



EE311 | Semester 1/2019 | Practice Questions for Quiz#1
Quiz date: Wednesday 6 November 2019, 13.00-13.30

4. A firm has a linear production function $Q = 10L + 2K$. For this production function $MP_L = 10$ and $MP_K = 2$. The price of labor, w , is £5 per unit and the price of capital services, r , is £2 per unit. Find the optimal input combination given that the firm wishes to produce 200 units of output.
7. A firm uses two inputs, capital and labor, to produce output. Its production function exhibits a diminishing marginal rate of technical substitution.
 - a) If the price of capital and labor services both increase by the same percentage amount (e.g., 20 percent), what will happen to the cost-minimizing input quantities for a given output level?
 - b) If the price of capital increases by 20 percent while the price of labor increases by 10 percent, what will happen to the cost-minimizing input quantities for a given output level?
9. Suppose the production of airframes is characterized by a CES production function: $Q = (L^{1/2} + K^{1/2})^2$. The marginal products for this production function are $MP_L = (L^{1/2} + K^{1/2})L^{-1/2}$ and $MP_K = (L^{1/2} + K^{1/2})K^{-1/2}$. Suppose that the price of labor is \$10 per unit and the price of capital is \$1 per unit. Find the cost-minimizing combination of labor and capital for an airframe manufacturer that wants to produce 121,000 airframes.
20. Consider the production function $Q = K + \sqrt{L}$. For this production function, $MP_L = 1/(2\sqrt{L})$ and $MP_K = 1$. Derive the input demand curves for L and K , as a function of the input prices w (price of labor services) and r (price of capital services). Show that at an interior optimum (with $K > 0$ and $L > 0$) the amount of L demanded does not depend on Q .
22. Suppose that the firm's production function is given by $Q = 10KL^{1/3}$. The firm's capital is fixed at K . What amount of labor will the firm hire to solve its short-run cost-minimization problem?
23. Suppose that the firm uses three inputs to produce its output: capital K , labor L , and materials M . The firm's production function is given by $Q = K^{1/3}L^{1/3}M^{1/3}$. For this production function, the marginal products of capital, labor, and materials are $MP_K = 1/3K^{-2/3}L^{1/3}M^{1/3}$, $MP_L = 1/3K^{1/3}L^{-2/3}M^{1/3}$, and $MP_M = 1/3K^{1/3}L^{1/3}M^{-2/3}$. The prices of capital, labor, and materials are $r = 1$, $w = 1$, and $m = 1$, respectively.
 - a) What is the solution to the firm's long-run cost minimization problem given that the firm wants to produce Q units of output?
 - b) What is the solution to the firm's short-run cost minimization problem when the firm wants to produce Q units of output and capital is fixed at K ?
 - c) When $Q = 4$, the long-run cost-minimizing quantity of capital is 4. If capital is fixed at $K = 4$ in the short run, show that the short-run and long-run cost-minimizing quantities of labor and materials are the same.

1. If the price of labor increases by 20 percent, but all other input prices remain the same, would the long-run total cost at a particular output level go up by more than 20 percent, less than 20 percent, or exactly 20 percent? If the prices of all inputs went up by 20 percent, would long-run total cost go up by more than 20 percent, less than 20 percent, or exactly 20 percent? Make sure to explain the logic behind.

8. A firm produces a product with labor and capital, and its production function is described by $Q = LK$. The marginal products associated with this production function are $MP_L = K$ and $MP_K = L$. Suppose that the price of labor equals 2 and the price of capital equals 1. Derive the equations for the long-run total cost curve.

11. A firm's long-run total cost curve is $TC(Q) = 40Q - 10Q^2 + Q^3$, and its long-run marginal cost curve is $MC(Q) = 40 - 20Q + 3Q^2$. Over what range of output does the production function exhibit economies of scale, and over what range does it exhibit diseconomies of scale?

16. When a firm uses K units of capital and L units of labor, it can produce Q units of output with the production function $Q = K\sqrt{L}$. Each unit of capital costs 20, and each unit of labor costs 25. The level of K is fixed at 5 units.

a) Find the equation of the firm's short-run total cost curve.

18. Consider a production function of three inputs, labor, capital, and materials, given by $Q = LKM$. The marginal products associated with this production function are as follows: $MP_L = KM$, $MP_K = LM$, and $MP_M = LK$. Let $w = 5$, $r = 1$, and $m = 2$, where m is the price per unit of materials.

a) Suppose that the firm is required to produce Q units of output. Show how the cost-minimizing quantity of labor depends on the quantity Q . Show how the cost minimizing quantity of capital depends on the quantity Q . Show how the cost-minimizing quantity of materials depends on the quantity Q .

b) Find the equation of the firm's long-run total cost curve.

19. The production function $Q = KL + M$ has marginal products $MP_K = L$, $MP_L = K$, and $MP_M = 1$. The input prices of K , L , and M are 4, 16, and 1, respectively. The firm is operating in the short run, with K fixed at 20 units. What is the short-run total cost of producing 400 units of output?

22. Suppose that the total cost of providing satellite television services is as follows:

$$TC(Q_1, Q_2) = \begin{cases} 0, & \text{if } Q_1 = 0 \text{ and } Q_2 = 0 \\ 1000 + 2Q_1 + 3Q_2, & \text{otherwise} \end{cases}$$

where Q_1 and Q_2 are the number of households that subscribe to a sports and movie channel, respectively. Does the provision of satellite television services exhibit economies of scope?