Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$$\frac{1}{R_f} = E[m_{01}]$$

$$\therefore R_f = \frac{1}{E[m_{01}]} \; ; \; m_{01} = \frac{\delta U'(C_1)}{U'(C_0)}$$

Given $U(C) = \ln(C)$

$$\therefore U'(C_0) = \frac{1}{C_0}$$

$$U'(C_1) = \frac{1}{C_1}$$

$$\text{so, } m_{01} = \frac{\delta C_0}{C_1}$$

$$\text{Then, } R_f = \frac{C_1}{\delta C_0}$$

when interest rate is high, so is expected growth in consumption

positive relationship between level of consumption and risk free rate in this economy

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$R_f = \frac{C_1}{\delta C_0}$$

$$\text{FOC}: \frac{\partial R_f}{\partial \frac{C_1}{C_0}} = \frac{1}{\delta}$$

$$= \frac{1}{\delta} \frac{R_f}{\frac{C_1}{C_0}} \quad \frac{\cancel{C_1}}{\delta \cancel{C_0}} \cdot \frac{\cancel{C_0}}{\cancel{C_1}}$$

$$\varepsilon \approx \frac{\% \Delta \frac{C_1}{C_0}}{\% \Delta R_f}, \quad \varepsilon \equiv \frac{R_f}{C_1/C_0} \cdot \frac{\partial \frac{C_1}{C_0}}{\partial R_f}$$

For primitive security s, $1 = E[m_{01} R_f]$

$1 = \pi m_{01} R_f$

$$\text{FOC}: \pi \left(\frac{\delta C_0}{C_1}\right) R_f = 1 \quad -(1)$$

Differentiate (1) ; $(\cancel{\pi})(\cancel{\delta} R_f) d\left(\frac{C_1}{C_0}\right) + (\cancel{\pi})\cancel{\delta}\left(\frac{C_1}{C_0}\right) dR_f = 0$

$$R_f \, d\frac{C_1}{C_0} = -\frac{C_1}{C_0} dR_f$$

$$\frac{R_f}{C_1/C_0} \cdot \frac{d\frac{C_1}{C_0}}{dR_f} = -1$$

$$\therefore \varepsilon = -1$$

elasticity of intertemporal substitution is -1.

$\therefore$ $\varepsilon = -1$ implied that it is inelasticity so, the income effect outweights the substitution effect.

If next year, the interest rate falling, individual's consumption level will consume less in both period.

Score....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

Optimal Price of asset $P_i = E[m_{01} X_1]$

$P_1 = E\left[\frac{\delta U'(C_1)}{U'(C_0)} X_i\right]$; $U'(C_0) = \frac{1}{C_0}$, $U'(C_1) = \frac{1}{C_1}$

$P_i = E\left[\frac{\delta C_0}{C_1} X_i\right]$ so, $m_{01} = \frac{\delta C_0}{C_1}$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

CAPM relation

$$E[R_i] = R_f + \beta_i (E[R_m] - R_f) \;;\; m_{01} = \frac{\delta U'(C_1)}{U'(C_0)} \;;\; E[R_i] = R_f - \frac{Cov[m_{01}, R_i]}{E[m_{01}]}$$

SO,
$$\frac{E[R_i] - R_f}{\sigma_{R_i}} = -\rho_{m_{01}, R_i} \frac{\sigma_{m_{01}}}{E[m_{01}]}$$

$$-1 \leq \rho_{m_{01}, R_i} \leq 1 \rightarrow \left| \frac{E[R_i] - R_f}{\sigma_{R_i}} \right| \leq \frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_{m_{01}} R_f \;;\; E[m_{01}] = \frac{1}{R_f}$$

$$m_{01} = \frac{\delta C_0}{C_1} = \delta e^{\ln(C_0/C_1)}$$

$$\therefore \frac{\sigma_{m_{01}}}{E[m_{01}]} = \frac{\sqrt{Var[e^{\ln(C_0/C_1)}]}}{E[e^{\ln(C_0/C_1)}]}$$

$$= \frac{\sqrt{E[e^{2\ln(C_0/C_1)}] - E[e^{\ln(C_0/C_1)}]^2}}{E[e^{\ln(C_0/C_1)}]} \;;\; \frac{\sqrt{\cdot}}{X} = \sqrt{\frac{\cdot}{X^2}}$$

$$= \sqrt{E[e^{-2\ln(C_1/C_0)}] / E[e^{-\ln(C_1/C_0)}]^2 - 1}$$

$$= \sqrt{e^{-(2\mu_c + 2(\sigma_c^2))} / e^{-(2\mu_c + \sigma_c^2)} - 1}$$

$$= \sqrt{e^{-2\sigma_c^2} - 1} \;\#\; \approx \pm 2\sigma_c^2$$

implied that very low $\frac{\sigma_{m_{01}}}{E[m_{01}]}$
= very risk aversion
= required high compensation to invest