Exercise 7

Integrals: Indefinite Integral/Integration by the u-Substitution/Area/Definite Integral/FTC

1. Evaluate the given indefinite integrals (these may require to use the substitution rule).

   (a) $\int \frac{(x-3)^2}{\sqrt{x}}\ dx$

   (b) $\int \left[e^x + \frac{1}{x} - \sin(x) + 3\sinh(x) + \pi^2 + \csc^2(x) - \sec(x)\tan(x)\right]\ dx$

   (c) $\int \sqrt{2+3x}\ dx$

   (d) $\int \frac{2\cos(\ln(x))}{x}\ dx$

   (e) $\int e^{5x}(1-e^{5x})^{1/3}\ dx$

   (f) $\int \frac{x}{9+4x^2}\ dx$

   (g) $\int \frac{1}{9+4x^2}\ dx$

   (h) $\int \frac{1-x}{\sqrt{3-x^2}}\ dx$

   (i) $\int \left[\sin^3(x)\cos(x) + \frac{\sec^2(x)}{\sqrt{\tan(x)}}\right]\ dx$

   (j) $\int [\tan(\pi x) + \sec(2\pi x)\ ]\, dx$

   (k) $\int \frac{1}{[1+x^2][\tan^{-1}(x)]}\ dx$

   (l) $\int 2x^3\sqrt{1+x^2}\ dx$

   (m) $\int e^{x+e^x}\ dx$

2. Determine the function $f(x)$ such that $f''(x) = (1+x)^4$, $f'(0) = 0$ and $f(0) = 0$.

3. Recall that the area A under the graph of a non-negative function $f(x)$ on a closed interval $[a,b]$ can be obtained from

$$A = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\triangle x_k,$$

   where $x_k^* = a + k\triangle x$ and $\triangle x_k = \triangle x = \frac{b-a}{n}$.

   (i) Use above equation to find the area A when $f(x) = x^2+2$, $a = 0$ and $b = 2$.

   (ii) Evaluate $\int_0^2 x^2 + 2\ dx$ by using *the Fundamental Theorem of Calculus* and compare to (i).

4. (a) Find the area between the function $f(x) = |x-2|$ and the x-axis on the interval $[0,2]$.

   (b) Find the area between the function $f(x) = x-2$ and the x-axis on the interval $[0,2]$.

   (c) Evaluate $\int_0^2 |x-2|\ dx$ and $\int_0^2 x-2\ dx$.

5. Use the Fundamental Theorem of Calculus (derivative form)to find the indicated derivative.

   (a) $\frac{d}{dx}\int_{2}^{x} \left[t^2e^t + \ln(|\sin(t)|)\right]\ dt$

   (b) $\frac{d}{dx}\int_{e^x}^{\pi} \ln(t)\tan(2t)\ dt$

   (c) $\frac{d}{dx}\int_{\ln(x)}^{\sin(x)} \frac{1}{1+t^5}\ dt$

   (d) $\frac{d}{dx}\int_{\pi}^{e^\pi} \ln(t)\ dt$

6. Evaluate the following definite integrals.

   (a) $\int_{1}^{2} \left[3x^2 - 2x + 1 - \frac{1}{x^2} + \frac{1}{3x} + e^x\right]\ dx$

   (b) $\int_{1}^{5} \frac{1}{1+2x}\ dx$

   (c) $\int_{1}^{2} \left[\frac{x^2+1}{x} + \frac{x}{(x^2+1)^2}\right]\ dx$

   (d) $\int_{0}^{\frac{1}{2}} \sin^2(\pi x)\ dx$

   (e) $\int_{0}^{\frac{\pi}{2}} \cos(x) + \cos^5(x)\sin(x)\ dx$

   (f) $\int_{-1}^{1} \left(\frac{x^5+x}{(x^4+4x^2+4)^3} + |x|\right)\ dx$

   (g) $\int_{0}^{\frac{3\pi}{2}} |\cos(x)|\ dx$

   (h) $\int_{\frac{1}{2}}^{e} \frac{[\ln(2x)]^5}{x}\ dx$

   (i) $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{t}{\sqrt{1-t^4}}\ dt$

   (j) $\int_{0}^{1} \frac{e^{3t}}{2-e^{3t}}\ dt$

7. Let $f(x) = \begin{cases} -2, & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$. Evaluate $\int_{-2}^{2} f(x)\ dx$ and $\int_{-2}^{2} |f(x)|\ dx$.