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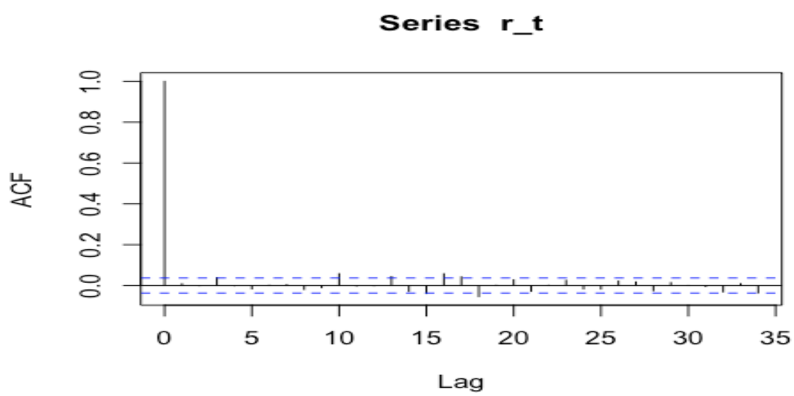
Assignment5.R

to_on

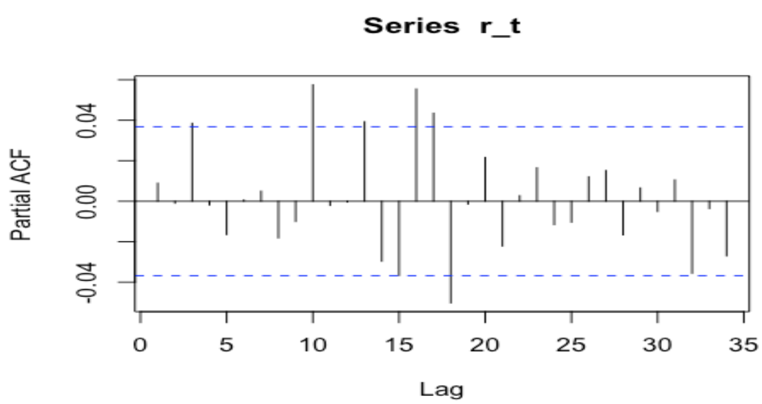
2021-04-29

```
getSymbols("CAT",from="2006-01-03",to="2017-04-14")

## [1] "CAT"

r_t<-diff(log(as.numeric(CAT[,6])))
#Question1
#1a
acf(r_t)
```

Question1




```
pacf(r_t)
```

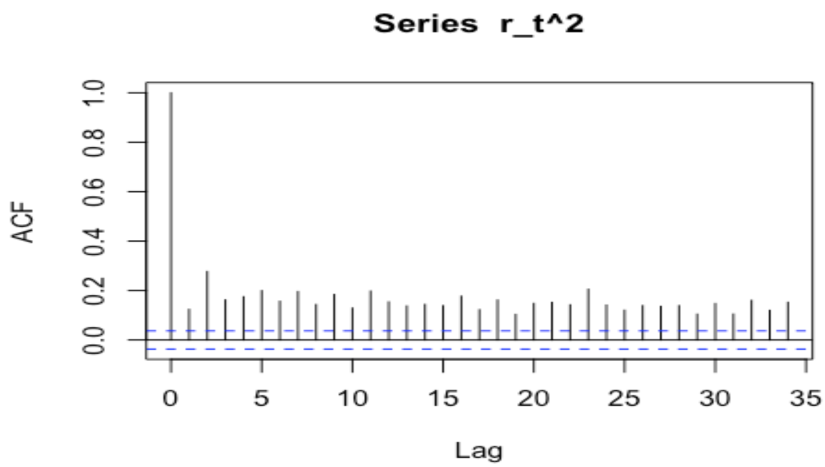



```
Box.test(r_t,lag=10,type='Ljung')

##
##  Box-Ljung test
```

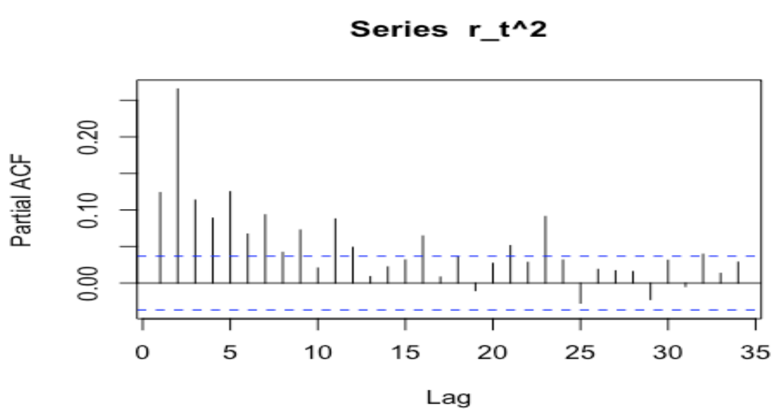
```
##
## data:  r_t
## X-squared = 16.229, df = 10, p-value = 0.09325
```

Ans1.a) There is no serial correlation in the log return(r_t) with 95% CI. Because the p-value of r_t is greater than 0.05. Then, we can not reject null hypothesis at 0.05 level of significance.

```
#1b
acf(r_t^2)
```




```
pacf(r_t^2)
```

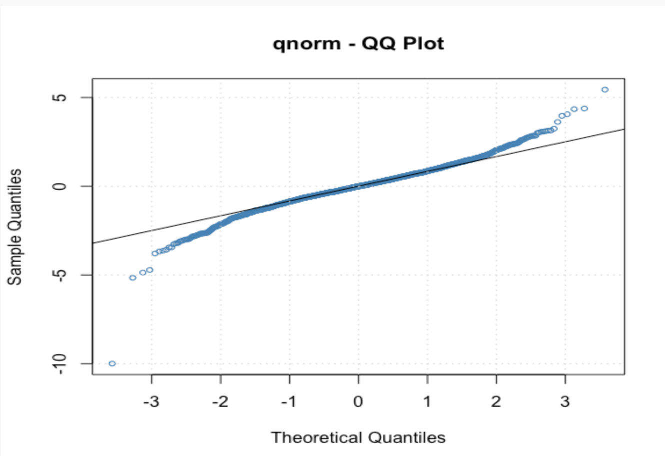



```
Box.test(r_t^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  r_t^2
## X-squared = 917.58, df = 10, p-value < 2.2e-16
```

