FN241: Session 7
Introduction to Value at Risk

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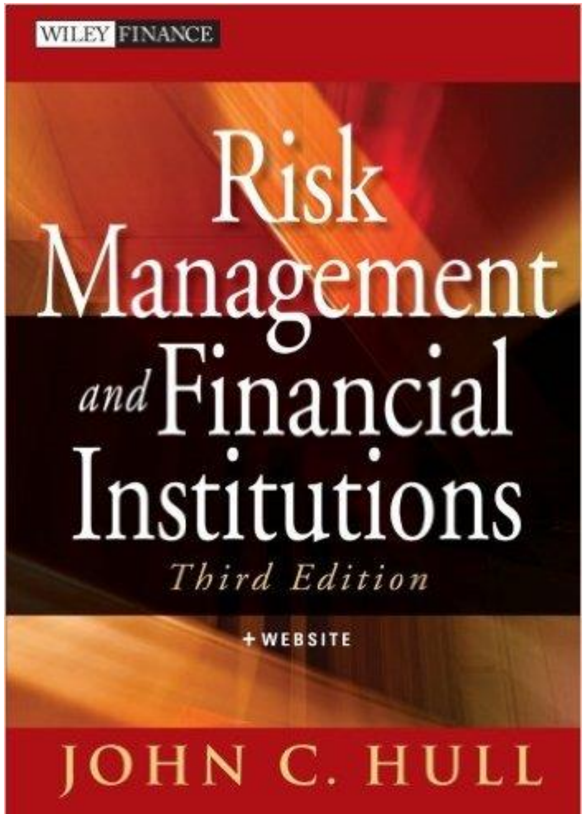
Outline

- Financial Risk
- Introduction to Value at Risk
- Example:

  Market Risk VaR: Model Building Approach

Reading

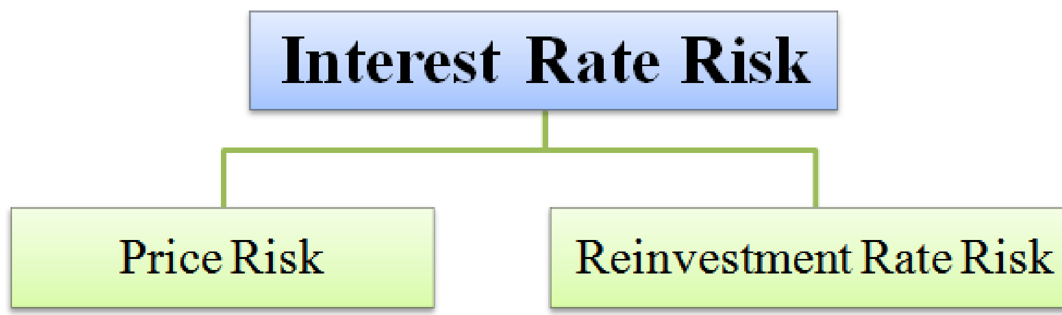


- Chapter 9 Value at Risk
- Chapter 15 Market Risk VaR:
  The Model-Building Approach

Financial Risk

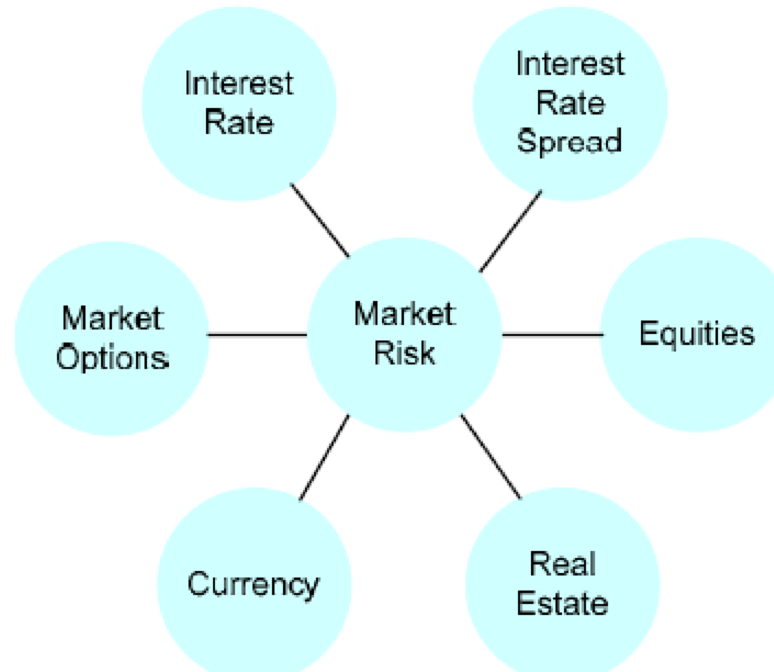
Major Risks of Financial Intermediation

- <u>Interest rate risk</u>
  - Mismatch in maturities of assets and liabilities.
  - Balance sheet hedge via matching maturities of assets and liabilities is problematic for FIs.
  - Refinancing risk.
  - Reinvestment risk.



