

Exercise: Solving Inequality

1 Part I

1. Let a , b , and c be real numbers. Show that if $a \geq b$ and $c < 0$, then $a^2 + b^2 + 5a - 5b - 4c > 0$.
2. Let x, y be real numbers. Suppose that $x < 0$ and $y > 3$. Determine whether each of the following inequalities is true or not. Explain your answer.

(a) $3x - xy > 0$.

(b) $\frac{3}{x} - \frac{y}{x} + x^2 + y^3 > 18$

3. Find the solution set for each of following inequalities.

(a)

$$\frac{2x}{1-x} \geq \frac{1-x}{2x}$$

(b)

$$\frac{16x^4 - 81}{6x^2 + x - 12} < 0$$

(c)

$$\frac{x^4 - 2x^2 - 8}{2x + 1} \geq 0$$

(d)

$$\frac{x^4 + x^3 + 2x^2 + 2x}{x - 1} \leq 0$$

(e)

$$\left(\frac{x}{x-3} - 2\right) \left(\frac{e^x}{\cos(x) + 2}\right) \geq 0$$

4. Let x and y be real numbers with $|x| < \frac{1}{2}$. Show that $|xy - x| < \frac{|y|+1}{2}$.
5. Let x be a real number with $|x| \leq 3$. Determine if the following inequality is true or not.

$$|x^2 - 4| \geq 5|x + 2|$$

Explain your answer.

6. Let x and y be real numbers with $x > 1$ and $y < -2$. Determine if the following inequality is true or not.

$$1 + \frac{|y|}{x} > \frac{4-y}{2|x|}$$

Explain your answer.

7. Find the solution set for each of following inequalities.

(a) $1 < |x - 1| \leq 6$

(b)

$$\frac{(|x| + 5)(x - 1)}{|x - 2| + x^2 + x + 1} \geq 0$$

(c)

$$2x + \frac{1}{|x|} \geq 1$$

(d)

$$\frac{|2 - x|}{x^3 + 3x^2 + 11x + 18} \leq 0$$

(e)

$$\frac{x^2 + 1 - |x - 1|}{5 - |x + 3|} \leq 0$$

2 Part II

1. Let a and b be real numbers with $0 < b < a < 1$. Determine whether each of the following inequalities is true or not. Explain your answer.

(a) $\frac{(b+1)^3}{|a|} < \frac{(a+1)^3}{b}$

(b) $\frac{b}{a} < \frac{a^3-1}{b^3-1}$

2. Let x and y be real numbers with $y > x > 1$. Show that

$$y(y - 2) > |x - 1|^2 - 1.$$

3. Find the solution set for each of following inequalities.

(a)

$$\frac{x^2 - 1}{x} < \frac{x + 1}{2x}$$

(b)

$$2 \leq \left| \frac{x - 1}{x} \right| \leq 7$$

(c)

$$\frac{x^2 + |x| + 1}{x^7 + x^5 + x^2 + 1} \leq 0$$

(d)

$$\frac{|x + 3| - 2}{5} + \frac{1}{|x - 1| + 1} \leq 1$$

(e)

$$\frac{|x - 1| - x^2 - 1}{5 - |x + 3|} \geq 0$$

3 Part III

1. Let a_1, a_2, a_3 , and a_4 be some constant real numbers such that

$$a_1 < a_2 < a_3 < a_4.$$

Determine the solution set for each of the following inequalities in terms of a_1, a_2, a_3 , and a_4 .

- (a) $(x - a_1)(x - a_2)(x - a_3)(x - a_4) \geq 0$
 (b) $(x - a_1)^2(x - a_2)(x - a_3)^2(x - a_4)^3 \geq 0$

2. Let a and b be real numbers with $|b| < |a| < 1$ and $a > 0$.

- (a) Show that $a - 1 < 0 < b + a < 2a < 1 + a$.
 (b) Show that

$$\frac{2a^2 - 2a}{b + a} > \frac{a^2 - 1}{2a}.$$

3. Find the solution set for each of following inequalities.

- (a)

$$\frac{1}{x} < 8$$

- (b)

$$\frac{9 - x}{x - 2} \geq \frac{2 - x}{x - 9}$$

- (c)

$$\frac{|2x + 5| - 2}{3|x| - 1} \leq 1$$

- (d)

$$\frac{|x^2 - x + 1|}{|3x + 1|} > 1$$

- (e)

$$\frac{|x^2 - 2| + 2}{x^2 - 3|x| + 2} < 0$$

4. A math class counts the midterm exam scores as $1/3$ of the grade and the final exam scores as $2/3$ of the grade. Paul scored 48% on the midterm.

To get an A, Paul must have the total score between 90% and 100% inclusive;
 to get a B, Paul must have the total score between 80% and 89% inclusive;
 to get a C, Paul must have the total score between 70% and 79% inclusive.

Suppose that all scores with decimal digits get rounded up to the closest higher integers.

- (a) What range of scores (in %) on the final exam would make Paul get a C?
 (b) What is the highest grade that Paul could get for this class?

5. The weekly demand (the number bought by consumers) for the a product is given by the formula

$$d = 9000 - 60p$$

where p is the price each in dollars.

- (a) What is the demand when the price is 30 each?
- (b) In what price range will the demand be above 6000?
6. A store's revenue $R(x)$ (in Baht) on the sale of x cupcakes is determined by the formula $R(x) = 50x - x^2$. The cost $C(x)$ (in Baht) for producing x cupcakes is given by the formula $C(x) = 2x + 400$. For what values of x is the store's profit positive? Note: profit = revenue - cost.

4 Additional Problems

1. Find the solution set for each of the following inequities.

(a) $|x - 1| < 7$

(b) $3 < |x + 2| < 4$

(c) $|2x + 3| \leq 2$

(d) $|3x - 1| \geq 1$

(e) $\frac{|x+2|}{2} > |x|$

(f) $|x + 1| - |2x - 1| < 0$

(g) $|2 - x| - x \leq 0$

(h) $|x + 1| > 3 - |x|$

(i) $x^2 - 2 \geq \frac{1}{2}|x - 1|$

(j) $|x + 3| \geq 2 - x$

(k) $|2x^2 + x - 1| \geq 2$

(l) $\frac{|x|}{2} + 3 > x^2$

(m) $|x| > \frac{2}{|x+1|}$

(n) $\frac{x^2}{x+1} < |x|$

(o) $\frac{4}{|x|} \leq \frac{1}{|x|}$

(p) $|x^2 + 1| < |x + 1|$

(q) $(x^2 + 1)(|x + 2| - |x|) \geq 0$

(r) $\frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$

2. Find the solution set for each of the following inequities.

(a) $|x - 1| < 7$

(b) $3 < |x + 2| < 4$

(c) $|2x + 3| \leq 2$

(d) $|3x - 1| \geq 1$

(e) $\frac{|x+2|}{2} > |x|$

(f) $|x + 1| - |2x - 1| < 0$

(g) $|2 - x| - x \leq 0$

(h) $|x + 1| > 3 - |x|$

(i) $x^2 - 2 \geq \frac{1}{2}|x - 1|$

(j) $|x + 3| \geq 2 - x$

(k) $|2x^2 + x - 1| \geq 2$

(l) $\frac{|x|}{2} + 3 > x^2$

(m) $|x| > \frac{2}{|x+1|}$

$$(n) \frac{x^2}{x+1} < |x|$$

$$(o) \frac{4}{|x|} \leq \frac{1}{|x|}$$

$$(p) |x^2 + 1| < |x + 1|$$

$$(q) (x^2 + 1)(|x + 2| - |x|) \geq 0$$

$$(r) \frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$$