Ans1b) There exist ARCH effect in log return(r_t) with 95% CI. Because p-value of r_t^2 computed by Ljung-Box test is less than 0.05. Then, we reject the null hypothesis.

```
#1c
m1=garchFit(formula = ~arma(1,0)+garch(1,1),data = r_t,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m1)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = r_t, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fc0d5b78ce8>
##  [data = r_t]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1
## 4.7490e-04  1.7677e-02  4.4860e-06  4.9755e-02  9.3861e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.749e-04   3.075e-04    1.544 0.122480
## ar1    1.768e-02   2.004e-02    0.882 0.377774
## omega  4.486e-06   1.280e-06    3.503 0.000459 ***
## alpha1 4.976e-02   8.200e-03    6.067  1.3e-09 ***
## beta1  9.386e-01   1.032e-02   90.934  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7381.067    normalized:  2.599883
##
## Description:
##  Thu Apr 29 00:29:00 2021 by user:
##
```

```
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3293.722  0
##  Shapiro-Wilk Test  R    W      0.9664334 0
##  Ljung-Box Test     R    Q(10)  12.36538  0.2613468
##  Ljung-Box Test     R    Q(15)  14.78355  0.4671183
##  Ljung-Box Test     R    Q(20)  19.40031  0.4959589
##  Ljung-Box Test     R^2  Q(10)  0.9807765 0.9998426
##  Ljung-Box Test     R^2  Q(15)  3.684778  0.9986004
##  Ljung-Box Test     R^2  Q(20)  6.924134  0.9969265
##  LM Arch Test       R    TR^2   2.720581  0.9972158
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.196243 -5.185762 -5.196250 -5.192463
```




```
#plot(m1)
```

The model yield adequate mean & variance equation, but the distribution is not normal.

The fitted model

Mean equation : $\hat{r}_t = 4.830e{-}04(1-0.01768) + 0.01768\, r_{t-1}$

$(3.075e{-}04) \qquad (2.006e{-}02)$

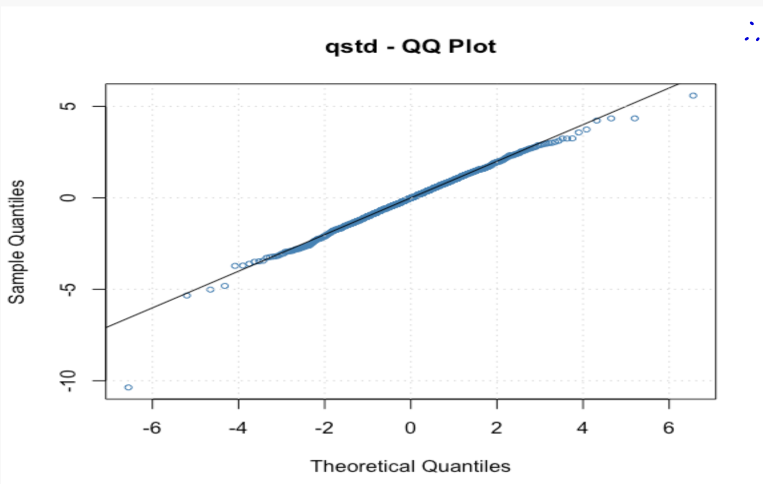
variance equation : $\sigma_t^2 = 4.486e{-}06 + 0.04476\, a_{t-1} + 0.9386\, \sigma_{t-1}^2$

$(1.28e{-}06) \qquad (0.0082) \qquad (0.01032)$

```
#1d
m2=garchFit(formular = ~garch(1,1),data=r_t,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(data = r_t, cond.dist = "std", trace = F, formular = ~garch(1,
##     1))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fc0d5e8ca30>
##  [data = r_t]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9008e-04  4.2138e-06  7.2389e-02  9.2028e-01  5.1049e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.901e-04   2.703e-04    2.183  0.02904 *
## omega  4.214e-06   1.574e-06    2.678  0.00741 **
## alpha1 7.239e-02   1.375e-02    5.265  1.4e-07 ***
## beta1  9.203e-01   1.474e-02   62.448  < 2e-16 ***
## shape  5.105e+00   4.839e-01   10.549  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7509.292    normalized:  2.645048
##
## Description:
##  Thu Apr 29 00:29:00 2021 by user:
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4047.363  0
##  Shapiro-Wilk Test  R    W      0.9639883 0
##  Ljung-Box Test     R    Q(10)  14.87685  0.1366167
##  Ljung-Box Test     R    Q(15)  16.83467  0.3288433
##  Ljung-Box Test     R    Q(20)  20.67317  0.4165887
##  Ljung-Box Test     R^2  Q(10)  2.955969  0.9824413
##  Ljung-Box Test     R^2  Q(15)  5.488358  0.9871219
##  Ljung-Box Test     R^2  Q(20)  9.459063  0.9769545
##  LM Arch Test       R    TR^2   4.275692  0.9779321
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.286574 -5.276093 -5.286580 -5.282793
```

```
#plot(m2)
```



∴ According to the Ljung-box test for both mean and volatility equation, the result of the test show that all of the p-value obtained are greater than 0.05. Thus, it can implied that the model is adequate for mean & variance equation as we do not reject any H_0.

```
#1e
```

Mean equation: $\hat{r}_t = 0.0003901$
$(2.702e-04)$

Variance equation: $\sigma_t^2 = 4.214e-06 + 0.07239\, a_{t-1}^2 + 0.9203\, \sigma_{t-1}^2$
$(1.574e-06)$ (0.01377) (0.01474)

```
#1f
predict(m2,5)

##   meanForecast  meanError standardDeviation
## 1 0.0005900796 0.01557857        0.01557857
## 2 0.0005900796 0.01565654        0.01565654
## 3 0.0005900796 0.01573356        0.01573356
## 4 0.0005900796 0.01580965        0.01580965
## 5 0.0005900796 0.01588481        0.01588481
```