Major Risks of Financial Intermediation

- <u>Market risk</u>
  - Incurred in trading of assets and liabilities (and derivatives).
  - Examples: Barings & decline in ruble.
  - Trend to greater reliance on trading income rather than traditional activities increases market exposure.



Major Risks of Financial Intermediation

- <u>Operational and technology risk</u>
  - Risk that technology investment fails to produce anticipated cost savings.
  - Risk that technology may break down.
  - Economies of scale.
  - Economies of scope.

Introduction to Value at Risk

The Question Being Asked in VaR

"What loss level is such that we are X% confident it will not be exceeded in N business days?"

Application: VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0
- Under Basel II, capital for credit risk and operational risk is based on a one-year 99.9% VaR

Advantages of VaR

- It captures an important aspect of risk in a single number
- It is easy to understand
- It asks the simple question: “How bad can things get?”

VaR Example Calculation

Example 1

- The gain from a portfolio during six month is normally distributed with mean \$2 million and standard deviation \$10 million
- The 1% point of the distribution of gains is 2−2.33×10 or − \$21.3 million
- The VaR for the portfolio with a six month time horizon and a 99% confidence level is \$21.3 million.

$$\text{VaR for the portfolio} = \mu - Z_{\alpha} \cdot \sigma$$

VaR Example Calculation

Example 2

- All outcomes between a loss of $50 million and a gain of $50 million are equally likely for a one-year project

What is VaR for a one-year time horizon at a 99% confidence level?

VaR Example Calculation

Examples 3 and 4

- A one-year project has:

  a 98% chance of leading to a gain of $2 million,

  a 1.5% chance of a loss of $4 million, and

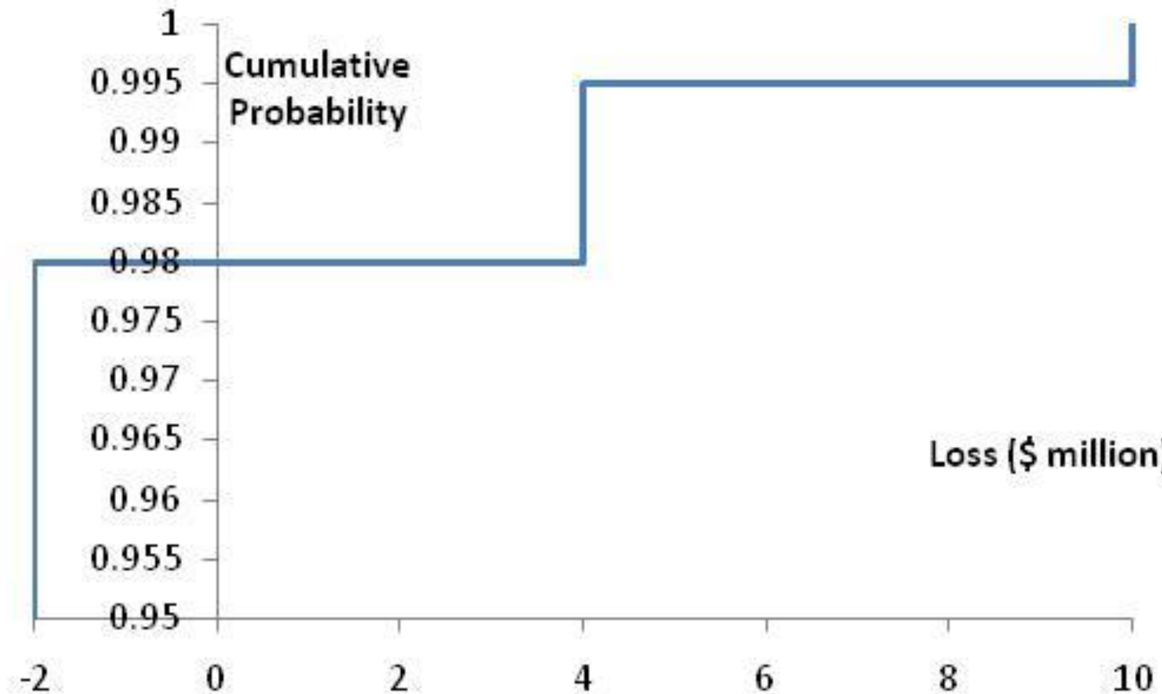
  a 0.5% chance of a loss of $10 million

- The VaR with a 99% confidence level is $4 million
- What if the confidence level is 99.9%?
- What if it is 99.5%?

