country B: $Y_B = C_B + I_B + X_B - M_B$

4.4) Simplify the model into a 2-variable system of equations where only Y_A *and* Y_B are included. Then rewrite the simplified model in the matrix form.

$Y_A = C_A + I_A + X_A - M_A$

$Y_A = 150 + 0.4Y_A + 250 + 0.4Y_B - 0.6Y_A$

$1.2Y_A - 0.4Y_B = 400$

$Y_B = C_B + I_B + X_B - M_B$

$Y_B = 100 + 0.5Y_B + 200 + 0.6Y_A - 0.4Y_B$

$-0.6Y_A + 0.9Y_B = 300$

$$\begin{bmatrix} 1.2 & -0.4 \\ -0.6 & 0.9 \end{bmatrix} \begin{bmatrix} Y_A \\ Y_B \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \end{bmatrix}$$

4.5) Solve for the equilibrium income using the *inverse matrix method*.

$$A = \begin{bmatrix} 1.2 & -0.4 \\ -0.6 & 0.9 \end{bmatrix} ==> A^{-1} = \begin{bmatrix} 1.07 & 0.47 \\ 0.71 & 1.42 \end{bmatrix}$$

$Y_A^* = 571.43 \ \ and \ \ Y_B^* = 714.29$

4.6) Under the equilibrium, how much is the *net export* in both countries?

$X_A - M_A = 0.4Y_B - 0.6Y_A = 0.4 * 714.29 \ - \ 0.6 * 571.43 = 285.71 - 342.85 = \ -57.14$

$X_B - M_B = 0.6Y_A - 0.4Y_B = 0.6 * 571.43 \ - \ 0.4 * 714.29 = 57.14$

Question 5

Given the following supply and demand functions:

$$Q_D = 100 - 3P$$

$$Q_S = 80 + 2P$$

a) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

Ans. Eq'm condition: $Q_D = Q_S$.

System of equations:

$$\begin{aligned} Q_D - Q_S &= 0 \\ Q_D \ + 3P &= 100 \\ Q_S - 2P &= 80 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix}$$

b) Use <u>matrix inversion</u> to solve for the equilibrium quantity and equilibrium price.

Ans.

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \rightarrow \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

c) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Use <u>Cramer's rule</u> to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

Ans. (i) $P_d^* = P_s^* - 5 = \$7 - \$5 = \$2$

(ii) $P_s^* = \$7$

(iii) $Q^* = 94$; $S^* = 94x\$5 = \$470$

Question 6

Examine for what values of the constants *a* and *b* the system of equations

$ax + y = 3$
$x + z = 2$
$y + az + bu = 6$
$y + u = 1$

has a unique solution in the unknown $x, y, z,$ and u. Find the unique solution (expressed in terms of a and b).

Ans. There is a unique solution provided that $a(b-2) \neq 0$.

Question 7

Consider an economy divided into an agricultural sector (A) and an industrial sector (I). To produce one unit in sector A requires 1/6 unit from A and 1/4 unit from I. To produce in sector I requires 1/4 unit from A and 1/4 unit from I. Suppose final demands in each of the two sectors are 60 units.

a) Write down the Leontif system for this economy.

Ans. Suppose x is the production in A, and y is the production in I.

$$x = \tfrac{1}{6}x + \tfrac{1}{4}y + 60$$

$$y = \tfrac{1}{4}x + \tfrac{1}{4}y + 60$$

b) Find the number of units that has to be produced in each sector in order to meet the final demands.

Ans. x = 320/3; y = 1040/9

Question 8

Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.

Ans. $Y = 800$, $r = 0.12$.