#1g

1 - step : $0.00059 \pm 1.96(0.01558) = [-0.0299, 0.0311]$

2 - step : $0.00059 \pm 1.96(0.01566) = [-0.0301, 0.0313]$

3 - step : $0.00059 \pm 1.96(0.01373) = [-0.0302, 0.0314]$

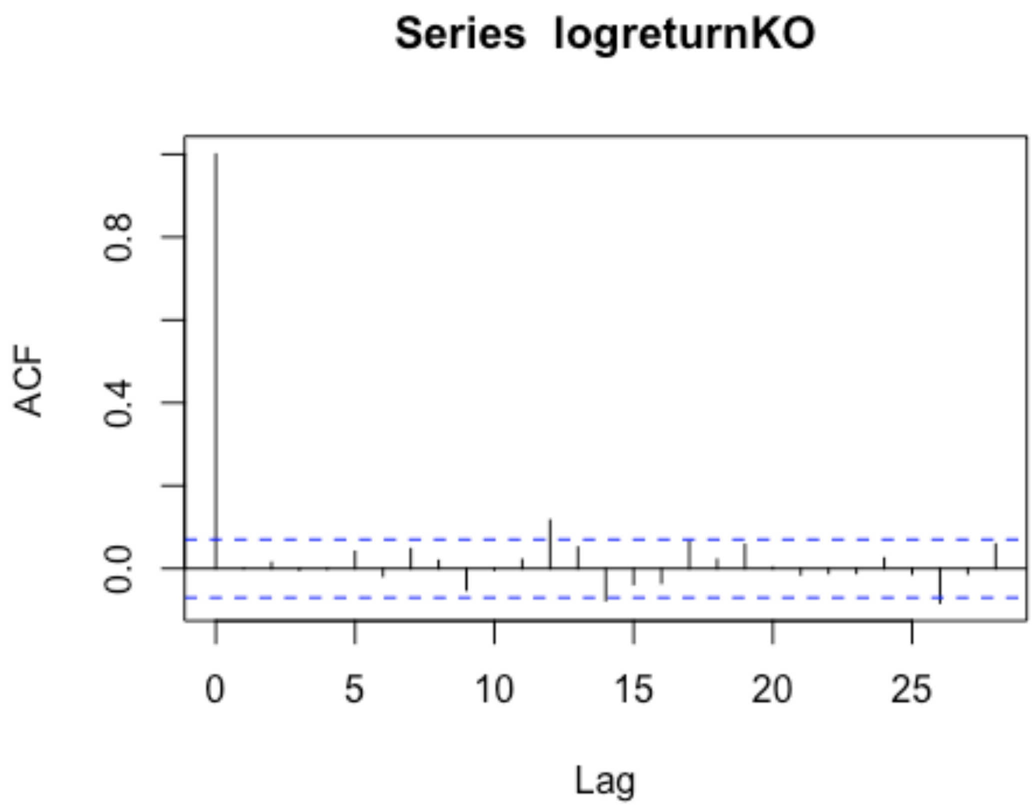
4 - step : $0.00059 \pm 1.96(0.01581) = [-0.0304, 0.0316]$

5 - step : $0.00059 \pm 1.96(0.01588) = [-0.0305, 0.0317]$

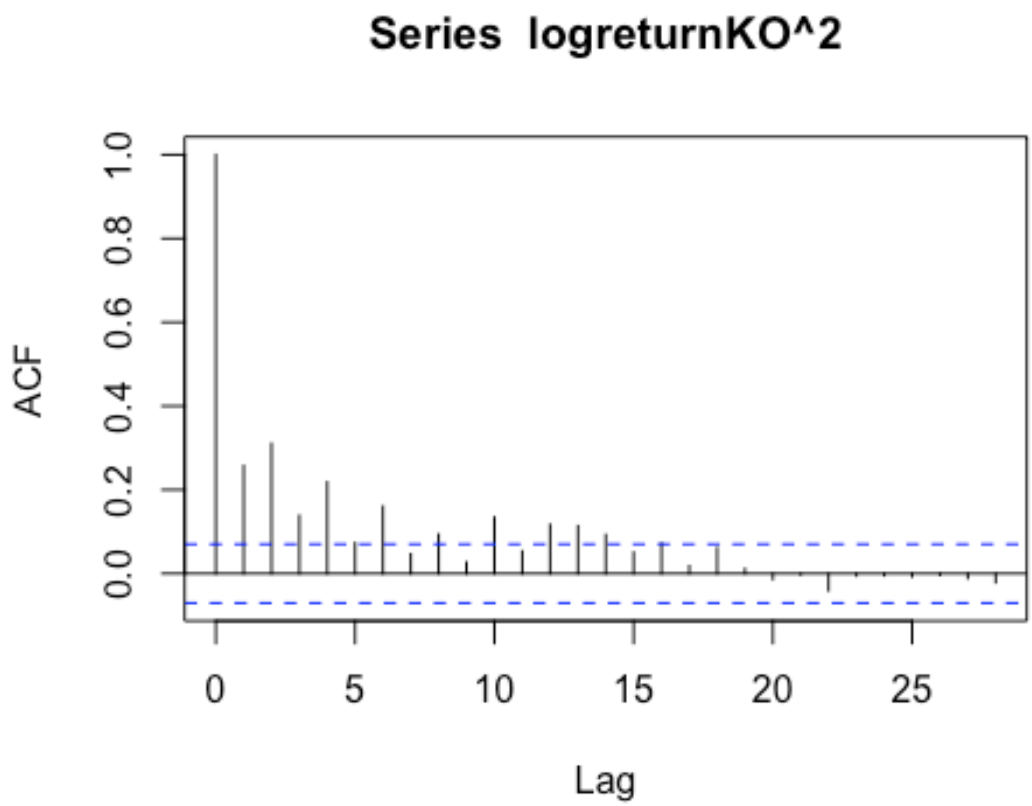
Question 2

```
#Question2
m.kovw=read.table("m-kovw-5116.txt",header = T)
ko=m.kovw[,3]
logreturnKO=log(ko+1)
#2a
t.test(logreturnKO)

##
##  One Sample t-test
##
## data:  logreturnKO
## t = 4.9853, df = 779, p-value = 7.628e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
```

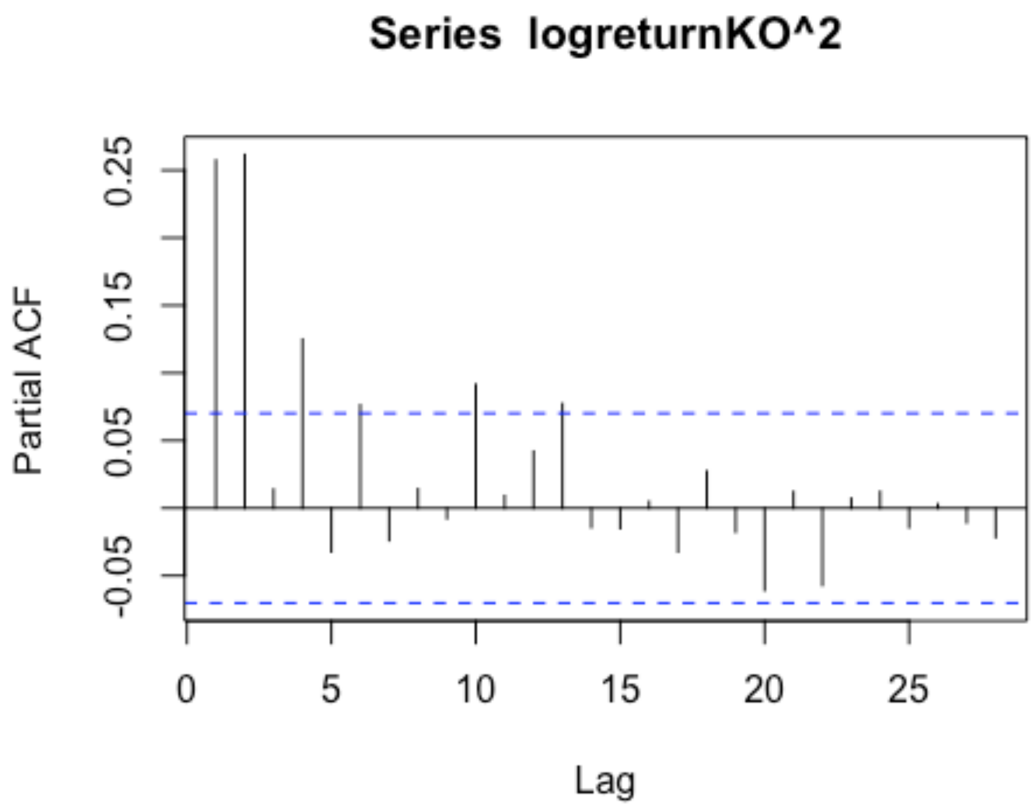
```
## sample estimates:
##  mean of x
## 0.01031992

acf(logreturnKO)
```




```
Box.test(logreturnKO,lag=10,type = 'Ljung')

##
##  Box-Ljung test
##
## data:  logreturnKO
## X-squared = 5.9201, df = 10, p-value = 0.8219

acf(logreturnKO^2)
```




```
pacf(logreturnKO^2)
```




```
Box.test(logreturnKO^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  logreturnKO^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16
```

