EE320 Exercise 7

Semester 2, 2014

Chapter 9: Optimization under Equality Constraint

1. Determine the extremum value of the following functions using Lagrange-multiplier and state whether the extremum value is the maximum or minimum by using Bordered Hessian

$a)\ Z = XY \quad \text{s.t.} \quad X + 2Y = 2$

$b)\ Z = X(Y + 4) \quad \text{s.t.} \quad X + Y = 8$

$c)\ Z = X - 3Y - XY \quad \text{s.t.} \quad X - Y = 6$

$d)\ Z = 7 - Y + X^2 \quad \text{s.t.} \quad X + Y = 0$

$e)\ Z = 3X^2 + 4Y^2 - XY \quad \text{s.t.} \quad 2X + Y = 21$

$f)\ Z = X^2 - 10Y^2 \quad \text{s.t.} \quad X - Y = 18$

2. Determine the extremum value of the following functions and determine how objective function will change if the constant value in the constraint function change by 1 unit.

$a)\ Z = XYW^2 \quad \text{s.t.} \quad X + Y + W = 56$
$b)\ Z = 5XY + 8XW + 3YW \quad \text{s.t.} \quad 2XYW = 1920$

3. A producer has the following production function
$Q = 10K^{0.7}L^{0.1}$ where $P_K = 28$, $P_L = 10$ and B = 4,000

a) What type of return to scale of this production function.
b) How should a producer utilize the factor of production such that the firm will maximize output. State the conditions and prove by Bordered Hessian determinant.[K = 125, L = 50]

4. Given that
Utility function: $U = X_1X_2$
Budget constraint : $P_1X_1 + P_2X_2 = M_0$

a) What are the value of X_1 and X_2 such that he will maximize the utility.
$[X_1 = \frac{M_0}{2P_1}, X_2 = \frac{M_0}{2P_2}]$

b)Determine the function of the Income Consumption Curve(ICC) from the given information. $[X_2 = (\frac{P_1}{P_2})X_1]$

c) Prove that X_1 is normal goods

d) Prove that X_2 satisfy the law of demand

e) Is the demand function for goods X_1 and X_2 the Homogeneous function ? If yes, what is the degree of homogeneous function ? State the economic meaning.

5. Let $U = X^{1.5}Y$
a) Prove that the slope of indifferent curve is negative and convex to the origin.
b) If her income is 100 Baht, price of X = 3 Baht and price of Y = 4 Baht, what is the equilibrium quantity of these two goods? [X = 20, Y =10]

6. Let the production function and cost function be $Q = 2x + 3y$ and $C = 45x^2 + 90xy + 90y^2$ respectively. (unit: Million Baht)

Determine the minimum cost and the level of x and y such that the producer minimize cost when he want to produce 60 units of goods. Prove your answer by using Bordered Hessian determinent. [x = 12, y = 12 and C = 23,760]

7. A producer has the following production function
$Q = 10K^{\frac{3}{5}}L^{\frac{2}{5}} where P_K = 20 and P_L = 3.$

a) Prove that the isoquant of this production function is convex to the origin.
b) Determine the ratio of labour to capital $(\frac{L}{K})$ at the minimum cost. $[\frac{L}{K} = 10]$
c) Determine Output Expansion Path

8. Prove that to maximize output given the constant input or to minimize input given the constant output will achieve the same efficiently maximum output.

9. The equation for an isoquant is $P_kK + P_LL = E$ where K is capital , L is labor and P_k is a price of capital , P_L is a price of labor and E is expenditure.

a) Derive the isoquant equation in the form of $K = f(L)$ and illustrate the equation by a graph.

b) Find the expenditure (E) when 1) $K = 0$ 2) $L = 0$

c) Derive the isoquant equation in the form of $K = f(L)$ when the expenditure increase to E' and illustrate the equation by a graph.

10. Mr. A has a budget of 120 Baht for buying X and Y in which the price is 3 and 5 Baht respectively.
a) Derive the budget line from the given information.
b) Derive the budget line from each situation.
(1) When Mr. A's budget is reduced by 25%
(2) When the price of X is double.
(3) When the price of Y is reduced to 4 baht.

11. The manager have to build cylindrical oil tank in which the material used to build the tank cost 10 Baht per square foot. And the budget for building the tank is 44,000 Baht. Maximize the size of cylindrical oil tank under the given budget constraint.

12. The manager want to build the villa by the sea such that the whole area equal to 100000 square foot and price of land in front of the sea is 5000 Baht per foot and 2000 Baht for the other sides. How much should he buy for each side so that he minimize cost? What is that minimum cost?