VaR Example Calculation

Examples 3 and 4

- What if the confidence level is 99.9%?
- What if it is 99.5%?



Coherent Risk Measures

- Define a coherent risk measure as the amount of cash that has to be added to a portfolio to make its risk acceptable
- Properties of coherent risk measure
    1. If one portfolio always produces a worse outcome than another its risk measure should be greater
    2. If we add an amount of cash K to a portfolio its risk measure should go down by K
    3. Changing the size of a portfolio by λ should result in the risk measure being multiplied by λ
    4. The risk measures for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged

VaR vs. Expected Shortfall

- VaR is the loss level that will not be exceeded with a specified probability
- Expected shortfall is the expected loss given that the loss is greater than the VaR level (also called C-VaR and Tail Loss)
- Two portfolios with the same VaR can have very different expected shortfalls
- How do the curves look like?

VaR vs Expected Shortfall

- VaR satisfies the first three conditions but not the fourth one
- Expected shortfall satisfies all four conditions.

Example:

- Each of two independent projects has a probability 0.98 of a loss of $1 million and 0.02 probability of a loss of $10 million
- What is the 97.5% VaR for each project?
- What is the 97.5% expected shortfall for each project?
- What is the 97.5% VaR for the portfolio?
- What is the 97.5% expected shortfall for the portfolio?

Example:

Market Risk VaR:

The Model-Building Approach

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the **probability distributions** of the returns on the market variables
- This is known as **THE MODEL BUILDING APPROACH** (or sometimes the **variance-covariance approach**)

Example: Microsoft

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is **2% per day** (about 32% per year)
- We use N=10 and X=99
- The standard deviation of the change in the portfolio in **1 day** is $200,000
- The standard deviation of the change in **10 days** is $200{,}000\sqrt{10} = \$632{,}456$
- We assume that **the expected change in the value** of the portfolio is zero (This is OK for short time periods). We assume that the change in the value of the portfolio is **normally distributed.**
- Since N(–2.33)=0.01, the VaR is $2.33 \times 632{,}456 = \$1{,}473{,}621$

Example: AT&T

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The SD per 10 days is $50{,}000\sqrt{10} = \$158{,}144$
- The VaR is $158{,}114 \times 2.33 = \$368{,}405$

Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3
- A standard result in statistics states that:

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case σ_X = 200,000 and σ_Y = 50,000 and ρ = 0.3. The standard deviation of the change in **the portfolio value** in one day is therefore 220,227.

VaR for Portfolio

- The 10-day 99% VaR for the portfolio is:

$$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$

- The benefits of diversification are:

$$(1{,}473{,}621 + 368{,}405) - 1{,}622{,}657 = \$219{,}369$$

- What is the incremental effect of the AT&T holding on VaR?

The Linear Model

We assume

- The **daily change** in the value of a portfolio is **linearly related to the daily returns** from market variables
- The returns from the market variables are **normally distributed**

Markowitz Result for Variance of Return on Portfolio

$$\text{Variance of Portfolio Return } = \sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}w_i w_j \sigma_i \sigma_j$$

w_i is weight of ith asset in portfolio

σ_i^2 is variance of return on ith asset in portfolio

ρ_{ij} is correlation between returns of ith and jth assets

Corresponding Result for Variance of Portfolio Value

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

σ_i is the daily volatility of the *i*th asset (i.e., SD of daily returns)

σ_P is the SD of the change in the portfolio value per day

$\alpha_i = w_i P$ is amount invested in *i*th asset

Covariance Matrix ($\text{var}_i = \text{cov}_{ii}$)

$$C = \begin{pmatrix} \text{var}_1 & \text{cov}_{12} & \text{cov}_{13} & \cdots & \text{cov}_{1n} \\ \text{cov}_{21} & \text{var}_2 & \text{cov}_{23} & \cdots & \text{cov}_{2n} \\ \text{cov}_{31} & \text{cov}_{32} & \text{var}_3 & \ldots & \text{cov}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{cov}_{n1} & \text{cov}_{n2} & \text{cov}_{n3} & \ldots & \text{var}_n \end{pmatrix}$$

Alternative Expressions for σ_P^2

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}_{ij}\, \alpha_i \alpha_j$$

$$\sigma_P^2 = \boldsymbol{\alpha}^{\mathbf{T}} C \boldsymbol{\alpha}$$

where $\boldsymbol{\alpha}$ is the column vector whose ith element is α_i and $\boldsymbol{\alpha}^{\mathbf{T}}$ is its transpose

Question?