Ans. 2a

T- test

$H_0 : \mu = 0$

$H_1 : \mu \neq 0$

The computed p-value is less than 0.05.

Then, we reject H_0 at 0.05 level of significant.

$\mu \neq 0$ with 95% CI

Ljung - Box test

$H_0 : e_1 = e_2 = e_3 = \dots = e_{10} = 0$

$H_1 : \exists e_i \neq 0 \; ; \; i = 1,2,3,\dots,10$

There is no serial correlation in log return KO with 95% CI. because computed p-value from Ljung-Box test is higher than 0.05. Then, we can not reject H_0 at 0.05 level of significant.

Test ARCH effect ($logreturnKO^2$)

Ljung - Box test

$H_0 : \rho_1 = \rho_2 = \rho_3 = \dots \rho_{10} = 0$ / No ARCH effect

$H_1 : \exists \rho_i \neq 0 \; ; \; i = 1,2,3,\dots,10$ / there exist ARCH effect

$\therefore$ There is an ARCH effect in log return KO with 95% CI. Because computed p-value from Ljung-Box test is less than 0.05. Then, we reject H_0 at 0.05 level of significant.

```r
#2b
m1=garchFit(formula = ~arma(1,0)+garch(1,1),data = logreturnKO,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = logreturnKO,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fc0dacff268>
##  [data = logreturnKO]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Thu Apr 29 00:29:00 2021 by user:
##
##
## Standardised Residuals Tests:
```

```
##                                   Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

∴ Mean & variance equation is adequate but the distribution is not normal.

Mean equation : $\hat{r}_t = \underset{(0.0019)}{0.0125}(1+0.02634) - \underset{(0.03881)}{0.02634}\, r_{t-1}$

variance equation : $\sigma_t^2 = \underset{(5.852e\text{-}05)}{0.0001811} + \underset{(0.01915)}{0.09535}\, a_{t-1}^2 + \underset{(0.02766)}{0.8486}\, \sigma_{t-1}^2$

```r
#2c
m2=garchFit(formula =
~arma(1,0)+garch(1,1),data=logreturnKO,cond.dist="std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = logreturnKO,
##     cond.dist = "std", trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fc0d8ca8228>
##  [data = logreturnKO]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
shape
##  0.01124020  -0.01887601   0.00017395   0.09642928   0.85044150
7.47877780
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1    -1.888e-02   3.691e-02   -0.511  0.60904
## omega   1.739e-04   6.596e-05    2.637  0.00836 **
## alpha1  9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1   8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape   7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
```

```
##  Thu Apr 29 00:29:00 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816988
##  LM Arch Test       R    TR^2   10.48089  0.5738501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```

∴ According to the Ljung-box test for both mean and volatility equation, the result of the test show that all of the p-value obtained are greater than 0.05. Then, it can implied that the model is adequate for mean & variance equation as we do not reject any H_0.

Mean equation : $\hat{r}_t = \underset{(0.00181)}{0.0124}(1+0.01888) - 0.01888\, r_{t-1}$

ariance equation : $\sigma_t^2 = \underset{(6.396e-05)}{1.739e-04} + \underset{(0.02338)}{0.09643}\, a_{t-1}^2 + \underset{(0.03267)}{0.08504}\, \sigma_{t-1}^2$

```r
#2d
m3=garchFit(formula = ~garch(1,1),data = logreturnKO,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m3)
```

```
##
## Title:
##  GARCH Modelling
```

```
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = logreturnKO, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fc0da964ba0>
##  [data = logreturnKO]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Thu Apr 29 00:29:00 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```

∴ Mean & variance is adequate but the distribution is not normal.

Mean equation: $\hat{r}_t = \underset{(0.001846)}{0.01098}$

variance equation: $\hat{\sigma}_t = \underset{(5.899e-05)}{1.85e-04} + \underset{(0.01912)}{0.0948}\, a_{t-1}^2 + \underset{(0.0287)}{0.8478}\, \sigma_{t-1}^2$

```
#2e
m4=garchFit(formula = ~garch(1,1),data=logreturnKO,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m4)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = logreturnKO, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fc0d2e03db0>
##  [data = logreturnKO]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Thu Apr 29 00:29:00 2021 by user:
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

Mean equation : $\hat{r}_t = \underset{(0.001757)}{0.0105}$

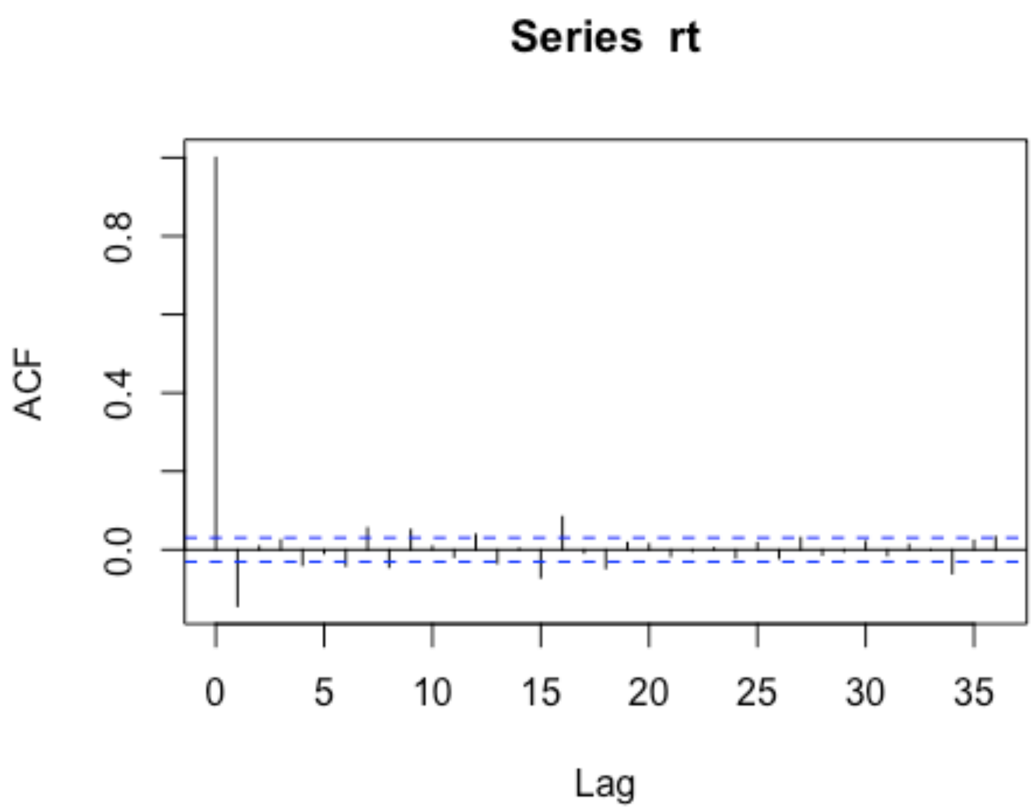
variance equation : $\hat{\sigma}_t = \underset{(6.627e-05)}{1.753e-04} + \underset{(0.02337)}{0.09633}\, a^2_{t-1} + \underset{(0.0327)}{0.8501}\, \sigma^2_{t-1}$

∴ According to the Ljung-box test for both mean and volatility equation, the result of the test show that all of the p-value obtained are greater than 0.05. Then, it can implied that the model is adequate for mean & variance equation as we do not reject any H_0.

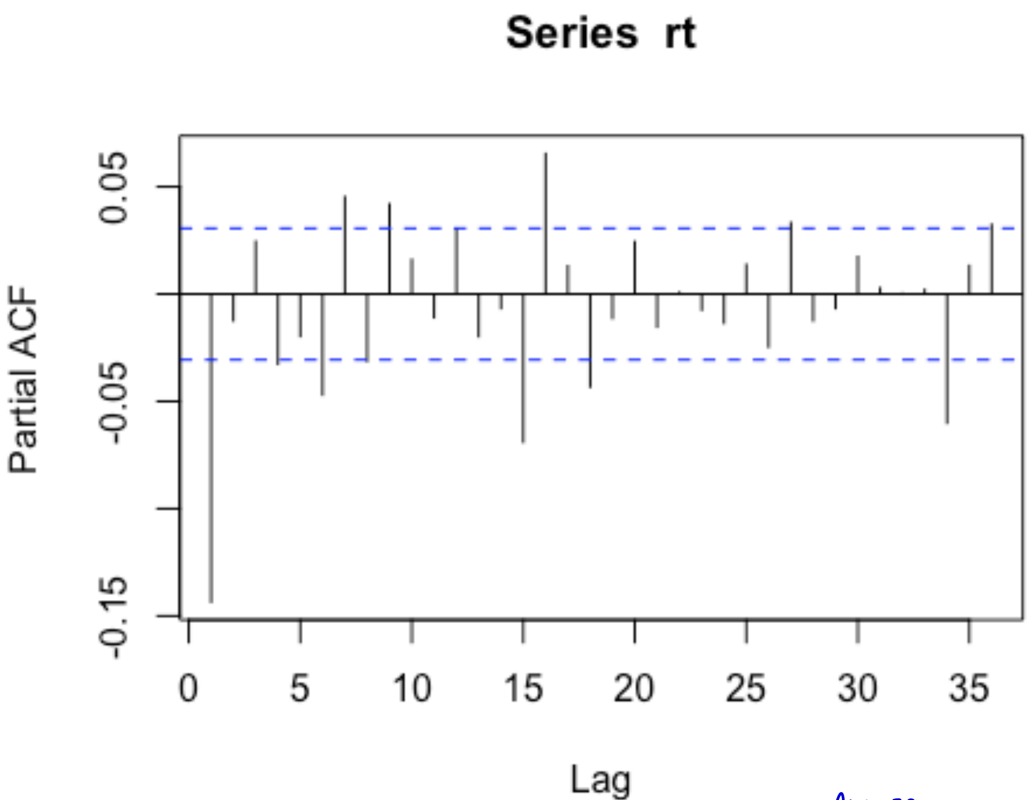
#2f

Model m4 (GARCH (1,1) with t-dist^n) is to be selected from its 1) Model adequacy for both mean & variance 2) significant coefficient estimates 3) lowest AIC & BIC among m1, m2, m3, m4.

```
#Question3
getSymbols("^GSPC",from="2005-01-02",to="2021-03-31")

## [1] "^GSPC"

rt<-diff(log(as.numeric(GSPC[,6])))
#3a
t.test(rt)

##
##  One Sample t-test
##
## data:  rt
## t = 1.4961, df = 4086, p-value = 0.1347
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -9.051939e-05  6.737464e-04
## sample estimates:
##    mean of x
## 0.0002916135

acf(rt)
```



```r
pacf(rt)
```




```r
Box.test(rt,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt
## X-squared = 131.85, df = 10, p-value < 2.2e-16
```

```r
#3b
auto.arima(rt)
```

```
## Series: rt
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##           ar1   mean
##       -0.1435  3e-04
## s.e.   0.0155  2e-04
##
## sigma^2 estimated as 0.0001521:  log likelihood=12166.05
## AIC=-24326.09   AICc=-24326.09   BIC=-24307.15
```

Ans. 3a

T- test

$H_0 : \mu = 0$

$H_1 : \mu \neq 0$

The computed p-value is higher than 0.05. Then, we can not reject H_0 at 0.05 level of significant. $\mu = 0$ with 95% CI

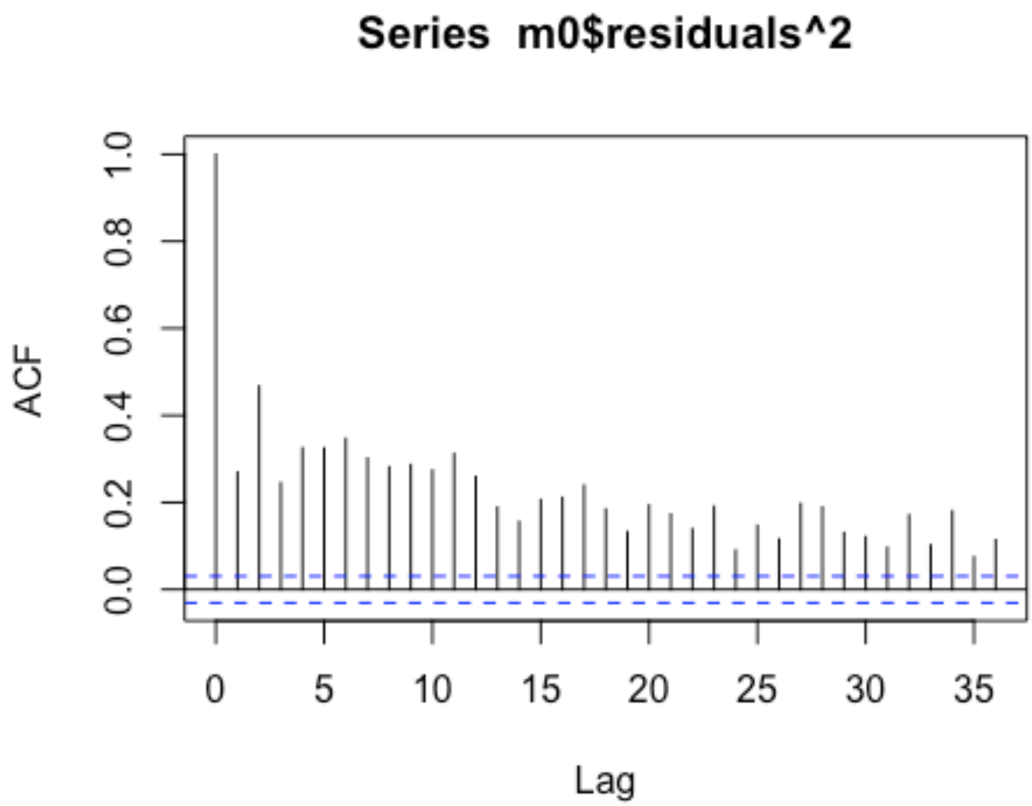
Ljung - Box test

$H_0 : e_1 = e_2 = e_3 = \ldots = e_{10} = 0$

$H_1 : \exists e_i \neq 0 \; ; \; i = 1,2,3,\ldots,10$

There is serial correlation in log return (rt) with 95% CI. because computed p-value from Ljung-Box test is lower than 0.05. Then, we reject H_0 at 0.05 level of significant.

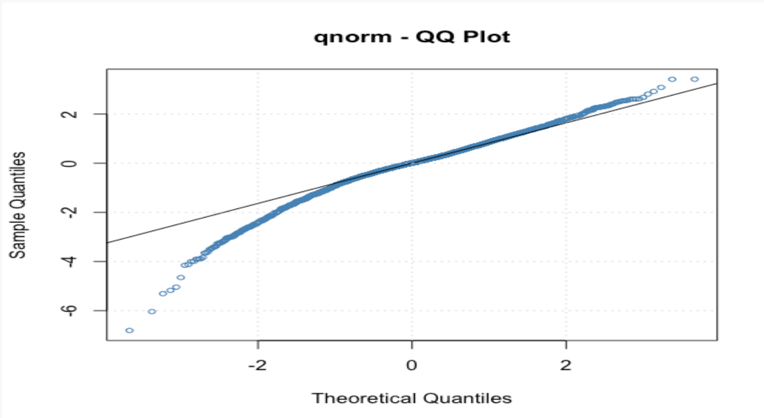
```
m0=auto.arima(rt)
acf(m0$residuals^2)
```




```
Box.test(m0$residuals^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  m0$residuals^2
## X-squared = 4132.1, df = 10, p-value < 2.2e-16

m1=garchFit(formula = ~arma(1,0)+garch(1,1),data = rt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m1)

##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fc0d2e259f8>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  7.4132e-04  -7.5408e-02   2.7169e-06   1.4182e-01   8.3740e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      7.413e-04   1.178e-04    6.292 3.14e-10 ***
## ar1    -7.541e-02   1.730e-02   -4.358 1.31e-05 ***
## omega   2.717e-06   3.504e-07    7.753 8.88e-15 ***
## alpha1  1.418e-01   1.202e-02   11.800  < 2e-16 ***
## beta1   8.374e-01   1.196e-02   70.032  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13428.16    normalized:  3.285579
##
## Description:
##  Thu Apr 29 00:29:01 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1128.594  0
##  Shapiro-Wilk Test  R    W      0.9719221 0
##  Ljung-Box Test     R    Q(10)  17.65633  0.061045
##  Ljung-Box Test     R    Q(15)  25.9899   0.03812896
##  Ljung-Box Test     R    Q(20)  31.68968  0.04671932
##  Ljung-Box Test     R^2  Q(10)  15.9892   0.09994212
##  Ljung-Box Test     R^2  Q(15)  18.25272  0.2496126
##  Ljung-Box Test     R^2  Q(20)  19.62331  0.4817041
##  LM Arch Test       R    TR^2   16.7807   0.1580346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.568712 -6.560985 -6.568715 -6.565976
```

```r
#plot(m1)
```



3b.

Mean equation : $\hat{r}_t = \underset{(1.178e-04)}{7.413e-04} + \underset{(1.730e-02)}{(-7.541e-02)}\, r_{t-1}$

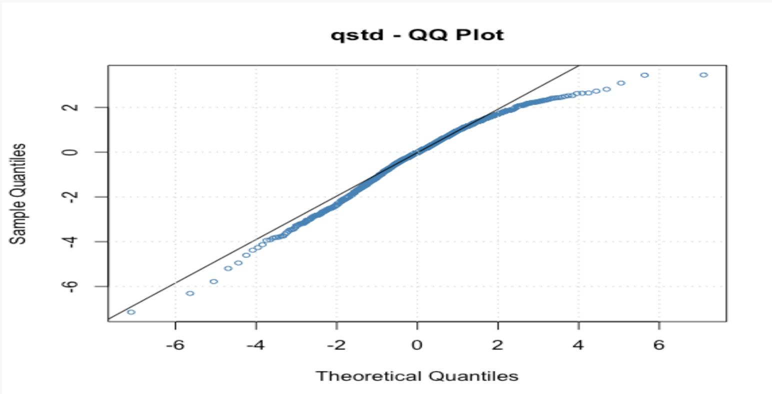
variance equation : $\hat{\sigma}_t^2 = \underset{(3.504e-07)}{2.717e-06} + \underset{(1.202e-02)}{(1.418e-01)}\, a_{t-1}^2 + \underset{(1.19e-02)}{(8.374e-01)}\, \sigma_{t-1}^2$

ARMA(1,0)+GARCH(1,1) is the most appropriate because it provide the lowest AIC, BIC.
The model is adequate except for the mean equation with 15 and 20 lags.

```r
#3c
m2=garchFit(formula = ~arma(1,0)+garch(1,1),data = rt,cond.dist =
"std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, cond.dist =
"std",
##     trace = F)
##
## Mean and Variance Equation:
```

```
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fc0d7826708>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
shape
##  9.1359e-04  -7.0189e-02   1.6949e-06   1.4129e-01   8.5658e-01
5.0787e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      9.136e-04   1.061e-04    8.612  < 2e-16 ***
## ar1    -7.019e-02   1.570e-02   -4.469 7.85e-06 ***
## omega   1.695e-06   3.625e-07    4.676 2.93e-06 ***
## alpha1  1.413e-01   1.460e-02    9.675  < 2e-16 ***
## beta1   8.566e-01   1.286e-02   66.618  < 2e-16 ***
## shape   5.079e+00   4.283e-01   11.857  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13563.45    normalized:  3.318682
##
## Description:
##  Thu Apr 29 00:29:02 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1426.546  0
##  Shapiro-Wilk Test  R    W      0.9697692 0
##  Ljung-Box Test     R    Q(10)  17.75546  0.05923155
##  Ljung-Box Test     R    Q(15)  26.09727  0.03701267
##  Ljung-Box Test     R    Q(20)  31.5407   0.04844502
##  Ljung-Box Test     R^2  Q(10)  13.10432  0.2178975
##  Ljung-Box Test     R^2  Q(15)  17.76262  0.275351
##  Ljung-Box Test     R^2  Q(20)  20.49609  0.4273063
##  LM Arch Test       R    TR^2   14.77776  0.2538157
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.634429 -6.625157 -6.634433 -6.631146
```

```r
#plot(m2)
```



<u>Fitted model</u>

Mean equation: $\hat{r}_t = 9.136e{-}04 + (-7.019e{-}02)\, r_{t-1}$

$(1.061e{-}04) \qquad (1.570e{-}02)$

variance equation: $\hat{\sigma}_t^2 = 1.695e{-}06 + (1.413e{-}01)(a_{t-1}^2) + (8.566e{-}01)\, \sigma_{t-1}^2$

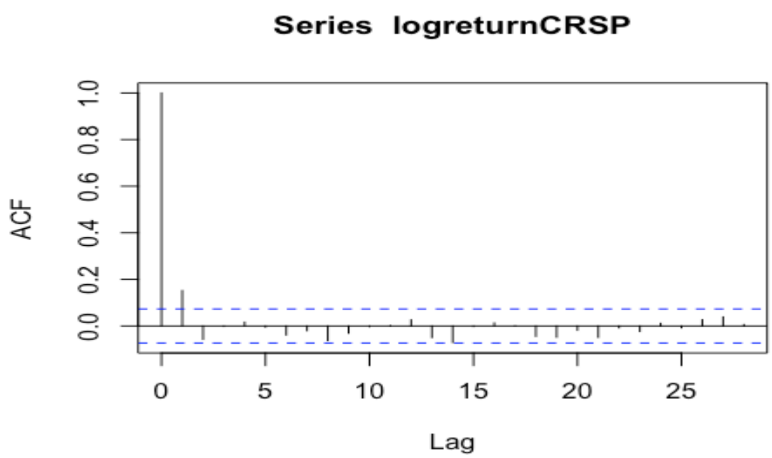
$(3.625e{-}07) \qquad (1.460e{-}02) \qquad (1.286e{-}02)$

```r
#3d
predict(m2,5)

##   meanForecast   meanError standardDeviation
## 1 0.0011355887 0.009054724       0.009054724
## 2 0.0008338888 0.009160372       0.009138299
## 3 0.0008550647 0.009243329       0.009220941
## 4 0.0008535784 0.009325271       0.009302675
## 5 0.0008536827 0.009406326       0.009383526
```

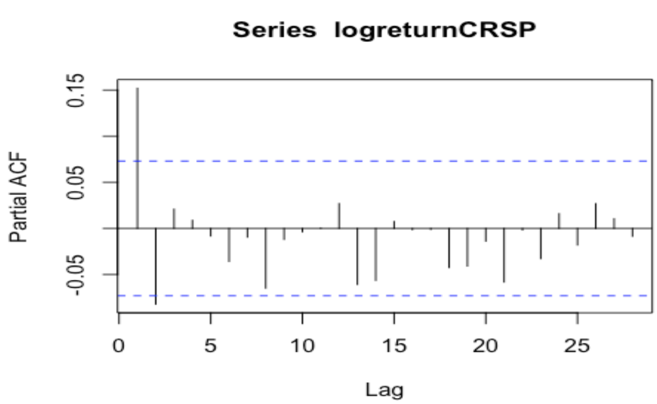
```
#Question4
CRSP=read.table("m-deciles.txt",header = T)
simplereturnCRSP=CRSP[,10]
logreturnCRSP=log(simplereturnCRSP+1)
#4a
t.test(logreturnCRSP)
```

```
##
##  One Sample t-test
##
## data:  logreturnCRSP
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054
```

```
acf(logreturnCRSP)
```




```
pacf(logreturnCRSP)
```



```r
Box.test(logreturnCRSP,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  logreturnCRSP
## X-squared = 24.257, df = 10, p-value = 0.006946
```

```r
auto.arima(logreturnCRSP)
```

```
## Series: logreturnCRSP
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
##          ma1      ma2    mean
##       0.1660  -0.0558  0.0096
## s.e.  0.0373   0.0370  0.0020
##
## sigma^2 estimated as 0.002392:  log likelihood=1152.66
## AIC=-2297.32   AICc=-2297.26   BIC=-2279
```

```r
m1<-arima(logreturnCRSP,order=c(0,0,2))
m1
```

```
##
## Call:
## arima(x = logreturnCRSP, order = c(0, 0, 2))
##
## Coefficients:
##          ma1      ma2  intercept
##       0.1660  -0.0558     0.0096
## s.e.  0.0373   0.0370     0.0020
##
## sigma^2 estimated as 0.002382:  log likelihood = 1152.66,  aic = -2297.32
```

```r
Box.test(m1$residuals,lag=10,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  m1$residuals
## X-squared = 4.3881, df = 10, p-value = 0.9281
```

$\hat{r}_t = 0.0096 + 0.166 a_{t-1} - 0.0558 a_{t-2}$

Checking the serial correlation in residual term (a_t)

Ljung-Box test: $H_0: e_1 = e_2 = \dots = e_{10} = 0$

$H_1: \exists e_i \neq 0$

The computed p-value = 0.92 > 0.05. Thus, we can not reject H_0 at 0.05 level of significant. This model is adequate (no serial correlation in a_t)

Ans. 4a

T-test

$H_0: \mu = 0$

$H_1: \mu \neq 0$

The computed p-value is higher than 0.05.
Thus, we can not reject H_0 at 0.05 level of significant. $\mu = 0$ with 95% CI Thus, expected value of log return of CRSP is equal to 0.

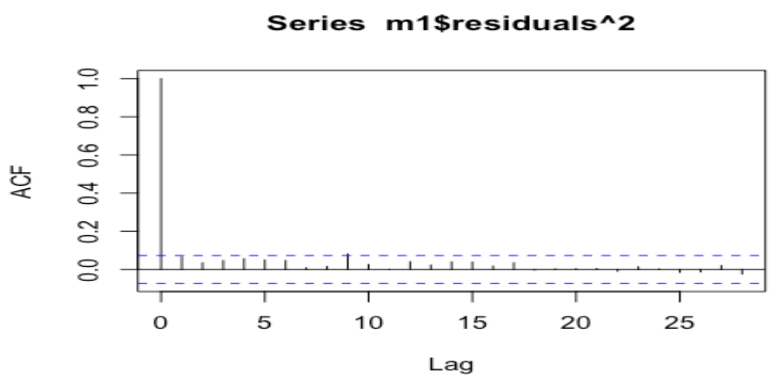
Ljung-Box test

$H_0: e_1 = e_2 = e_3 = \dots = e_{10} = 0$

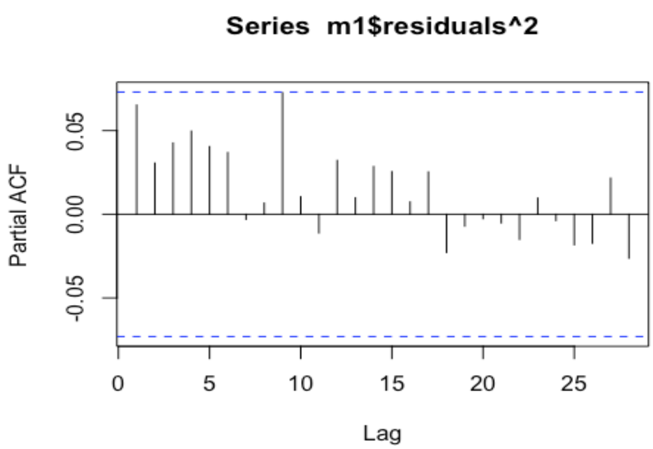
$H_1: \exists e_i \neq 0$; $i = 1, 2, 3, \dots, 10$

There is serial correlation in log return of CRSP with 95% CI. because computed p-value from Ljung-Box test is lower than 0.05. Thus, we reject H_0 at 0.05 level of significant.

```r
#4b
acf(m1$residuals^2)
```




```r
pacf(m1$residuals^2)
```




```r
Box.test(m1$residuals^2,lag = 10,type = 'Ljung')

##
##  Box-Ljung test
##
## data:  m1$residuals^2
## X-squared = 16.677, df = 10, p-value = 0.08183
```

4b. <u>Test ARCH effect</u> (log return of CRSP)

Ljung - Box test

H_0 : $\rho_1 = \rho_2 = \rho_3 = \ldots \rho_{10} = 0$ / No ARCH effect

H_1 : $\exists \rho_i \neq 0$; $i = 1, 2, 3, \ldots, 10$ / there exist ARCH effect

$\therefore$ There is no ARCH effect in log return of CRSP with 95% CI. Because computed p-value from Ljung-Box test is higher than 0.05. Thus, we cannot reject H_0 at 0.05 level of significant.

```
#4c
m2<-garchFit(~arma(1,0)+garch(1,0),data=logreturnCRSP, trace = F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = logreturnCRSP,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x7fc0d64942f8>
##  [data = logreturnCRSP]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu      ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1    0.1470670   0.0424301    3.466 0.000528 ***
## omega  0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Thu Apr 29 00:29:02 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
```

```
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480
```

Mean & variance equation is adequate but the distribution is not normal.

Fitted equation

Mean : $\underset{}{\hat{r}_t} = \underset{(0.00192)}{0.01053} + \underset{(0.042)}{0.147\, r_{t-1}}$

variance: $\hat{\sigma}_t^2 = \underset{(0.0001)}{0.002} + \underset{(0.065)}{0.1815\, a_{t-1}^2}$

```
#4d
m3=garchFit(~arma(1,0)+garch(1,0),data=logreturnCRSP,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m3)

##
## Title:
##  GARCH Modelling
```

```
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = logreturnCRSP,
##     cond.dist = "std", trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x7fc0d8e3b8b0>
##  [data = logreturnCRSP]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115897    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Thu Apr 29 00:29:02 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
##
## Information Criterion Statistics:
```

```
##       AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

Mean & variance equation is adequate with t-distribution

Fitted eq

$$\text{Mean}: \quad \hat{r}_t = \underset{(0.00177)}{0.01162} + \underset{(0.040317)}{0.1076982}\, r_{t-1}$$

$$\text{variance}: \quad \sigma_t^2 = \underset{(0.00013)}{0.00192} + \underset{(0.0714)}{0.19008}\, a_{t-1}^2$$

```
#4e
m4=garchFit(formula = ~garch(1,0),data = logreturnCRSP,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m4)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = logreturnCRSP, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7fc0da8d8890>
##  [data = logreturnCRSP]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
```

```
##       mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Thu Apr 29 00:29:02 2021 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

4e.

$\therefore$ Variance equation is adequate while mean equation is not fully adequate, and not normally distributed.

<u>Fitted equation</u> Mean: $\underset{(0.0019)}{\hat{r}_t = 0.01235}$

Variance: $\sigma_t^2 = \underset{(0.000145)}{0.002016} + \underset{(0.0622)}{0.194126}$

```r
#4f
m5=garchFit(formula = ~garch(1,0),data = logreturnCRSP,cond.dist =
"std",trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = logreturnCRSP, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7fc0d71e5d68>
##  [data = logreturnCRSP]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Thu Apr 29 00:29:02 2021 by user:
##
##
## Standardised Residuals Tests:
```

```
##                                    Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705
##  Ljung-Box Test     R    Q(15)  22.99926  0.08415548
##  Ljung-Box Test     R    Q(20)  27.3947   0.1245202
##  Ljung-Box Test     R^2  Q(10)  6.667871  0.7563837
##  Ljung-Box Test     R^2  Q(15)  10.0157   0.8187514
##  Ljung-Box Test     R^2  Q(20)  11.63085  0.928196
##  LM Arch Test       R    TR^2   6.819315  0.8693193
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.275969 -3.250529 -3.276031 -3.266148

#4g
```

4f. Variance equation is adequate while mean equation is not fully adequate.

Fitted eq

Mean : $\hat{r}_t = 0.013356$
(0.0016881)

variance : $\sigma_t^2 = 0.001918 + 0.204163\, a_{t-1}^2$
$(0.000185) \quad (0.0724)$

4g The most appropriate model is m3 : AR(1) - ARCH(1) with t-distribution because

1. adequate mean & variance equation
2. lowest AIC & BIC
3. All coefficient estimate are significant.